AZIMUTHAL TAU-P ANALYSIS IN A WEAK ORTHORHOMBIC MEDIUM

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ABSTRACT

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Nine elastic coefficients are needed to describe a traveltime curve in an orthorhombic medium. The problem of estimating all the elastic parameters by iterative fitting of traveltime data from a single azimuth recording is highly non-unique. More anisotropic parameters in an orthorhombic medium than in a transversely isotropic medium make the parameter estimation process more challenging. A solution can possibly be achieved by simultaneous fitting of multiple azimuth travel time data. However that would require picking travel time and accurate estimation will require numerical ray tracing for multi-layered media. To circumvent these difficulties we propose analysis of plane wave transformed azimuthal gathers interactively using a single azimuth data at a time and a new P-wave delay time equation which is a function of two parameters at each azimuth. Results from independently estimated multi-azimuth gathers can be combined to estimate anisotropic parameters. Azimuthal τ-p analysis also avoids numerical ray tracing resulting in a rapid algorithm. We demonstrate the applicability of our method using a set of P-wave synthetic seismograms from a multi-layered medium consisting of isotropic and orthorhombic layers. Azimuth dependent anisotropy parameters are derived by delay time fitting and NMO correction. The reflections from the bottom interface of an isotropic layer with an anisotropic overburden show apparent anisotropic travel time behavior which is easily accounted for by our layer-stripping based azimuthal NMO analysis.

KEY WORDS: anisotropy, delay time, NMO, numerical approximation, orthorhombic, azimuth.

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INTRODUCTION

Sen and Mukherjee (2003) developed a reduced parameter delay-time slowness (τ-p) domain normal moveout (NMO) correction equation for VTI medium for P-wave data. Recently Sil and Sen (2007, 2008) generalized that equation to HTI media. In this paper we generalize the same equation further to include orthorhombic media. Even though it is easy to describe a fractured reservoir model as an HTI medium, realistically it is an orthorhombic medium, or even a medium of lower symmetry (Schoenberg and Helbig, 1997; Bakulin et al., 2000). An orthorhombic medium can be described by nine anisotropic parameters and three planes of symmetry (Tsvankin, 1997). Those parameters are V_{P0}, V_{S0}, $\epsilon^{(2)}$, $\delta^{(2)}$, $\gamma^{(2)}$, $\epsilon^{(1)}$, $\delta^{(1)}$, $\gamma^{(1)}$ and $\delta^{(3)}$, where V_{P0}, V_{S0} are the vertical P- and S-wave velocities, and the other parameters are functions of elements of the orthorhombic medium stiffness matrix, defined by Tsvankin, (1997) in the style of Thomsen parameters (Thomsen, 1986). A typical schematic of the orthorhombic medium is shown in Fig. 1.

