# ACCELERATING WAVEFIELD EXTRAPOLATION ISOCHRON-RAY MIGRATION

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(Received April 19, 2008; revised version accepted August 20, 2008)

ABSTRACT

Silva, E.F.F. and Sava, P., 2009. Accelerating wavefield extrapolation isochron-ray migration. *Journal of Seismic Exploration*, 18: 21-41.

Wavefield extrapolation isochron-ray migration (WEIM) is a wave-equation depth migration method for common-offset sections. The method uses the concepts of isochron-rays and equivalent-velocity media to extend the application of the exploding-reflector model to finite-offset imaging. WEIM is implemented as a trace-by-trace algorithm in three steps: 1) equivalent-velocity computation, 2) data-conditioning for zero-offset migration, and 3) wavefield-extrapolation migration. WEIM is attractive for migration velocity analysis (MVA) because it permits the migration of common-offset sections using standard zero-offset wave-equation algorithms, which, in turn, is favorable for development of algorithms for parallel processing. The problem, however, is that WEIM is computationally expensive. The objective here is to present strategies to improve the computational performance of WEIM, perhaps making the method feasible for MVA. The reduction in computation of WEIM is achieved by redatuming and migration of beams rather than single traces, and by reducing the migration aperture. The procedures of beam formation and redatuming differ from the conventional processes described in the literature and are simultaneously applied in the data-conditioning step. The use of beams and redatuming improves the computational performance of WEIM particularly for larger offsets. In a successful application of these strategies to a field-data example, the effective cost reduction obtained is about seven times.

KEY WORDS: depth migration, isochron ray, isochron surface, equivalent velocity, exploding reflector model, wave equation, wavefield extrapolation, beam migration, redatuming, commom offset.

0963-0651/09/$5.00

## INTRODUCTION

Isochron rays are lines associated with propagating isochrons, which are surfaces related to seismic reflections all having the same two-way traveltime. Iversen (2004) introduced the term *isochron ray* for trajectories associated with surfaces of equal two-way time, i.e., isochron surfaces and suggested the potential use of the isochron rays in future implementations of prestack depth migration.

Silva and Sava (2008a) exploit this idea in a methodology that uses the isochron ray concept to perform prestack depth migration. This methodology uses the additional concept of *equivalent velocity* to extend the use of zero-offset algorithms to finite offset. The method is implemented in a trace-by-trace algorithm in three steps: equivalent-velocity computation, data conditioning, and migration.

Wavefield extrapolation isochron-ray migration (WEIM) is attractive for several reasons, among them we can list:

- It extends the use of zero-offset algorithms;
- It permits migrating a single finite-offset trace using a 3D wave-equation algorithm, which is a favorable feature for the development of parallel processing algorithms; and
- It permits the migration of small pieces of common-offset gathers using the wave equation, which is attractive for migration-velocity analysis.

In this article, we are focused on reducing the computational cost of common-offset isochron-ray migration by wavefield extrapolation, using the acronym WEIM to refer to this common-offset approach exclusively. The cost of WEIM is reduced by adopting the following procedures: migrating beams instead of single traces, performing the redatuming during the data-conditioning step, and using limited aperture.

Liu et al. (2007) combine redatuming and beams for efficient target oriented migration velocity analysis. The theory for wavefield redatuming is presented by Berryhill (1979). Sun et al. (2000) present a strategy to migrate groups of traces using Kirchhoff approach and Wu et al. (2003) introduce procedures of beam-source synthesis for wave-equation migration.

Mulder (2005) defines redatuming as an operation on seismic data that in effect translates the positions of sources or receivers, or both. Here, we perform this operation by applying additional static time-shifts to the input trace during the data-conditioning step. For each position where the input trace is repeated, the time-shift corresponds to the difference of two-way traveltimes for this position and the initial position of the corresponding isochron rays. For homogeneous media, time-shifts are analytically calculated.

Although we apply Gaussian tapering to form beams, the concept of beams for WEIM is essentially different from that of Gaussian beams used by Hill (1990) for Gaussian-beam migration. In WEIM, each input trace is migrated with an equivalent velocity that depends on the source and receiver position. The beam formation for WEIM is a data operation that transforms the data to be migrated with a translated equivalent velocity field, i.e., prepares each trace of the beam to be migrated with the equivalent velocity of the central position of the beam.

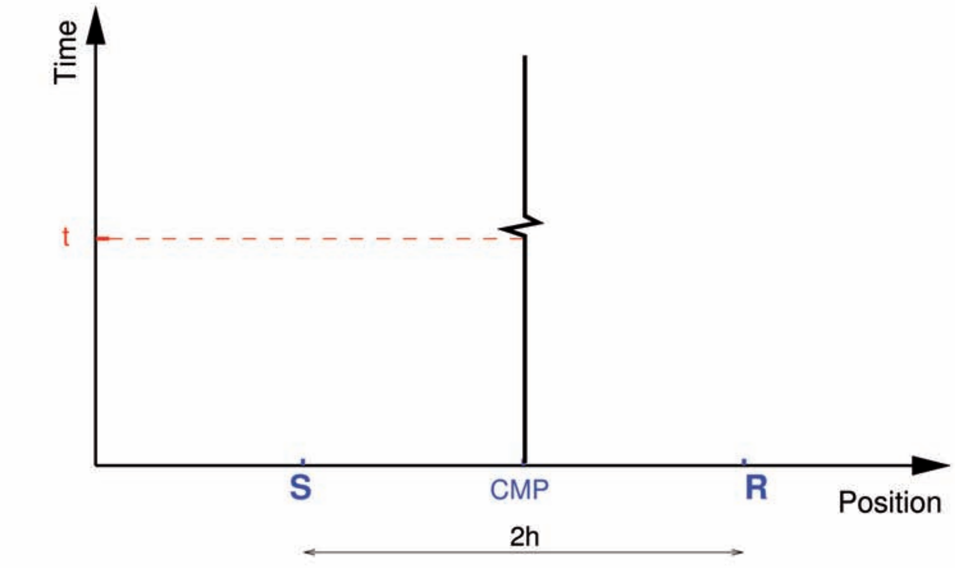
## THEORY

In this section, we present the basic concepts used to improve the performance of wavefield extrapolation isochron-ray migration method. First, we review the method, then we introduce the concept of redatuming for isochron extrapolation, and explain how to form beams for common-offset isochron-ray migration.

