COMPARISON BETWEEN THE NEARLY PERFECTLY MATCHED LAYER AND UNSPLIT CONVOLUTIONAL PERFECTLY MATCHED LAYER METHODS USING ACOUSTIC WAVE MODELING

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ABSTRACT

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The unsplit convolutional perfectly matched layer (CPML) and nearly perfectly matched layer (NPML) methods both have been proven to be very efficient algorithms for eliminating artificial reflections from the edges of the synthetic seismic wave models. Their absorbing performance and efficiency have been studied in separate works in several papers. Obviously, if we provide numerical comparisons between CPML and NPML in seismic modeling, it is very helpful to understand their performances and differences. In this paper, we will carry out these comparisons using 2D acoustic wave modeling codes with staggered-grid finite-difference schemes. For the implementation of the finite-difference operator, we employ fourth-order accuracy methods in space and second-order accuracy methods in time. In the model tests, we demonstrate that NPML has the same absorbing performance as CPML, with some very minor differences. We suggest that more analysis will be needed to study how these methods perform in the wide varieties of complex media that are typically used in seismic modeling.

KEY WORDS: nearly perfectly matched layer, unsplit convolutional perfectly matched layer, acoustic modeling, staggered-grid finite-difference.

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INTRODUCTION

When we simulate seismic wave propagation in the earth's interior, we have to treat the seismic model as a truncated domain with boundaries. The computational resources are finite and limit the size of seismic simulations. Thus, we must face the new problem: how can we eliminate the reflections from the edges of the model because of the limitation of computational domain?

During the last several decades, absorbing boundary conditions (ABC) and absorbing layers techniques have been developed for dealing with the problem of existing artificial reflections mentioned above. Many authors have introduced various absorbing boundary conditions (ABC) in numerical modeling, for example, sponge boundary conditions (Cerjan et al., 1985), paraxial conditions (Clayton and Engquist, 1977; Higdon, 1991; Quarteroni et al., 1998), the eigenvalue decomposition method (Dong et al., 2005), and continued fraction absorbing boundary conditions (Guddati and Lim, 2006). However, all of these local condition operators exhibit poor behavior under some circumstances, especially, there can exist a large amount of spurious energy at grazing incidence angles. In addition, some researchers also provided improved methods by globally minimizing the amount of energy (Madariaga, 1976; Virieux, 1986; Bording, 2004; Chen et al., 2010). They can get the nearly optimal results, but these methods need more computing time in the search.

Absorbing layers techniques were developed a little later than ABC, but they revealed significant advantages over ABC. Bérenger (1994) introduced a technique called the perfectly matched layer (PML) that had the remarkable property of generating no reflection at the interface between the free medium and the artificial absorbing medium. This method has been proven to be more efficient and has become widely used (Collino and Tsogka, 2001). The improved PML absorbing behavior has been developed for Maxwell's equations in Kuzuoglu and Mittra (1996) and Roden and Gedney (2000), named unsplit convolutional perfectly matched layer (CPML). Subsequently, Komatitsch and Martin (2007) and Martin et al. (2008) introduced this technique to seismic wave modeling. In the CPML, it is only necessary to store the memory variables in the absorbing layers, and its cost in terms of memory storage is similar to that of the classical PML (Komatitsch and Martin, 2007). All numerical results reveal that the CPML has better performance than classical PML. In addition, another novel PML named nearly perfectly matched layer (NPML) was introduced in Cummer's paper (Cummer, 2003). This alternative PML also has been shown better performance in removing the reflections from artificial model boundaries than classical PML. Some authors further provided the details of the valuable advantages of the NPML over classical PML (Hu and Cummer, 2004, 2006; Bérenger, 2004; Hu et al., 2007). The NPML does not modify the original form of the governing equations in any linear media, and the

NPML uses fewer auxiliary variables and fewer extra ordinary differential equations (Hu et al., 2007). Some authors conclude the NPML has similar advantages as the CPML, and can be a valuable alternative to other PML implementations (Bérenger, 2004; Hu et al., 2007).