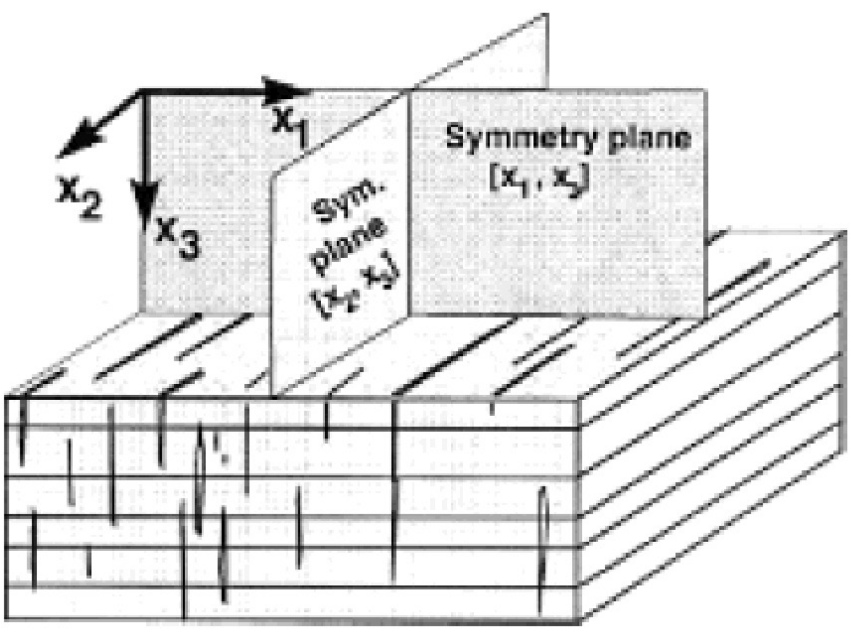

Fig. 1. Schematic of an orthorhombic model caused by parallel vertical fractures within a finely layered medium. Three planes of symmetry are [x1, x3], [x2, x3] and [x1, x2]. VTI medium equivalence described in the text is for the [x1, x3] plane shown here. From Tsvankin (2001).

One of the major derivations by Tsvankin (1997, 2001) is that as long as the wave propagation is restricted to one of the vertical symmetry planes of the orthorhombic medium (say [x1, x3] plane in Fig. 1), all of the kinematic signatures (i.e., phase velocity, group velocity, etc.) can be described by corresponding VTI equations after replacing the anisotropic coefficients of the VTI medium with the coefficients of an orthorhombic medium (Tsvankin, 2001). This simple rule of thumb makes it easy to derive an NMO equation in the τ-p domain for the orthorhombic medium from an existing VTI equation. In this paper we first show that this rule for the [x1, x3] plane is valid when we express the P-wave phase velocity as a function of horizontal slowness p. We then generalize it for all azimuths following Tsvankin (1997, 2001).Then using the phase velocity expression V(p), we develop our orthorhombic NMO correction equation following a procedure outlined by Sen and Mukherjee (2003) and Sil and Sen (2007). We also test our model for three different azimuths of an orthorhombic medium using a set of full-wave anisotropic synthetic seismograms.

Reduced parameter NMO correction equations for the transversely isotropic (TI) medium in the τ-p domain show better fit to the theoretical curve, and hence better NMO correction and parameter estimation ability over the x-t domain (van der Baan and Kendall, 2003). The layer striping approach of NMO correction in the τ-p domain also helps to reduce the additional ambiguity of pseudo anisotropic signature from an isotropic layer when it is overlain by an anisotropic layer and tedious work of ray tracing. Thus the importance of τ-p domain cannot be ignored. Even though NMO correction equations exist for TI media, not much work (one work is reported by Mah and Schmitt, 2001) has been done on the orthorhombic medium. Here we report on the development of quasi-P-wave NMO equation for the orthorhombic medium considering weak anisotropy and horizontal reflector. Just like the x-t domain NMO correction equation, this equation also consists of two reduced parameters (Al-Dajani et al., 1998; Grechka and Tsvankin, 1999; Bakulin et al., 2000; Vasconcelos and Tsvankin, 2006). It is not possible to include all the steps in our derivation due to space limitation; they draw upon the results from the cited references.

THEORY

For the purpose of τ-p analysis in an orthorhombic medium, we need to obtain a P-wave phase velocity expression V(p), which is a function of horizontal slowness p. To derive that expression, we make use of the fact that the phase velocity $V(\theta)$ (where θ is the angle of incidence of a plane wave) of the orthorhombic medium in its [x1,x3] plane (Fig. 1) can be derived by substituting the suitable anisotropic parameters from the VTI phase velocity expression (Tsvankin, 2001). That means that from any $V(\theta)$ expression of the