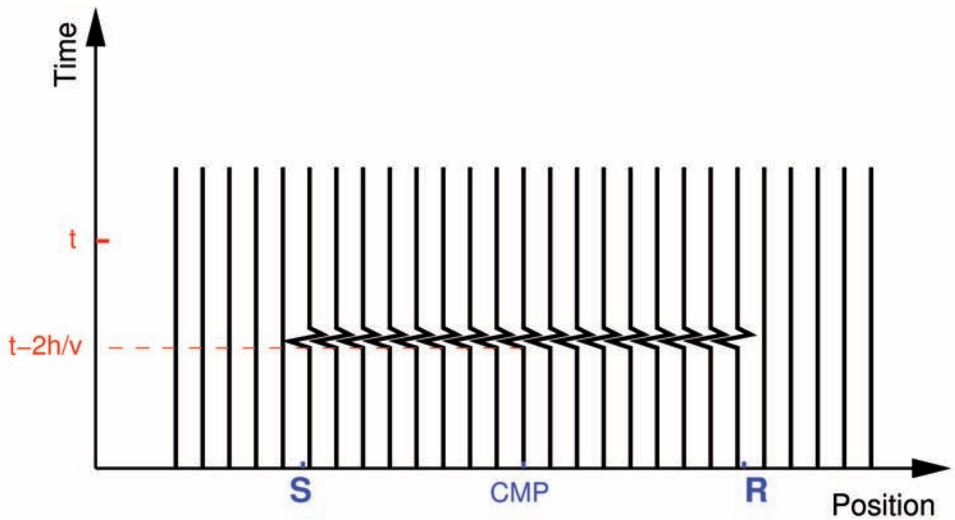
### Wavefield extrapolation isochron-ray migration

This WEIM method (Silva and Sava, 2008b) is implemented in a trace-by-trace algorithm in three steps: equivalent velocity computation, data conditioning, and migration using a zero-offset wavefield extrapolation (ZOWE) algorithm. Here, we review the concept of equivalent velocity and focus on the data-conditioning step, which is closely related to the adopted accelerating strategies.

Equivalent velocity is the speed of a hypothetical propagating isochron that "moves" into the model as reflection time increases. This speed, which is related to medium velocity, is variable even for homogeneous media. In the absence of caustics, the isochron-field can be reproduced by a hypothetical experiment in which the actual source-receiver pair is replaced by a *pseudo seismic source*, which has a constant amplitude along a line connecting the source and receiver, and the propagation medium has the equivalent velocity. As the source segment shrinks to a point, the zero-offset case, the speed of propagation becomes unique and equal to the half-velocity.




(a)




(b)

Fig. 1. Data-conditioning illustration: a) input trace, b) conditioned data for zero-offset migration.

Data conditioning is the procedure by which a single finite-offset trace is transformed into a gather of traces for zero-offset migration using the equivalent velocity field. For wavefield-extrapolation migration, the input trace is time-shifted by a negative amount that corresponds to the traveltime measured along the raypath connecting the source and receiver. The conditioned data gather is obtained by repeating the shifted trace at every grid position between the source and receiver, while the remaining positions are filled with zeros. Figs. 1(a) and 1(b) show a 2D example of data conditioning. In this example, the seismic model is homogeneous so that the applied time-shift is $2h/v$, where $h$ is the half-offset and $v$ is the medium velocity.

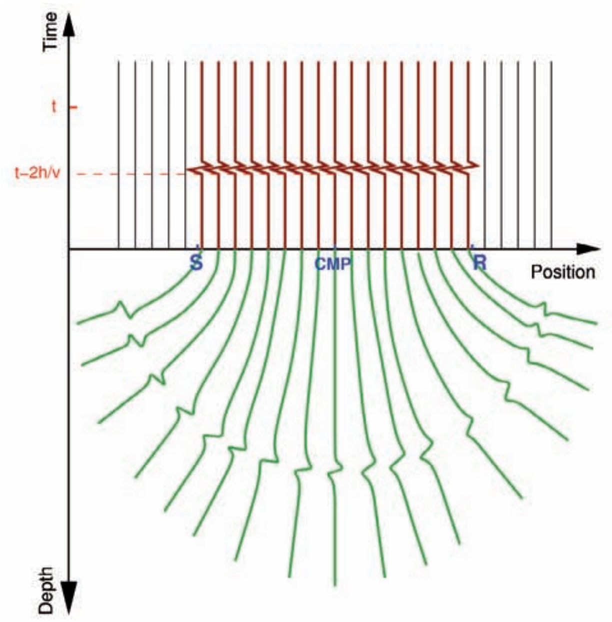
In WEIM, each non-null trace of the conditioned data is associated with the isochron ray that starts in the position where the trace is located. During the migration, the field is extrapolated along the isochron rays. Fig. 2(a) illustrates the isochron-ray migration process. Observe that horizontal lines in conditioned data are mapped into isochrons in depth.

## Redatuming

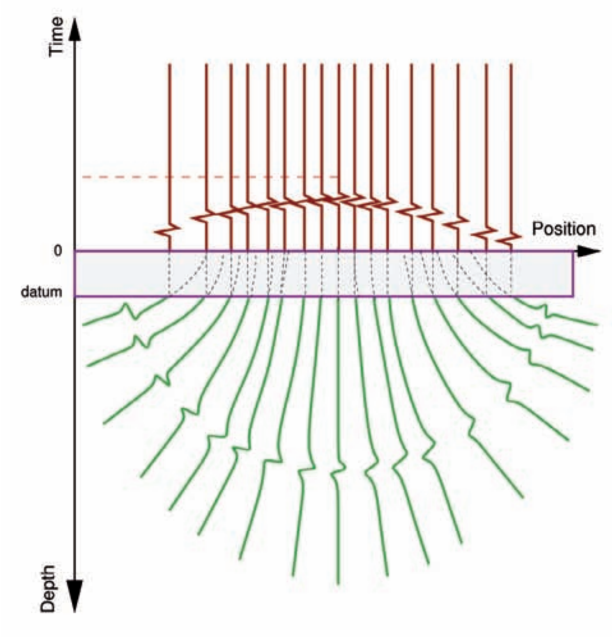
In general, redatuming refers to a data transformation that simulates the translation of the source and the receiver from one datum (reference plane) to another. Here, we use the term *redatuming* to refer to a data transformation associated with an isochron extrapolation from one horizontal level to another, without any assumption about the new location of the source-receiver pair.

The isochron redatuming is achieved by applying additional static time-shifts to the input trace during the data-conditioning step. For each position where the input trace is repeated, the time-shift is the difference of the two-way traveltime between this position and the initial position of the corresponding isochron ray. Time-shift calculation involves a mapping procedure along isochron rays, which is analytically determined by doing the redatuming in a homogeneous layer.

Fig. 3 illustrates isochron redatuming for an arbitrary trace of the conditioned data. In this example, the source-receiver pair is moved from the surface $z = 0$ to the level $z =$ datum. The selected trace is originally located at A, which is the initial point of the isochron ray $\Omega$ in the plane $z = 0$. This trace is moved to location C after it is properly shifted in time. C is the vertical projection of B, which is the intersection point between the ray $\Omega$ and the plane $z =$ datum. The applied time-shift, $t_b - t_a$, is the traveltime computed along the isochron ray $\Omega$ from A to B.




(a)




(b)

Fig. 2. Conditioned data for WEIM: a) without redatuming, b) with redatuming.

Fig. 2(b) shows the conditioned data of Fig. 1(b) after redatuming. Observe two important modifications in the data-conditioned space: the horizontal event becomes curved, and the trace distribution becomes irregular. In practice, redatuming and data conditioning are performed together, and the trace distribution is regularized, i.e., made uniform by using interpolated isochron rays.

