PREDICTIVE DECONVOLUTION BY FREQUENCY DOMAIN WIENER FILTERING

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(Received March 15, 2009; accepted June 30, 2009)

ABSTRACT

Broadhead, M.K., Liner, C.L., Ulrych, T.J. and Treitel, S., 2009. Predictive deconvolution by frequency domain Wiener filtering. *Journal of Seismic Exploration*, 18: 347-356.

Predictive deconvolution operator design and application is normally accomplished in the time domain. We study the problem of implementing this algorithm in the frequency domain, the key to which is an alternative formulation of prediction filtering than is normally presented. We find that a significant speed-up is possible, but only for longer than normal operator lengths. However, we give evidence that such operator lengths can improve multiple attenuation. We also discuss some other possible advantages that are still under investigation.

KEYWORDS: deconvolution, linear prediction, multiples, FFT, frequency domain, prediction filter.

INTRODUCTION

Predictive deconvolution has long been a routine step in the processing of seismic reflection data, where improvement in resolution can be accomplished,

0963-0651/09/$5.00

along with reduction of multiples (Robinson and Treitel, 2000). Traditionally, the method has been implemented in the time domain, however, an implementation in the frequency domain is possible. This paper discusses one approach to this and why it might be beneficial. The key idea is to exploit an alternate but equivalent formulation of the problem. Robinson (1967) first demonstrated this alternate point of view by designing the prediction error filter in the special case of unit prediction (spiking decon), where he showed that the spiking deconvolution filter was equivalent to an inverse filter for the effective assumed wavelet, which is minimum phase, and has an autocorrelation function equivalent to the trace autocorrelation. The latter is true under the assumption of white reflectivity.

Peacock and Treitel (1969) partially extended this alternate view to the case for prediction distance greater than the sample rate by noting that predictive deconvolution is equivalent to designing a Wiener least-squares shaping filter that converts this minimum phase estimated wavelet to a desired output with length of the prediction distance, without identifying the desired output. This was clarified by Ulrych and Matsuoka (1991), who showed that this desired output was precisely the estimated minimum phase wavelet truncated to a length of the prediction distance. Using different terminology, this was also discussed by Robinson (1998). In an expanded abstract, Ulrych et al. (1988) exploited this alternate formulation to arrive at a frequency domain design of the prediction filter, which was validated on synthetic data. This point-of-view on predictive deconvolution does not appear to be widely known.

In this paper we recapitulate this alternate point of view, and extend it slightly so as to directly obtain the prediction-error filter. We then apply the method to synthetic and real data in order to demonstrate that it works and to highlight some potential advantages over the time domain, which include: faster design of long operators, which in turn can give improved multiple reduction; ease of incorporating additional filtering steps into the operator design such as signal-to-noise ratio (SNR) enhancement; and relaxation of the desired output from only being a truncated version on the estimated input wavelet.

PREDICTIVE DECON THEORY

Here we present a brief outline of predictive deconvolution theory. For a detailed explanation, see Robinson and Treitel (2000). We consider an N point seismic trace $\mathbf{x}$ defined by

$$\mathbf{x} = \mathbf{w} * \mathbf{u} \quad , \tag{1}$$

where **w** is the wavelet and **u** is the reflection coefficient series, both assumed unknown. A prediction filter, **f**, for prediction distance α, returns a predicted value of the trace $x_{j+\alpha}$ at time index $j+\alpha$ using M values at times j and earlier, or

$$\mathbf{Xf} \approx \mathbf{x}_\alpha \ , \tag{2}$$

where **X** is the convolutional matrix form for the trace **x**, and $\mathbf{x}_\alpha$ is **x** time shifted forward by α points. The normal equations for the least-squares design of **f** are given by

$$\mathbf{X}^T\mathbf{Xf} = \mathbf{X}^T\mathbf{x}_\alpha \ , \tag{3}$$

which is equivalent to

$$\mathbf{Rf} = \mathbf{r}_\alpha \ , \tag{4}$$

or

$$\begin{pmatrix} r_0 & r_1 & \cdots & r_{M-1} \\ r_1 & r_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_1 \\ r_{M-1} & \cdots & r_1 & r_0 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_M \end{pmatrix} = \begin{pmatrix} r_\alpha \\ r_{\alpha+1} \\ \vdots \\ r_{\alpha+M-1} \end{pmatrix} , \tag{5}$$