As mentioned above, CPML and NPML as excellent absorbers both have better absorbing ability, but were introduced independently. Currently, we can not find any published papers or reports discussing them together for seismic modeling applications. A comparison between CPML and NPML would very helpful to understand the differences of their absorbing performances. In this paper, we will provide this kind of comparison using 2D acoustic models.

Numerical modeling with the finite-difference method has been used to simulate wave propagation in complex media (Hassanzadeh, 1991; Dai et al. 1995; Carcione, 1996). In this paper, we apply staggered-grid finite-difference operator due to its higher accuracy (Faria and Stoffa, 1994; Carcione, 1998; Moczo et al., 2000; Zeng and Liu, 2001; Mittet, 2002; Sheen et al., 2006). For the implementation of the staggered-grid finite-difference scheme, fourth-order accuracy method in space and second-order accuracy method in time are used.

We must recognize that the 2D acoustic model used in our research is simple and straightforward, but the comparison between CPML and NPML is very interesting topic and we do not want to introduce media differences. In future studies, we will conduct further discussion and analysis for complex media.

GOVERNING EQUATIONS

The governing equations describing acoustic wave propagation are introduced by (Fokkema and van den Berg, 1993). They are written in 2D first-order pressure-velocity form in time domain by

$$\partial P/\partial t = -\rho V^2[(\partial v_x/\partial x) + (\partial v_z/\partial z)] \quad , \tag{1}$$

$$\partial v_x/\partial t = -(1/\rho)(\partial P/\partial x)] \quad , \tag{2}$$

$$\partial v_z/\partial t = -(1/\rho)(\partial P/\partial z)] \quad , \tag{3}$$

where P is the acoustic pressure (Pa), V is the acoustic velocity (m/s), and ρ denotes the volume density of mass (kg/m^3). In addition, v_x and v_z are the acoustic velocity components along x- and y-coordinates, respectively.

THE IMPLEMENTATION OF CPML

Komatitsch and Martin (2007) and Martin et al. (2008) introduced the key idea of the CPML (Roden and Gedney, 2000) to a more general choice for s_x (along the x-coordinate) in seismic modeling by introducing a real variable $k_x \geq 0$ and $\alpha_x \geq 0$, so we get

$$s_x = k_x + d_x/(\alpha_x + iw) \quad , \tag{4}$$

where d_x is the PML decay factor along the x-direction, $i^2 = -1$, and w is the angular frequency. According to Collino and Tsogka (2001), for the PML decay factor, we select $d_x = d_0(x/L)^N$, where L is the width of the absorbing layers, and d_0 is a function of the theoretical reflection coefficient R_c and is expressed by $d_0 = -(N + 1)V\log(R_c)/(2L)$, where V is the acoustic velocity and N is a integer parameter. In the particular case of $k_x = 1$ and $\alpha_x = 0$ in eq. (4), we get the classical PML form of the coordinate transformation. Using some simple mathematical algorithms and combining the recursive convolution method (Luebbers and Hunsberger, 1992), the generalized form can be carried out by introducing a memory variable ψ_x updated at each time step according to the following expression

$$\psi_x^{n+1} = b_x\psi_x^n + a_x(\partial_x)^{n+1/2} \quad , \tag{5}$$

where ∂_x denotes the spatial derivative to x, $b_x = e^{-(d_x/k_x+\alpha_x)\Delta t}$ and $a_x = [d_x/k_x(d_x + k_x\alpha_x)](b_x - 1)$. Then, in the implementation of staggered-grid finite-difference code without PML, we only replace the spatial derivative ∂_x with $(\partial_x/k_x) + \psi_x$ and update ψ_x in time according to the expression (5).