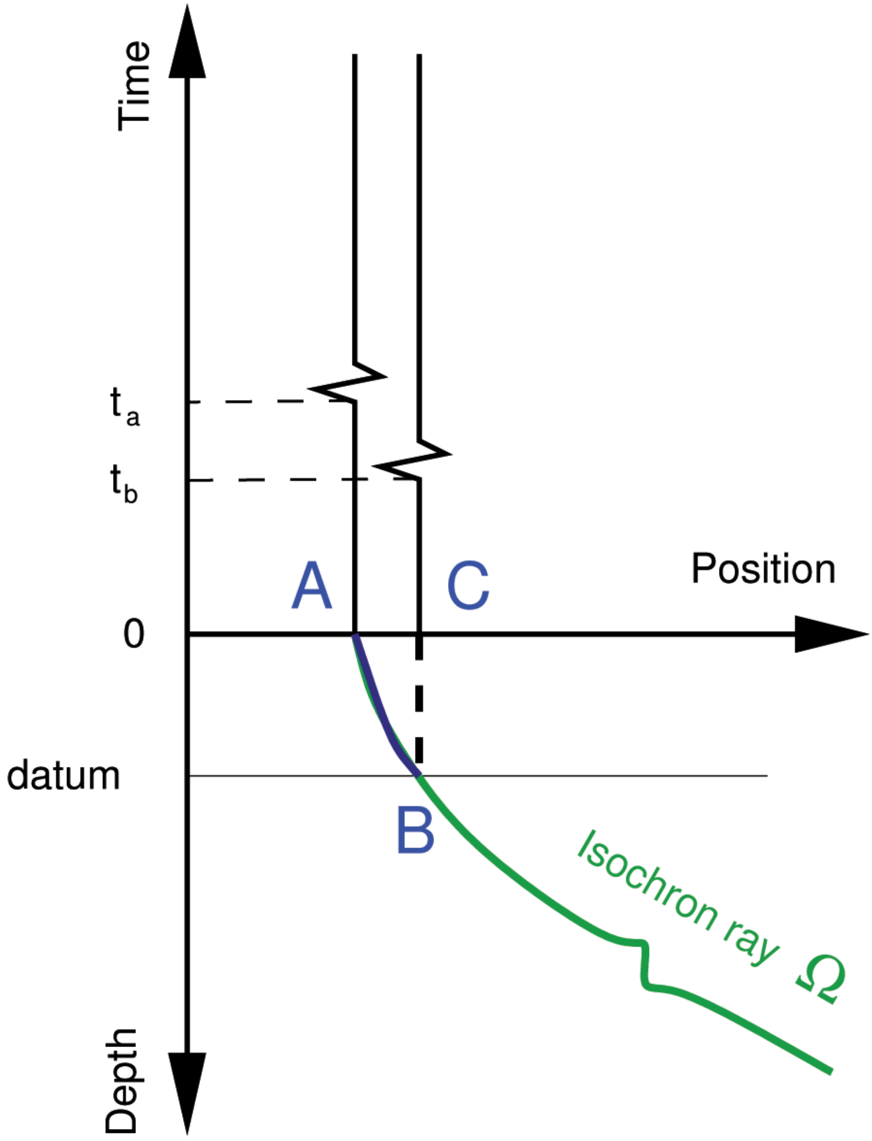



Fig. 3. Cartoon illustrating the redatuming procedure for one trace of conditioned data.

## Beam formation

In common-offset WEIM, each input trace is transformed into a gather (conditioned data), which is used as input for ZOWE migration with equivalent velocity field. The basic idea is to reduce the computational cost by migrating groups of traces instead of individual traces. The input common-offset section is divided in groups with equal numbers of traces and with overlap of adjacent groups. Fig. 4 shows an example of beam distribution for a 2D common-offset section. In this example, three groups have central traces located at A, B, and C. In order to keep amplitudes balanced after superposition, traces are multiplied by a weight function with an exponential decay; i.e., each group is windowed using Gaussian tapering.

The beam formation for WEIM is performed during the data-conditioning step in a such a way that the conditioned data gather contains information from all beam traces. It is equivalent to compute gathers for individual traces and stack them using a proper weight.

The problem of imaging a group of traces by WEIM is that the equivalent velocity depends on the source and receiver positions; thus we have to choose one source-receiver pair to compute this velocity field. The beam center is a natural choice for the equivalent velocity computation. Consequently, traces located away from beam center are migrated using an improper velocity. To overcome this drawback, we create conditioned data for ZOWE migration using a laterally shifted equivalent velocity field.

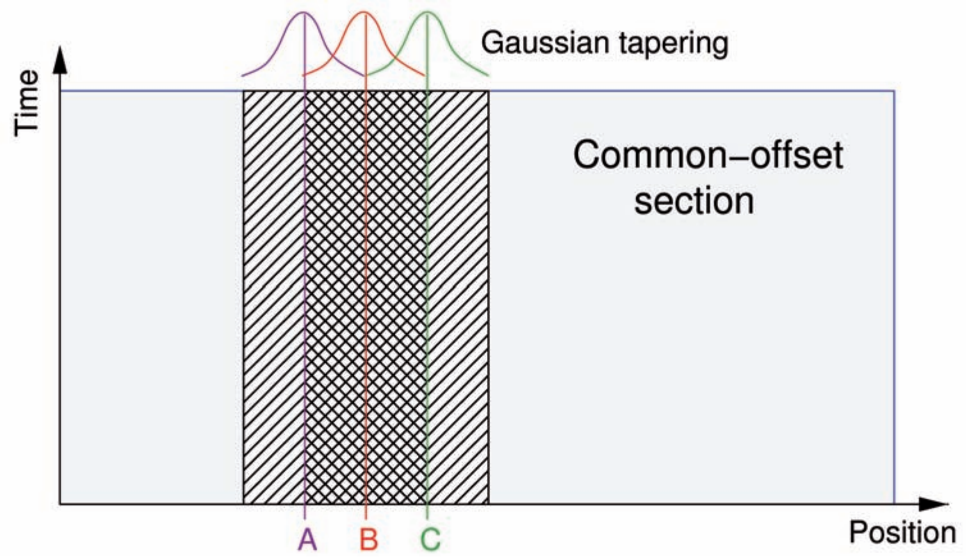



Fig. 4. Illustration of beam distribution for 2D common-offset isochron-ray migration.

For the trace located at the beam center, the data conditioning consists of a time-shift and a trace repetition. This operation is equivalent to mapping a point from the input trace into a horizontal line in the conditioned data. After ZOWE migration, this horizontal line is mapped into an isochron. Fig. 5(a) illustrates this procedure. First, the point M of the central trace $tr_3$ is mapped into the horizontal line, $\tau_3$, which is mapped into isochron $\zeta_3$ after migration. Isochron $\zeta_3$ and horizontal line $\tau_3$ are used as reference in the conditioning of the other traces.