where r_i is the i-th lag of the seismic trace autocorrelation function. This filter is convolved with the input seismic trace, the result of which is then time shifted and differenced with the input trace in order to obtain an estimate of the prediction errors. Alternatively, the prediction error filter can be designed and applied to the input trace,

$$\mathbf{f}_{pe} = 1,0,\ldots,0,-f_1,\ldots,-f_M \ , \tag{6}$$

with $\alpha-1$ zeros. Note, when $\alpha = 1$, the process is called spiking deconvolution. M is the filter (sometimes called operator) length.

FREQUENCY DOMAIN FORMULATION

We can re-state the predictive deconvolution problem as: 1) Assuming the reflection coefficient series is white, estimate the wavelet $\mathbf{w}$ from the trace autocorrelation function by minimum phase spectral factorization. This gives a minimum phase wavelet, which we will call $\mathbf{w}_m$. 2) Obtain the prediction filter by designing a least-squares shaping filter that converts $\mathbf{w}_m$ to the α point forward shifted version of $\mathbf{w}_m$ that we label $\mathbf{w}_{m_\alpha}$. Alternatively, obtain the prediction-error filter by shaping $\mathbf{w}_m$ to a truncated (at the prediction distance α) version of $\mathbf{w}_m$ that we call $\mathbf{w}_{m_T}$.

Eqs. (4) and (6) implement this algorithm (implicitly) by using one-sided filters and autocorrelation functions. However, this formulation does not translate well into the frequency domain (e.g., eq. (4) is not a convolution). However, Robinson (1980) provides a two-sided infinite lag formulation of a Wiener filter, which is what we need. Let our estimated minimum phase wavelet $\mathbf{w}_m$ be the input to the Wiener filter, and either $\mathbf{w}_{m_\alpha}$ or $\mathbf{w}_{m_T}$ be the desired output, which we will call $\mathbf{w}_{m_d}$ for now. Then, the least-squares filter $\mathbf{f}$ can be obtained as the solution to the linear system of equations

$$\sum_{s=-\infty}^{\infty} f_s \phi_{w_m w_m}(\tau - s) = \phi_{w_m w_{m_d}}(\tau) \quad , \tag{7}$$

for all integers τ, where $\phi_{w_m w_m}$ is the two-sided autocorrelation of $\mathbf{w}_m$ and $\phi_{w_m w_{m_d}}$ is the two-sided cross correlation of $\mathbf{w}_m$ and $\mathbf{w}_{m_d}$. This convolution easily translates into the frequency domain as

$$F(\omega) = \bar{W}_m(\omega) W_{m_d}(\omega) / \| W_m(\omega) \|^2 \quad , \tag{8}$$

where capitals indicate the Fourier transform. This is equivalent to

$$F(\omega) = W_{m_d}(\omega) / W_m(\omega) \quad . \tag{9}$$

In other words, the filter replaces $\mathbf{w}_m$ with $\mathbf{w}_{m_d}$. Then, for example, we could implement the prediction-error filter with

$$F_{pe}(\omega) = \bar{W}_m(\omega) W_{m_T}(\omega) / [\| W_m(\omega) \|^2 + \epsilon] \quad , \tag{10}$$

where ϵ is a small number for filter stability. To compute $W_m(\omega)$ directly in the frequency domain, we use the Kolmogorov method of spectral factorization (Robinson, 1967; Claerbout, 1976).