VTI medium if we replace ϵ, δ and γ VTI parameters by $\epsilon^{(2)}$, $\delta^{(2)}$ and $\gamma^{(2)}$ orthorhombic parameters, we obtain the V(θ) expression for the orthorhombic medium in the [x1, x3] plane. Details of these parameters are described in several papers (Tsvankin, 1997; Bakulin et al., 2000; Tsvankin, 2001; Vasconcelos and Tsvankin, 2006). Even though from the above statement it is obvious, here we do show that changing the V(p) expression for VTI medium with orthorhombic parameters can return the V(p) expression for the orthorhombic medium in the [x1, x3] plane. We start our analysis with the V(θ) expression for the VTI medium given by Tsvankin (1996). We exchange all the VTI anisotropic parameters for orthorhombic parameters and obtain:

$$V^2(\theta)/V_{P0}^2 = 1 + \epsilon^{(2)}\sin^2\theta - f/2$$

$$+ (f/2)\sqrt{\{1 + (4\sin^2\theta/f)(2\delta^{(2)}\cos^2\theta - \epsilon^{(2)}\cos2\theta) + (4\epsilon^{(2)}\sin^4\theta/f^2)\}} \quad , \quad (1)$$

where

$$f = 1 - (V_{S0}^2/V_{P0}^2) \quad , \quad (2)$$

Eq. (1) is the V(θ) expression for the orthorhombic medium in [x1, x3] plane. To convert this expression to V(p) we follow the steps described by Cohen (1997). Following Cohen (1997), using eq. (1) and considering weak anisotropy we can write:

$$V(p) = V_{PO}[1 + \delta^{(2)}z + (\epsilon^{(2)} - \delta^{(2)})z^2] \quad , \quad (3)$$

where

$$z = V_{P0}^2p^2 \quad . \quad (4)$$

Notice that we can easily obtain this expression just by changing the anisotropic parameters from the VTI phase velocity expression obtained by Cohen (1997).

Tsvankin's (2001) equivalent expression for P-wave phase velocity of our eq. (3) is:

$$V(\theta) = V_{P0}[1 + \delta^{(2)}\sin^2\theta\cos^2\theta + \epsilon^{(2)}\sin^4\theta] \quad . \quad (5)$$

A general expression of V(θ,φ) for any azimuth is obtained by replacing the anisotropic parameters of eq. (5) with the new azimuth dependent anisotropic coefficients (Tsvankin, 2001). The expression is:

$$V(\theta,\varphi) = V_{P0}[1 + \delta(\varphi)\sin^2\theta\cos^2\theta + \epsilon(\varphi)\sin^4\theta] \quad . \quad (6)$$

Similarly our general expression for $V(p,\varphi)$ can be written as:

$$V(p,\varphi) = V_{P0}[1 + \delta(\varphi)z + \{\epsilon(\varphi) - \delta(\varphi)\}z^2] \ , \tag{7}$$

where

$$\delta(\varphi) = \delta^{(1)}\sin^2\varphi + \delta^{(2)}\cos^2\varphi \ , \tag{8}$$

and

$$\epsilon(\varphi) = \epsilon^{(1)}\sin^4\varphi + \epsilon^{(2)}\cos^4\varphi + [2\epsilon^{(2)} + \delta^{(3)}]\sin^2\varphi\cos^2\varphi \ . \tag{9}$$

Eqs. (8) and (9) are defined by Tsvankin (2001).

Similarly an exact expression for V(p) in the [x1, x3] plane can be obtained for the orthorhombic medium if we change the exact V(p) expression of the VTI medium derived by van der Baan and Kendall (2003).

Once we derive eq. (7), following Sen and Mukherjee (2003) and Sil and Sen (2007), we can write:

$$\tau(p,\varphi) = \sum_{i=1}^{nl} \tau_0^i(1 - p^2\alpha_{el}^{i2})^{1/2}[1 - 2p^4\alpha_{el}^{i4}\kappa^i/(1 - p^2\alpha_{el}^{i2})]^{1/2} \ , \tag{10}$$

where τ_0^i is the two-way normal time, α_{el}^i is the elliptical velocity, κ^i is the anisotropic parameter for layer i and nl is the total number of layers. The terms α_{el} and κ are defined as:

$$\alpha_{el}^2(\phi) = V_{P0}^2[1 + 2\delta(\phi)] \ , \tag{11}$$

$$\kappa(\phi) = [\epsilon(\phi) - \delta(\phi)]/[1 + 2\delta(\phi)]^2 \ , \tag{12}$$

Eq. (11) is same as the NMO velocity of weak orthorhombic medium defined by Tsvankin (2001).