For a trace located away from the beam center, the conditioning also starts with a time-shift, but it is followed by a dynamic trace operation instead of a simple repetition. In this operation, a point of the input trace is mapped into a curve instead of a horizontal line. We refer to this curve as a *spreading line*. The spreading line is defined in such a way that it is mapped into an isochron after ZOWE migration with the beam-center equivalent velocity.

Fig. 5(b) explains how the spreading line is defined. From the input trace, $tr_4$, the point N is selected at the time location of M. N is mapped into the spreading line, $\tau_4$, which must be migrated to isochron $\zeta_4$. Thus, for each trace of the conditioned data, the difference between $\tau_4$ and $\tau_3$ is the traveltime between two points of the corresponding isochron ray, which are the points where the isochron ray intersects isochrons $\zeta_3$ and $\zeta_4$. For example, for the trace corresponding to the last isochron ray on the right, the traveltime is measured between points A and B.

Spreading lines vary in time and depend on the relative position relative to the beam center. Thus, each sample of each trace of the beam has a corresponding spreading line, and each spreading line is mapped into a corresponding isochron.

## EXAMPLE

In this section, we present the results of application of the WEIM method, using beams and redatuming, in a 2D synthetic common-offset (CO) section. The objective here is not yet to analyze the computational cost, but to illustrate how the method works.

Fig. 6(a) shows the CO section and the beam-center locations. The beam width is 200 m and, the distance between beam centers is 100 m. The input section is composed of five linear events, which were artificially inserted, without concern about physical meaning. We assume that: a) the source and receiver are located at the surface z = 0, b) the offset is 1 km, c) the velocity is 2 km/s, and d) the distance between traces is 10 m. Fig. 6(b) shows the superposition of all beam images using redatuming to z = 200 m.

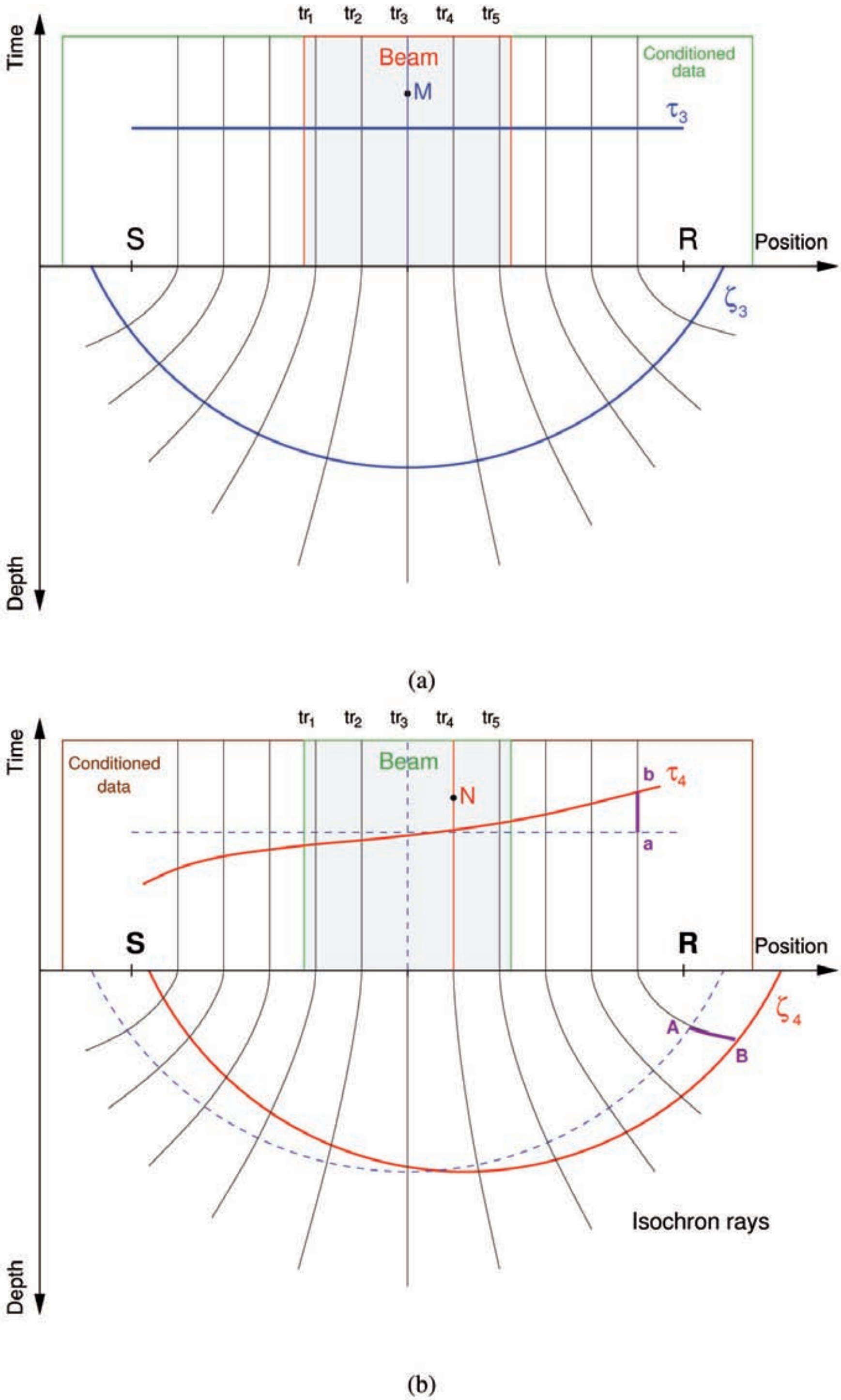
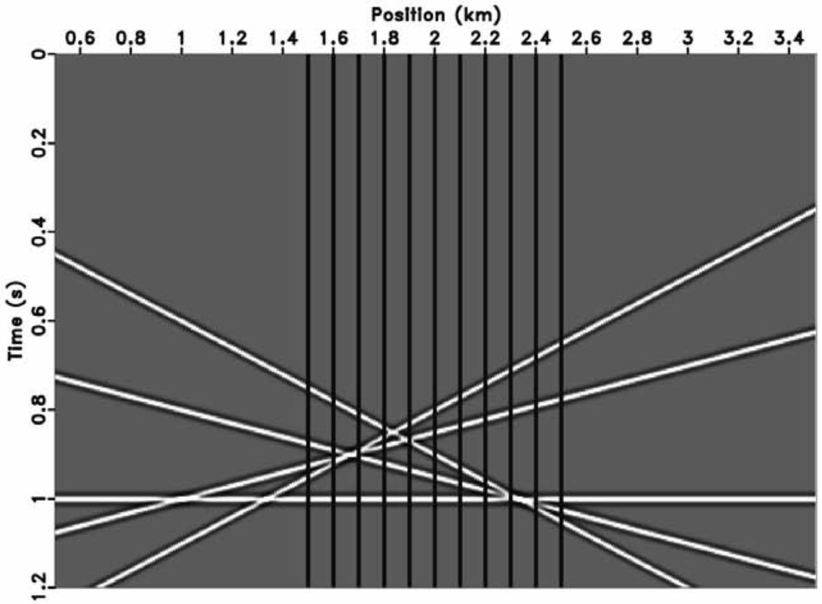
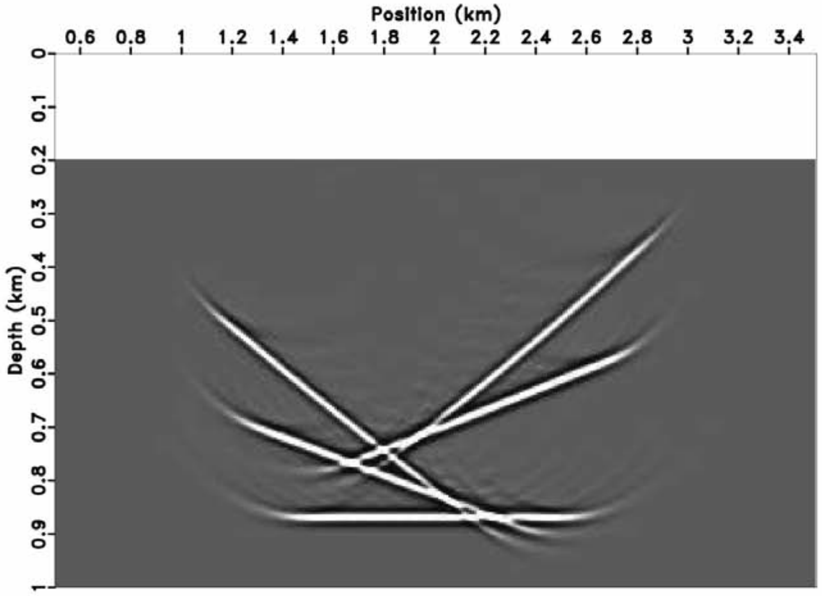