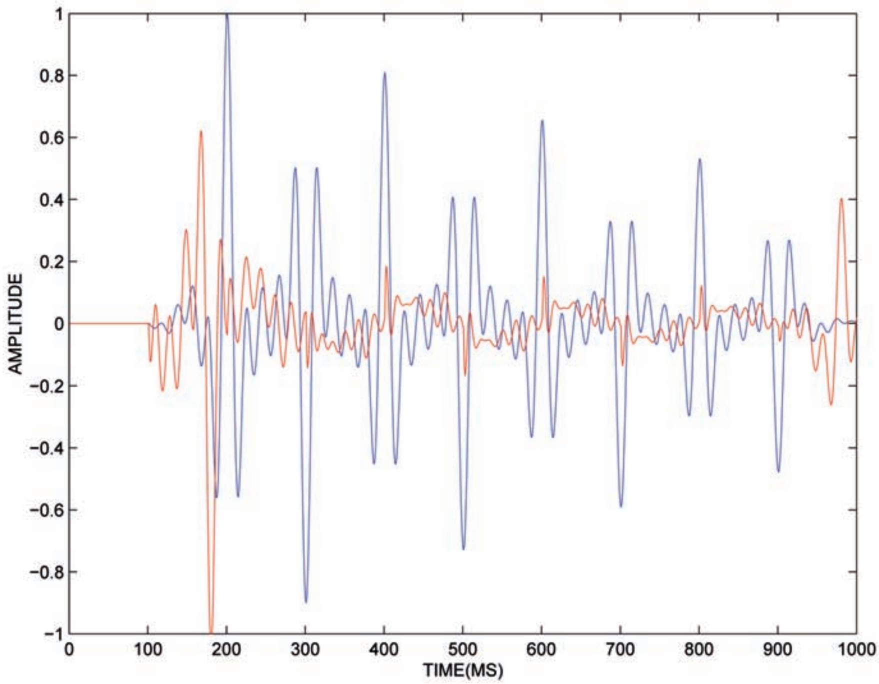

Fig. 1. Time domain spiking deconvolution on 100 ms period multiples with Klauder wavelet. Operator length = 150 ms.

SYNTHETIC DATA EXAMPLES

In Fig. 1, we display synthetic data to simulate multiples with a vibroseis Klauder source wavelet (blue), overlaid with the results of applying traditional deconvolution in the time domain (red), using a typical operator length of 150 ms. Note the poor performance, which is to be expected due to violation of the minimum phase assumption.

Two simple parameter changes drastically improve performance: increasing the prediction distance, which relaxes the minimum phase assumption on the Klauder wavelet (source signature) part of the wavelet, and increasing the operator length, which provides the algorithm more of the autocorrelation function to work with and hence, making a better estimate of the multiples for removal. We demonstrate this in Fig. 2, where the data example is the same as in Fig. 1, but the prediction distance is now 50 ms and the operator length is now 850 ms. In Fig. 3 we show the same case as Fig. 2, but the frequency domain algorithm is used, verifying that it produces essentially the same result. If we want longer operators than normal, however, the frequency domain will allow us to compute them faster.