Thus considering weak anisotropy, the τ-p curve for P-wave reflection from horizontal orthorhombic medium at a given azimuth can be written as a function of two reduced parameters α_{el} and κ, where the first one is the elliptical velocity and the second one is an anelliptic term, responsible for the far offset non-elliptic behavior of the τ-p curve over anisotropic medium (Sen and Mukherjee, 2003; Sil and Sen, 2007). We can use eq. (10) for the VTI and HTI media as well; in case of VTI medium $\epsilon^{(1)} = \epsilon^{(2)} = \epsilon$; $\delta^{(1)} = \delta^{(2)} = \delta$ and $\delta^{(3)} = 0$, then eq. (10) reduces to a two-term approximate equation for the VTI medium, obtained by Sen and Mukherjee (2003). For the HTI medium $\epsilon^{(1)} = \delta^{(1)} = 0$, then eq. (10) reduces to a two-term approximate equation for the HTI medium. When the medium is isotropic, all the anisotropic parameters and φ will be zero. Thus eq. (10) can be used for NMO correction and parameter estimation in isotropic, VTI, HTI and orthorhombic media although the out of the plane propagation is limited to weak anisotropy.

SYNTHETIC EXAMPLE

To show the performance of our equation, we generate exact synthetic seismograms using an anisotropic reflectivity code for a given "standard" anisotropic orthorhombic medium sandwiched between two isotropic media (Table 1), at azimuths 0°, 45° and 90°. The parameters are obtained from Xu and Tsvankin (2006). This model is used to demonstrate the effectiveness of our

Table 1. "Standard" orthorhombic Parameter of the 3 layered case used in this study.

	Layer 1	Layer 2	Layer 3	Layer 4
System Type	ISO	ORTHO	ISO	ISO
Thickness (km)	0.710	1.300	1.000	∞
Density (g/cm^3)	1.000	2.500	3.000	3.500
V_{P0} (km/sec)	2.960	3.330	3.500	5.200
V_{S0} (km/sec)	0.000	2.110	2.220	3.300
$\epsilon^{(1)}$	0.000	0.329	0.000	0.000
$\delta^{(1)}$	0.000	0.083	0.000	0.000
$\gamma^{(1)}$	0.000	0.046	0.000	0.000
$\epsilon^{(2)}$	0.000	0.258	0.000	0.000
$\delta^{(2)}$	0.000	−0.078	0.000	0.000
$\gamma^{(2)}$	0.000	0.182	0.000	0.000
$\delta^{(3)}$	0.000	−0.106	0.000	0.000

equation by fitting the data from orthorhombic layer as well as the data from the underlying isotropic layer, which shows a pseudo azimuthal anisotropic effect because of transmission through the orthorhombic medium. First we fit the seismogram using eq. (10) (Figs. 2, 3 and 4). Then we find the best fit parameters for our approximate equation by a conventional grid search method for 0° azimuth (Fig. 5). The tradeoff between the two reduced parameters to fit the curve is negligible. The results show excellent match between the exact and the approximated curves.