Fig. 5. Data conditioning and beam formation: a) central trace, b) non-central trace.




(a)




(b)

Fig. 6. WEIM using redatuming and beams: a) input common-offset section with eleven beam-center locations, b) final image: superposition of all beam contributions.

Figs. 7(a) and 7(b) show the conditioned data for the single trace located at x = 2 km, with and without redatuming, respectively. Notice that the horizontal events become curved after redatuming. Figs. 8(a) and 8(b) are the corresponding conditioned data for the beam, which center is located at x = 2 km. Notice that events become stretched and asymmetric, being the asymmetry related to the time-slope in common-offset input section. Figs. 7(c) and 8(c) show the migration of the single trace and the beam, respectively. In the beam image, notice that the energy is concentrated around the points where the isochrons are tangent to the reflectors.

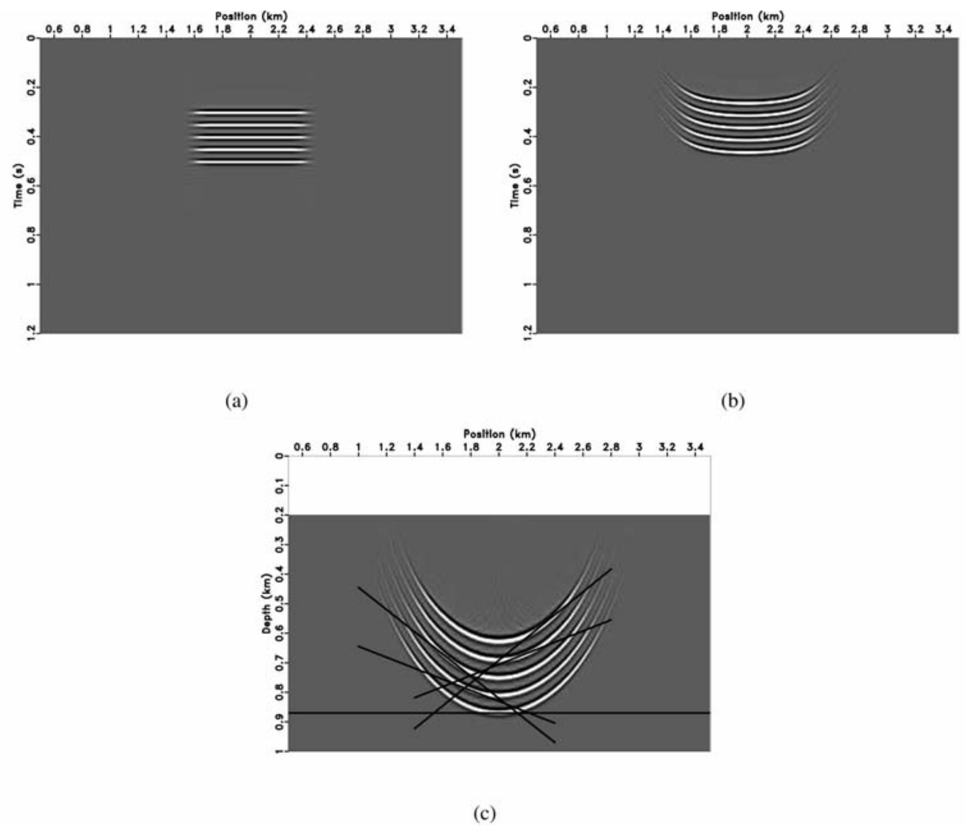



Fig. 7. WEIM of a single trace: a) conditioned data without redatuming, b) conditioned data with redatuming from z = 0 to z = 200 m, c) image obtained by ZOWE migration.