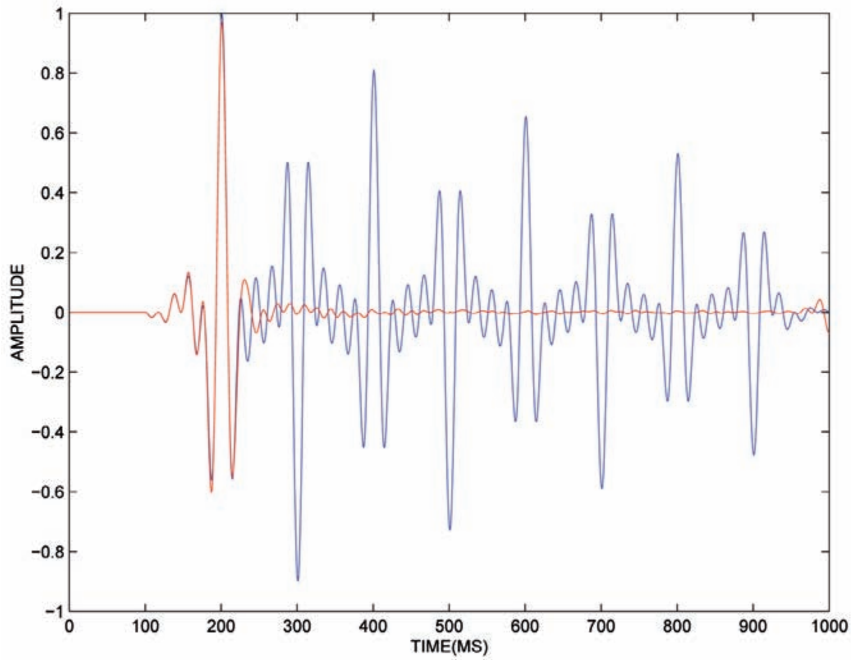

Fig. 2. Time domain predictive decon for data in Fig. 1, but with prediction distance = 50 ms and operator length = 850 ms.

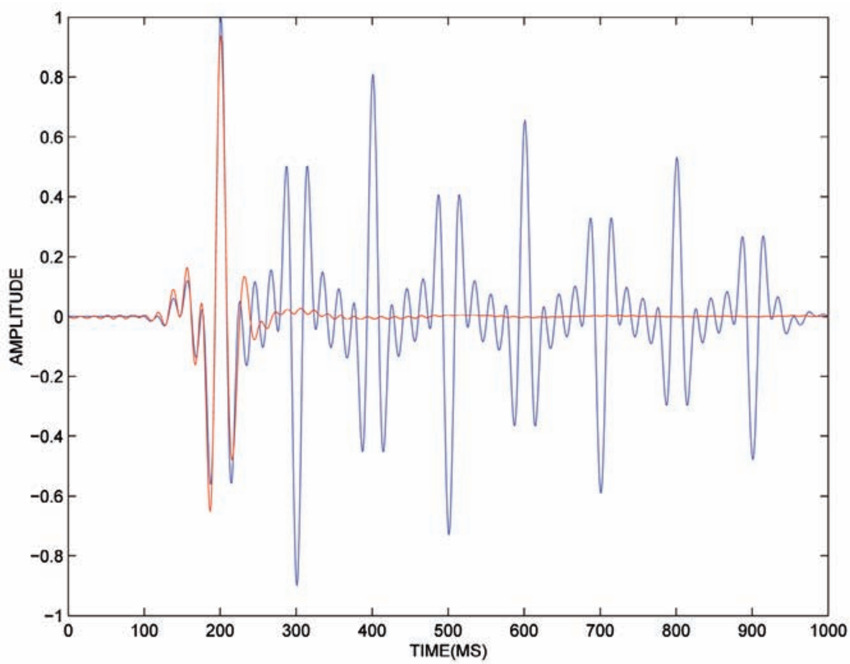

Fig. 3. Same as Fig. 2, but with the frequency domain algorithm.

MEASURED DATA EXAMPLES

We now consider land seismic vibrator data that suffers from a significant multiple problem. All of our calculations are done post-stack. First, we display the data in Fig. 4a. In Fig. 4b we show the result of applying predictive deconvolution with a 50 ms prediction distance and a 5500 ms operator length (designed and applied in the frequency domain). Results of the same operator designed in the time domain were very similar (depending on exact implementation details of the two algorithms, and parameter settings). The frequency domain method ran about 11.5 times faster.

In Fig. 5a, we show results of the predictive deconvolution with a 50 ms prediction distance and a 250 ms operator length. In Fig. 5b, the long operator frequency domain decon result of Fig. 4b is displayed again, for comparison.