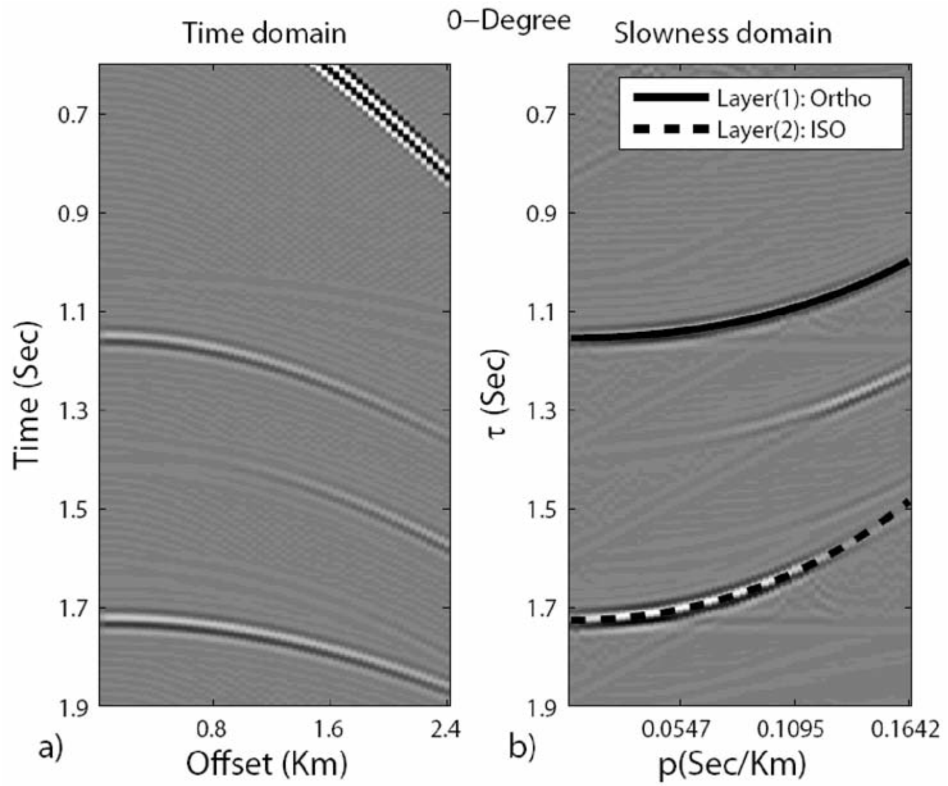

Fig. 2. Seismogram generated using "standard" orthorhombic model (table 1) for 0° azimuth in x-t domain (a) and τ-p domain (b). In the τ-p domain we fit the seismogram using our approximation equation 10 (solid black line). The same equation is used to fit the isotropic layer below (dashed black line).

CONCLUSIONS

We have developed an azimuth dependent two parameter delay-time equation for reflection delay-time from a flat interface in a weak orthorhombic medium. The equation reduces nicely to the limiting cases of isotropy, HTI and VTI and therefore, it is suitable for NMO analysis and parameter estimation in layered media consisting of isotropic, VTI, HTI and orthorhombic layers. Since