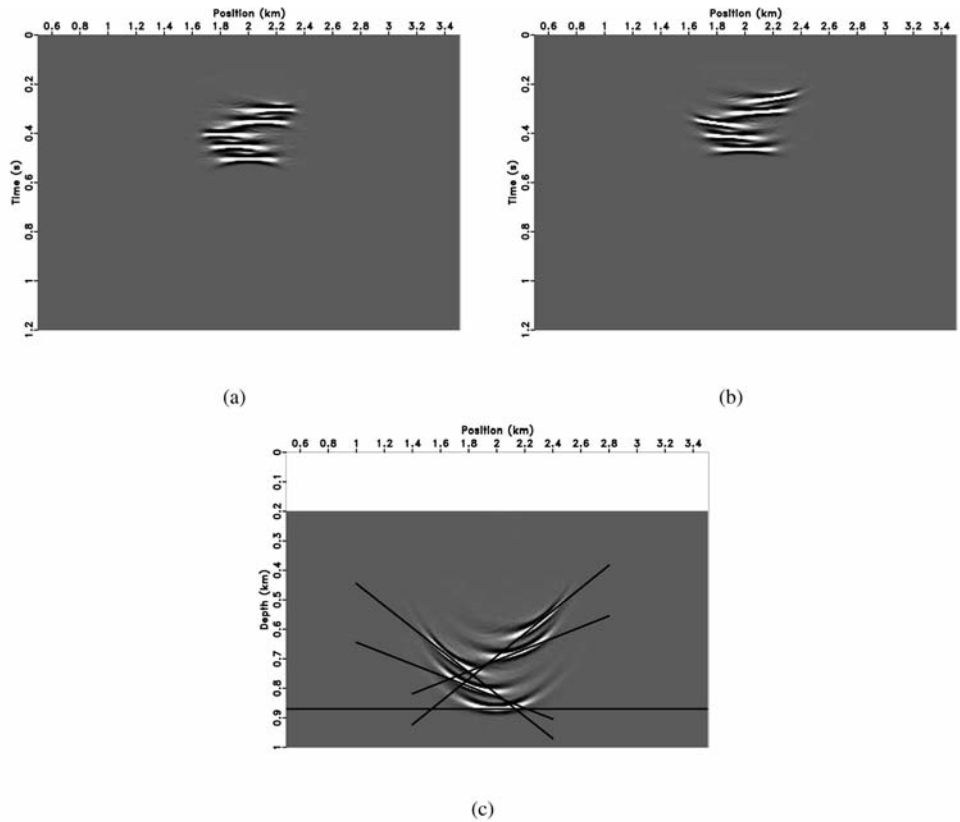



Fig. 8. WEIM of a 21 traces beam: a) conditioned data without redatuming, b) conditioned data with redatuming from z = 0 to z = 200 m, c) image obtained by ZOWE migration.

COMPUTATIONAL COST

The computational cost depends on several aspects, including required memory, access to disk and paralleling strategy. Here, we only consider the CPU time for serial programs.

The cost of applying WEIM in common-offset section is proportional to the dataset size and depends on each step involved in WEIM. A reasonable estimate is given by the expression

$$C_{co}^{0} = n_t(C_e + C_d + C_{zo}) \quad , \tag{1}$$

where $n_t$ represents the number of traces of the input section and C stands for the computational cost of each involved process, the subscript e pertaining to equivalent-velocity computation, d to data conditioning, and zo to the zero-offset wavefield extrapolation migration. $C^0_{co}$ stands for the computational cost of common-offset WEIM without using beams and redatuming.

For a fixed offset, $C_e$ and $C_d$ do not significantly vary from trace to trace, and both are much smaller than $C_{zo}$; thus we can write

$$C_e + C_d = \alpha C_{zo} \quad , \tag{2}$$

with $\alpha < 1$.

When inserting expression (2) into equation (1), we obtain

$$C^0_{co} = (1 + \alpha)n_t C_{zo} \quad . \tag{3}$$

Expression (3) explicitly shows that the cost of common-offset WEIM can be reduced by either decreasing the number of traces or reducing $C_{zo}$. The use of redatuming and beams influence this cost in different ways. While redatuming reduces the number of steps in the wavefield extrapolation, the use of beams is equivalent of decreasing the number of traces. We can estimate the cost of WEIM using beams and redatuming by the expression

$$C^{rb}_{co} = (1 + \beta)n_t C^r_{zo} \quad , \tag{4}$$

where $n_b$ represents the number of beams, and $\beta$ is the cost of the equivalent-velocity computation plus the cost of the conditioning-data step with redatuming and beam formation. $C^r_{zo}$ stands for the cost of zero-offset wave-equation migration with redatuming. Since $C_{zo}$ is proportional to the size of the wavefield, we can write

$$C^r_{zo} = [(z_{max} - z_{SR} - z_{datum})/(z_{max} - z_{SR})]C_{zo} \quad , \tag{5}$$

with $z_{max}$ being the maximum depth, $z_{SR}$ is the depth of the plane where the source-receiver pair is located, and $z_{datum}$ is the depth to which the source-receiver pair is translated after redatuming.

The cost-reduction factor due from of beams and redatuming is the ratio between expressions (4) and (2). Assuming, with no loss in generality, that $z_{SR} = 0$, the cost-reduction factor can be expressed by

$$f_{red} = C^{rb}_{co}/C^0_{co} = [(1 + \beta)/(1 + \alpha)](n_b/n_t)[(z_{max} - z_{datum})/z_{max}] \quad . \tag{6}$$

For a 2D common-offset section with regular geometry, the ratio $n_b/n_t$ is

$2/(n_{trb} - 1)$, where $n_{trb}$ is the number of traces per beam. Typically, $\alpha$ stays in the range from .01 to .05 in 2D computations, $\beta$ varies from .03 to .1, so the ratio $(1 + \beta)/(1 + \alpha)$ is close to unity. The cost reduction factor for 2D can be well estimated by the expression

$$f^{2D}_{red} = [2/(n_{trb} - 1)][(z_{max} - z_{datum})/z_{max}] \ . \tag{7}$$

In addition to the gain from use of beams and redatuming, an extra cost reduction can be obtained by using reduced migration aperture, which is equivalent of reducing the conditioned data size.

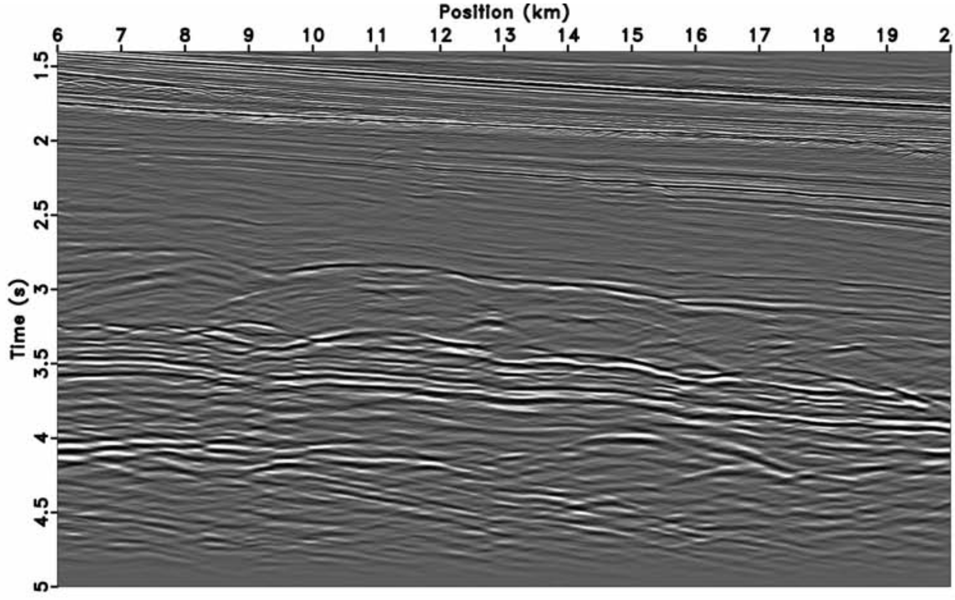
Although we do not present a formula to estimate the cost reduction for 3D data, we can infer that it is close to the square of $f^{2D}_{red}$ for square grids.