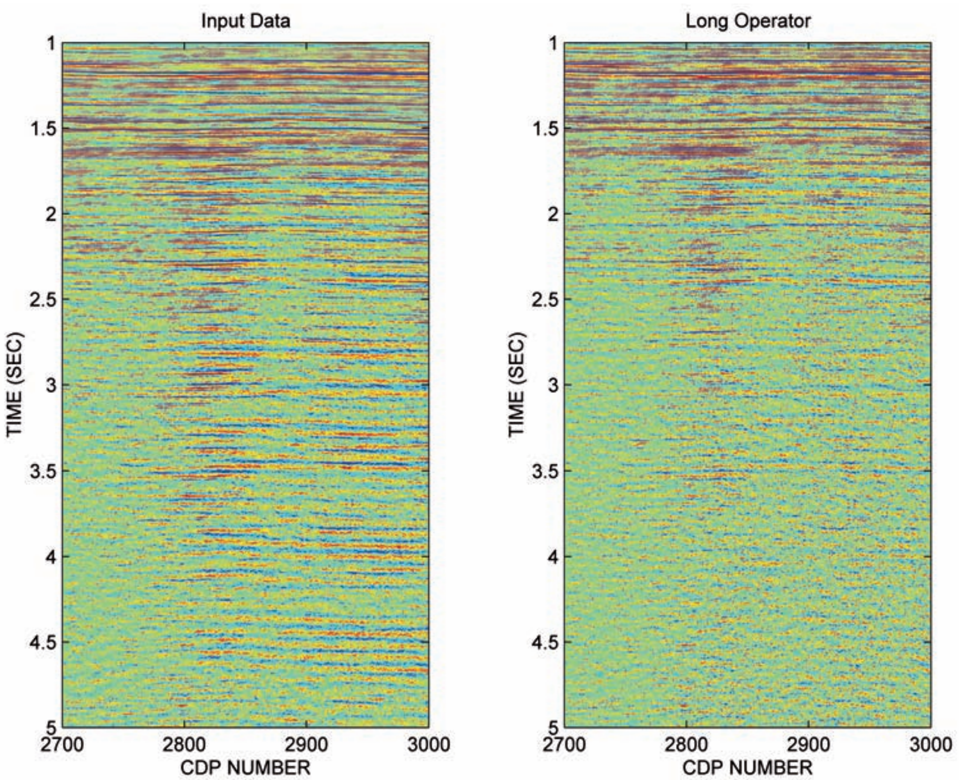

Fig. 4. a) Left. Multiple contaminated land seismic data. b) Right: Frequency domain post-stack predictive deconvolution with 5500 ms operator length. Ran 11 times faster than time domain.

DISCUSSION

We now have a somewhat generalized view of predictive deconvolution which consists of several distinct components. First, wavelet estimation, then choosing a desired output, and finally, designing a shaping filter to replace the estimated wavelet with the desired wavelet. This viewpoint gives us more control and options about how we accomplish predictive deconvolution.

We should say a word about minimum phase estimation for the wavelet - why not something else? One feature that makes predictive deconvolution distinct from other deconvolution approaches is that we are implicitly including an estimate of the multiples in our wavelet - or an earth reverberatory response. This response is minimum phase. If the source is zero phase, then we have a mixed phase wavelet. However, the multiples are still captured to a degree by making the minimum phase assumption. This is what, in our opinion, gives predictive deconvolution its famous robustness. For non-minimum phase wavelets such as Klauder, larger prediction distances seem adequate. Resolution enhancement can be sought using other methods.