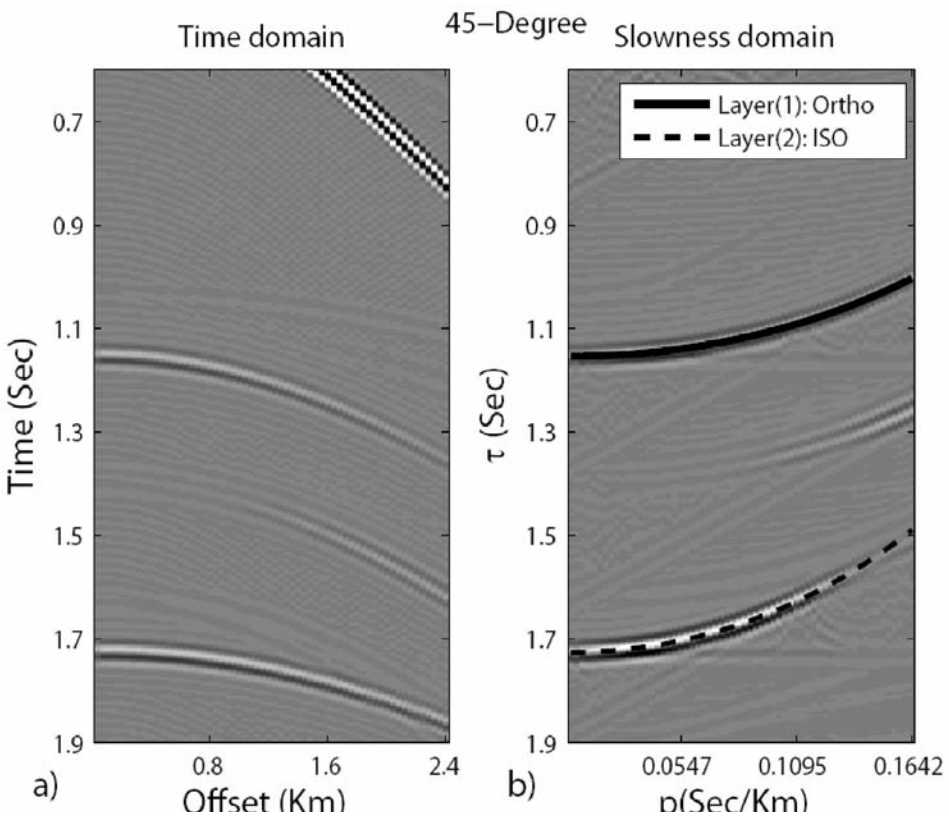

Fig. 3. Time domain (a) and τ-p domain (b) seismogram for 45° azimuth, with the plot of eq. (10) (same as Fig. 2).

this equation contains fewer parameters, a simple grid search method can be applied for determining the value of the parameters and further the extent of non-uniqueness is less. The equation is developed in the τ-p domain and therefore, a layer stripping approach to data fitting can be used to overcome the problem of an overlying anisotropic medium. Based on this equation we have also developed an interactive code for velocity picking for application to real data. The interactive code will be used in field data analysis.