## APPLICATION TO FIELD-DATA

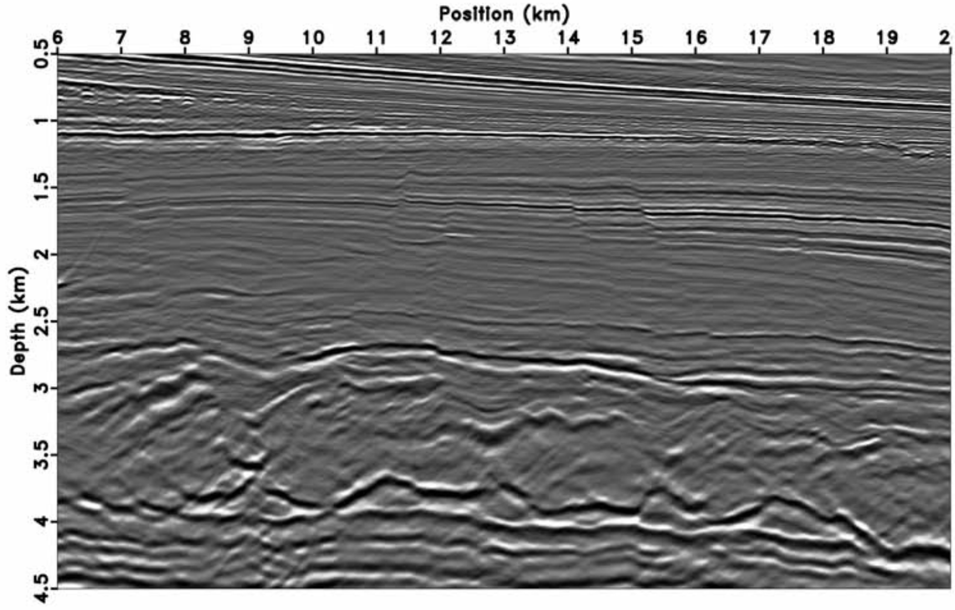
The WEIM using isochron rays was applied to a pseudo 2.5D dataset that consists of 22 common-offset gathers extracted from a 3D dataset following the sequence. (1) the input traces were organized in 22 groups, using as sorting criteria the source-receiver offset, (2) a 3D Kirchhoff time-migration algorithm was applied, and (3), a 2.5D Kirchhoff time-demigration procedure was applied to each image. In the sorting procedure, each input trace was scaled by an areal factor in order to compensate acquisition irregularities. The weight function used in the 3D time-migration algorithm produces a true-amplitude image gather when the medium velocity is constant, i.e., the output amplitudes are proportional to the reflection coefficients. Also, the applied demigration program uses a true-amplitude weight function that produces a 2D common-offset gather in which amplitudes are reduced by a 3D geometrical spreading factor that is correct for homogeneous media.

The minimum offset is 160 m, and the increment between offsets is 200 m. Each common-offset gather has 1351 traces, and the distance between them is 18.75 m. The traces are 5 s long, and the time sampling interval is 4 ms. From this dataset, we selected the section with offset of 1960 m for experiments using beams and redatuming. Fig. 9(a) shows the selected common-offset section, and Fig. 9(b) shows the image from standard WEIM, i.e., without using beams and redatuming. This image is used as reference in our migration comparisons. The migration aperture and the spatial length of the conditioned data are 7480 m.

The experiments consist of ten migrations with different beam sizes and with redatuming for the same depth. Both redatuming and beam formation were carried out by a constant-velocity algorithm. The number of traces per beam is odd, varying from 3 to 21, while the depth for redatuming is 400 m.




(a)




(b)

Fig. 9. Field-data experiment: a) input common-offset section (offset = 1960 m), and b) image from standard WEIM (without using beams and redatuming).

From visual inspection over the ten images (only two of which are shown here), we observe that

- there are no significant kinematic differences between the beam images and the reference image,

- artifacts are present in all beam images, as well as in the reference image, and

- the greater the beam size, the stronger the artifact amplitudes.

Here, we select for illustration the images with 13 and 17 traces/beam, shown in Figs. 10(a) and 10(b), respectively. From the smallest to the largest beam size studied, the 13 traces/beam image is the first where the artifacts become evident, while the 17 traces/beam is the last useful image for migration velocity analysis. Figs. 11(a) and 11(b) show a window where the artifact amplitudes become stronger.

Two possible reasons for the presence of artifacts are the use of a constant velocity algorithm to form beams and lack of an appropriated weight function to compensate differential geometrical spreading associated with different trajectories.

The measured CPU time reduction is a factor of 6.5 for 13 traces/beam and 8.4 for 17 traces/beam, while the expected factor according to relation (7) are about seven and nine, respectively.