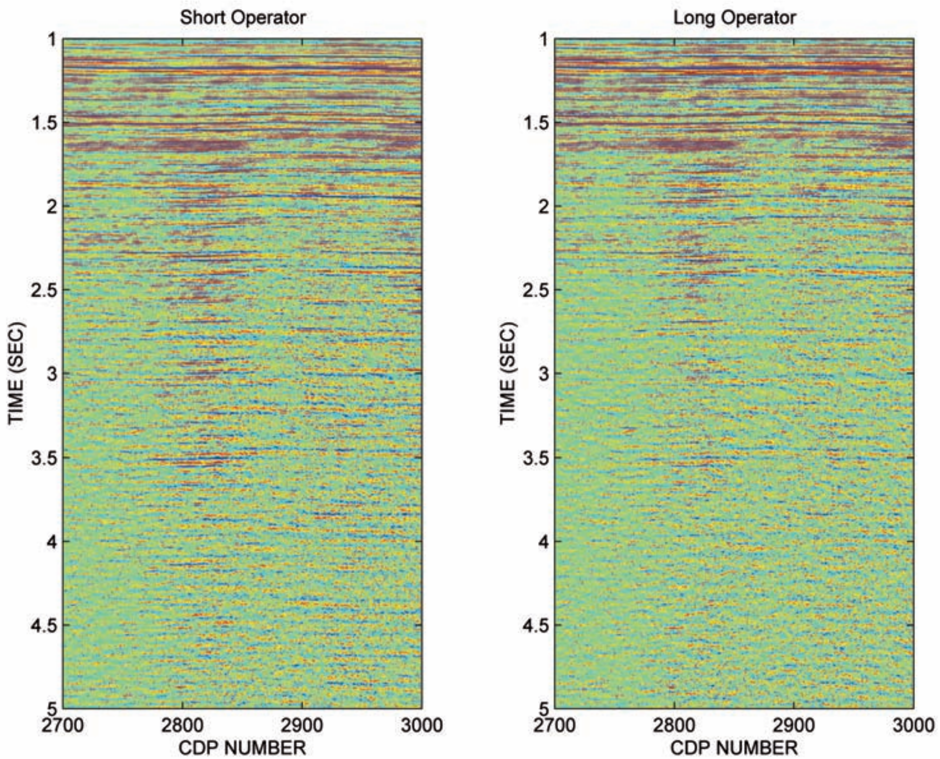

Fig. 5. a) Right. Poststack predictive decon with 250 ms operator length. b) Left. Compare with long (5500 ms) operator length.

The Wiener filter view of predictive deconvolution states that the desired output is a truncated version of the estimated minimum phase wavelet. In principle, however, we could allow the desired output to be something else (example, apply a Hamming taper rather than truncate). We are currently investigating this idea. Also, since we are working in the frequency domain, we recognize that each frequency is treated independently. Thus, if there are additional filtering operations we wish to perform, they can be incorporated into the frequency domain operator design algorithm - such as enhancement of SNR. If we let

$$\mathbf{x} = \mathbf{s} + \mathbf{n} \quad , \tag{11}$$

where **s** is signal and **n** is noise (**s** uncorrelated with **n**), and if we seek a Wiener filter to apply to the trace **x** that recovers **s** optimally, in a least-squares sense, we arrive at (Robinson, 1980)

$$F_{SNR}(\omega) = \Phi_s(\omega)/[\Phi_s(\omega) + \Phi_n(\omega)] \quad , \tag{12}$$

where Φ_s and Φ_n are the signal and noise power spectra. Applying the same approach to the predictive deconvolution problem leads to

$$F_{pe}(\omega) = \bar{W}_m(\omega)W_{m_T}(\omega)/[\Phi_s(\omega) + \Phi_n(\omega)] \quad , \tag{13}$$

where $\bar{W}_m$ is now the frequency domain spectral factorization of Φ_s. For information on one method for estimating the signal and noise autocorrelation functions (or, equivalently, power spectra), see Dash and Obaidullah (1970).

CONCLUSIONS

We have presented a alternative formulation of predictive deconvolution from a wavelet perspective. This alternate view is not entirely new, but not well known, and never fully explicate in one place. It then became straightforward to translate these results to the frequency domain. Several advantages of doing this are given, where the primary one would be speed-up of operator design (comparing an $N_{fft} \log_2 N_{fft}$ operation count to an order of M^2 count for Levinson recursion). However, this advantage is only appreciable for operator lengths that are longer than normally used.

We then demonstrated with real and synthetic data that improved multiple reduction might be expected with longer operators. Two other potential advantages for going to the frequency domain include: ease of obtaining the optimal filter in the presence of colored noise, and the possibility of other desired outputs than just a truncated version of the input. We also noted improved performance for vibrator data by using a longer prediction distance than typical (this help to avoid phase distortion).

ACKNOWLEDGMENTS

We would like to thank Yi Luo and Panos Kelamis of Saudi Aramco for helpful discussions, test data and review of this manuscript. We also benefited from discussions with Jerry Schuster of the University of Utah while he was on sabbatical at Saudi Aramco. Note, C. Liner performed this work while employed by Saudi Aramco.

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