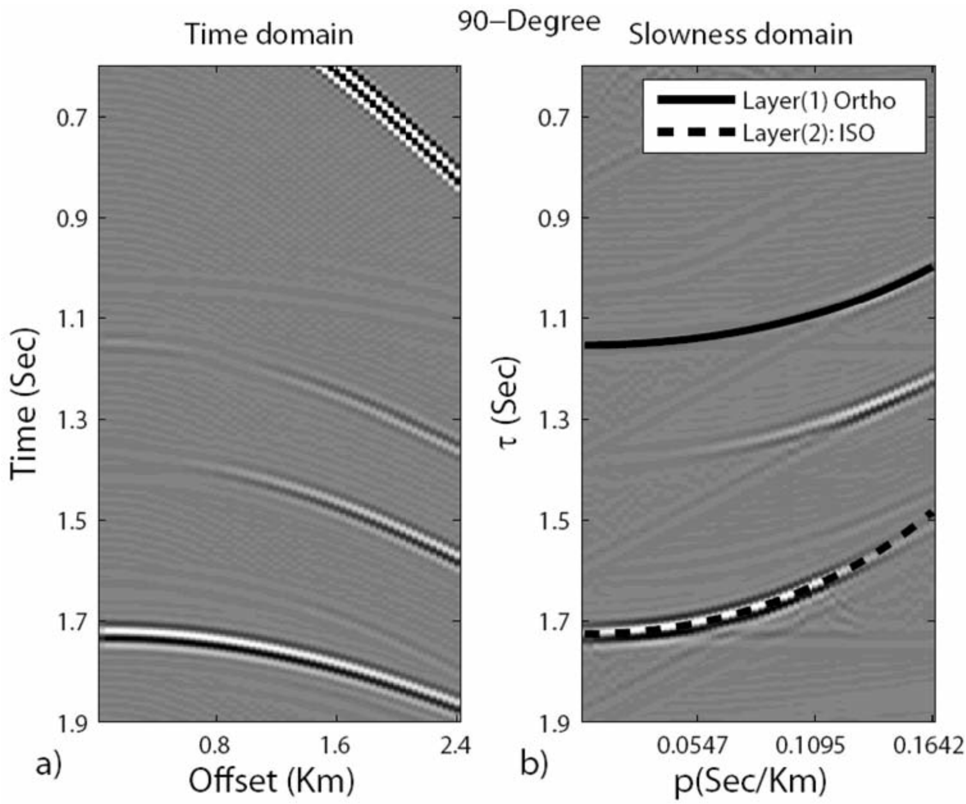

Fig. 4. Time domain (a) and τ-p domain (b) seismogram for 90° azimuth, with the plot of eq. (10) (same as Figs. 2 and 3).

So far several researches have been done on τ-p analysis of the VTI medium (Van der Baan and Kendall, 2002, 2003; Sen and Mukherjee, 2003; Douma and Van der Baan, 2008). All this research came up with an approximate equation for the τ-p analysis based on weak anisotropy assumption. Following the process described in this paper [i.e., changing the VTI anisotropic parameters in the approximate equations with the azimuth dependent anisotropic parameters, eqs. (8) and (9)] all these VTI medium works can be extended for the orthorhombic medium.

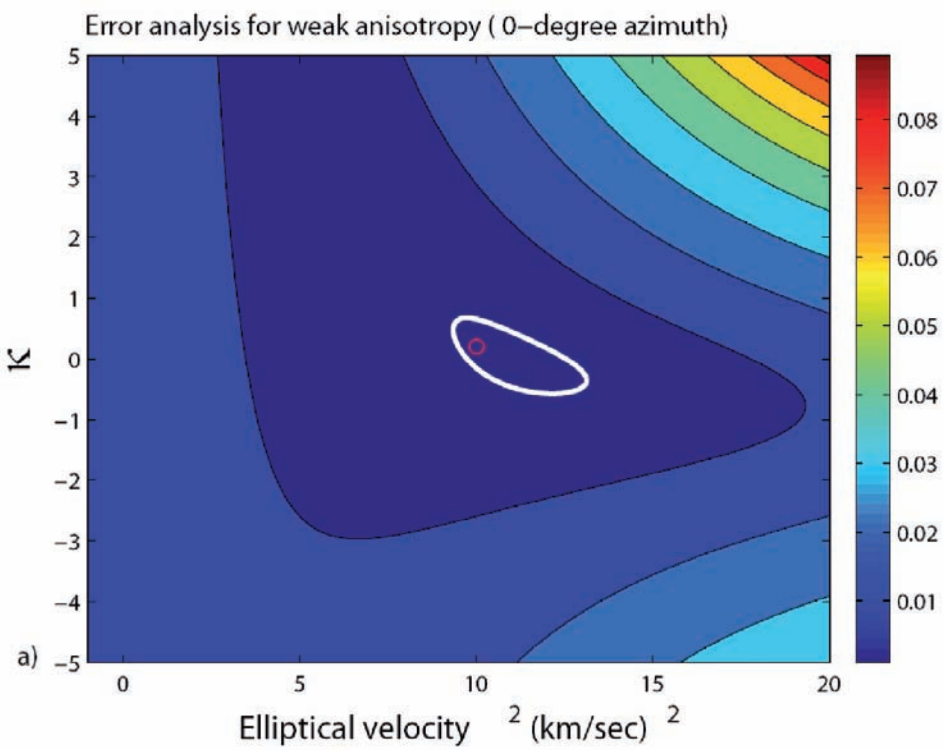

Fig. 5. Sensitivity of delay time to elliptical velocity and anisotropy parameter κ, for 0° azimuth. A tradeoff exists between both the parameters (white line). The red circles represent the original value of the parameters. Similar analysis can be performed for other azimuth also.

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