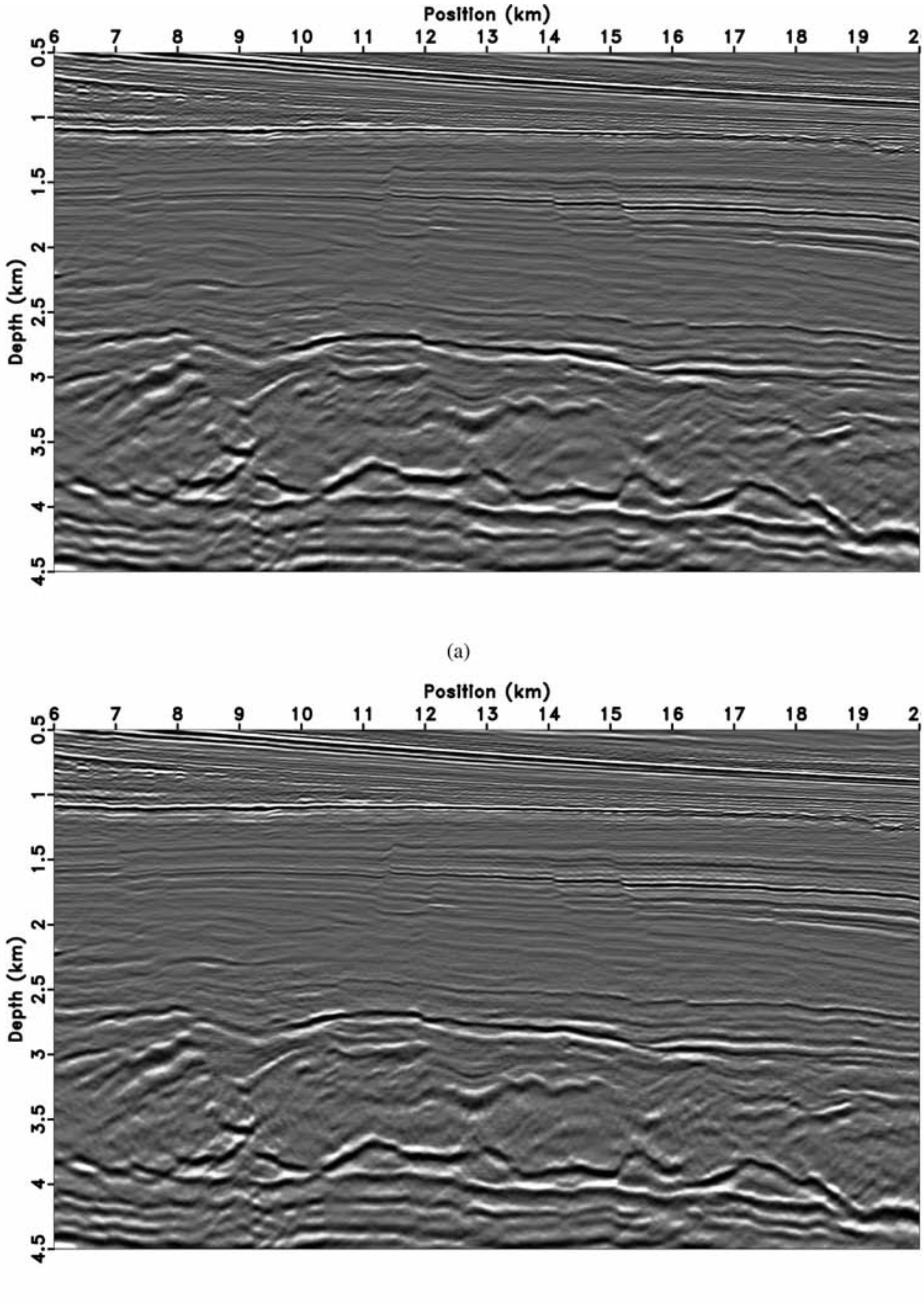
## DISCUSSION

These cost reductions from use of beams and redatuming can make WEIM feasible for MVA. Let us compare an estimate of the MVA-WEIM cost with the cost of conventional zero-offset wavefield-extrapolation (ZOWE) migration.

The computational cost reduction from beams depends on offset because the number of traces per beam increases with offset. To simplify, we consider a linear relationship for this dependency and assume that the average reduction is approximated well by the cost reduction for the average offset.

Consider an MVA algorithm that consists of analyzing image strips from several common-offset sections. The average computational cost of imaging one common-offset section is given by

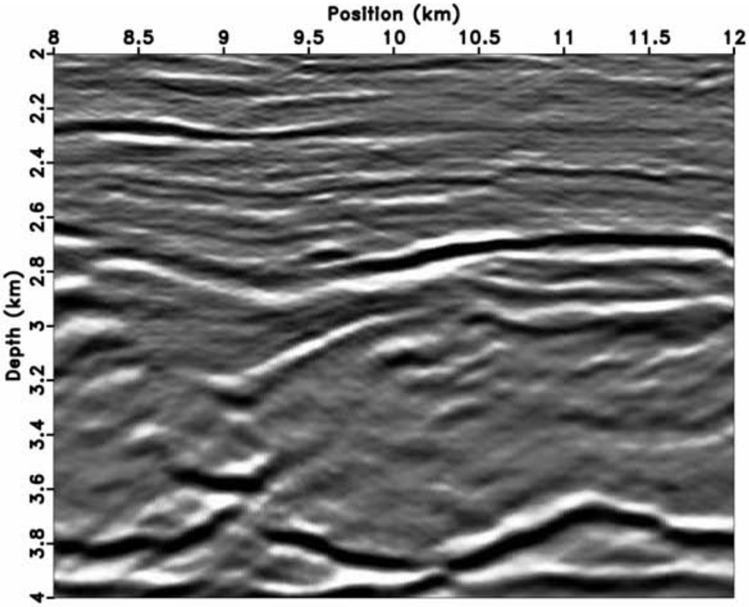
$$C_{mva} = f_{br}\, f_s\, f_a\, n_t\, C_{zo} \quad , \tag{8}$$



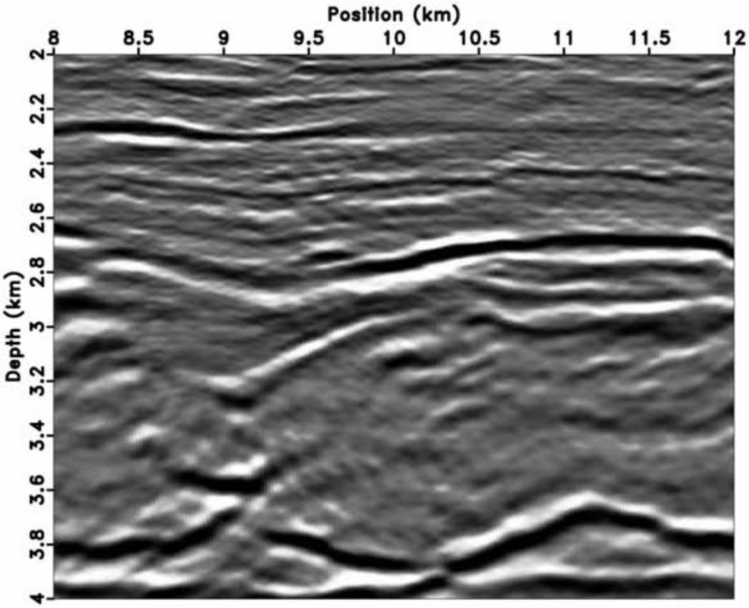

(a)

(b)

Fig. 10. WEIM images using beam and redatuming: a) 13 traces/beam, and b) 17 traces/beam.




(a)




(b)

Fig. 11. Zoomed WEIM images using beam and redatuming: a) 13 traces/beam, and b) 17 traces/beam.

where $n_t$ is the number of traces in the common-offset section, $f_{br}$ is the factor of cost reduction from beams and redatuming, $f_s$ is the reduction from the use of strips, $f_a$ is the ratio between migration aperture and spatial size of the input section, and $C_{zo}$ is the ZOWE-migration cost.

For the presented field-data example, $f_{br}$ is close to 1/8, $f_a$ is roughly 1/4, and $n_t$ is 1351. Suppose that the use of strips reduces the number of input traces to 20%, i.e., $C_{mva} = 8C_{zo}$. This result suggests that the computational cost of individual WEIM experiments for MVA has the same order of magnitude of ZOWE migration, indicating that MVA using WEIM is feasible.

## CONCLUSION

The objective of improving the computational performance of WEIM was achieved by using beams and redatuming. The effective reduction of the computational cost observed in a 2D field-data example is in accordance with the reduction predicted by a simplified formula. In this example, the maximum speedup from beams is around 7.5 times, while the gain from redatuming is close to 10%. The obtained results indicate that MVA using WEIM is feasible.

Numeric experiments indicate that the higher the density of traces, the better the relative performance expected for WEIM using beams. This fact is particularly useful for the application of MVA based on WEIM to 3D surveys with dense acquisition grid. We expect more significant gains for deep-water data, in which case the redatuming can efficiently extrapolate isochrons from the surface to a deep level in one step, perhaps providing a gain of 50% instead of the 10% observed here.

## ACKNOWLEDGEMENT

We thank Petrobras for the financial support and permission to publish this article. We also acknowledge the support of the Center for Wave Phenomena at Colorado School of Mines.

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