THE IMPLEMENTATION OF NPML

Hu et al., (2007) introduced the NPML method in acoustic wave modeling. Here, we give the detail implementation of NPML in acoustic media. According to Cummer's paper (Cummer, 2003) and combining the particular case of eq. (4), $s_x = 1 + (d_x/iw)$, we are able to use four auxiliary variables and equations to rewrite the eqs. (1), (2) and (3) as the following formulations,

$$\partial P/\partial t = -\rho V^2[(\partial\xi_{vx,x}/\partial x) + (\partial\xi_{vz,z}/\partial z) \quad , \tag{6}$$

$$\partial v_x/\partial t = -(1/\rho)(\partial\xi_{P,x}/\partial x) \quad , \tag{7}$$

$$\partial v_z/\partial t = -(1/\rho)(\partial\xi_{P,z}/\partial z) \quad , \tag{8}$$

where $\xi_{vx,x}$, $\xi_{vz,z}$, $\xi_{P,x}$, and $\xi_{P,z}$ are auxiliary variables, the corresponding four auxiliary expressions are $\xi_{vx,x} = v_x/s_x$, $\xi_{vz,z} = v_z/s_z$, $\xi_{P,x} = P/s_x$, and $\xi_{P,z} = P/s_z$. We can redefine these expressions above from frequency domain to time domain by using the inverse Fourier transform,

$$(\partial\xi_{vx,x}/\partial t) + d_x\xi_{vx,x} = \partial v_z/\partial t \quad , \tag{9}$$

$$(\partial\xi_{vz,z}/\partial t) + d_x\xi_{vz,z} = \partial v_z/\partial t \quad , \tag{10}$$

$$(\partial\xi_{P,x}/\partial t) + d_x\xi_{P,x} = \partial P/\partial t \quad , \tag{11}$$

$$(\partial\xi_{P,z}/\partial t) + d_x\xi_{P,z} = \partial P/\partial t \quad . \tag{12}$$

So far we have obtained all the needed eqs. (6)-(12) in 2D acoustic media. It is very easy to use the staggered-grid finite-difference method.

NUMERICAL TEST

We consider a 2D acoustic model of size 4000 × 4000 m. This model is discretized using a grid comprising of $N_x \times N_z = 400 \times 400$ nodes. The spatial grid spacings Δx and Δz are selected by the same value 10 m, whereas the time step is 1 ms. The acoustic media is homogeneous, and has the acoustic velocity of 3300 m/s, and a density of 2800 kg/m^3. The implementation of staggered-grid finite-difference meets the Courant-Friedrichs-Lewy stability condition (Komatitsch and Martin, 2007). The point source is a velocity vector oriented at 135° in the (x,z) plane, and is located at the grid position (150,150) of the model (Fig. 1). This selected point source is the first derivative of a Gaussian (Yao and Margrave, 2000) of dominant frequency $f_0 = 25$ Hz shifted by $t_0 = 1.2/f_0 = 0.048$ seconds from time $t = 0$ to have null initial conditions. For checking the absorbing ability of CPML and NPML layers, we record the seismograms by x- and z-components of the velocity vectors at two receivers (R_1 and R_2) in our model, respectively. The interior edge of the absorbing region we call the first PML layer. The two receivers are placed at the grid points (20,170) and (380, 375) close to the PML layers (Fig. 1). Thus it is very easy to find the strongly spurious reflections from the PML layers if the PML layers are not working well. In Fig. 1, the black cross indicates the source S, the two black-filled triangles indicate the receivers R_1 and R_2, respectively.

In modeling tests, we set the same width of CPML and NPML layers at 100 m, i.e., 10 grids. Following Komatitsch and Martin (2007), we take N=2, $R_c = 0.1\%$, $\alpha_{max} = \pi f_0$ and $k_x = k_z = 1$ in the implementation of CPML, and $N = 2$, $R_c = 0.1\%$ in the implementation of NPML, respectively. Fig. 2 illustrates the snapshots of the acoustic wave propagation at 0.4 s, 0.6 s, 0.8 s and 1.0 s by using CPML. Fig. 2a represents the x-component of the velocity vectors v_x, and Fig. 2b represents the z-component of the velocity vectors v_z.

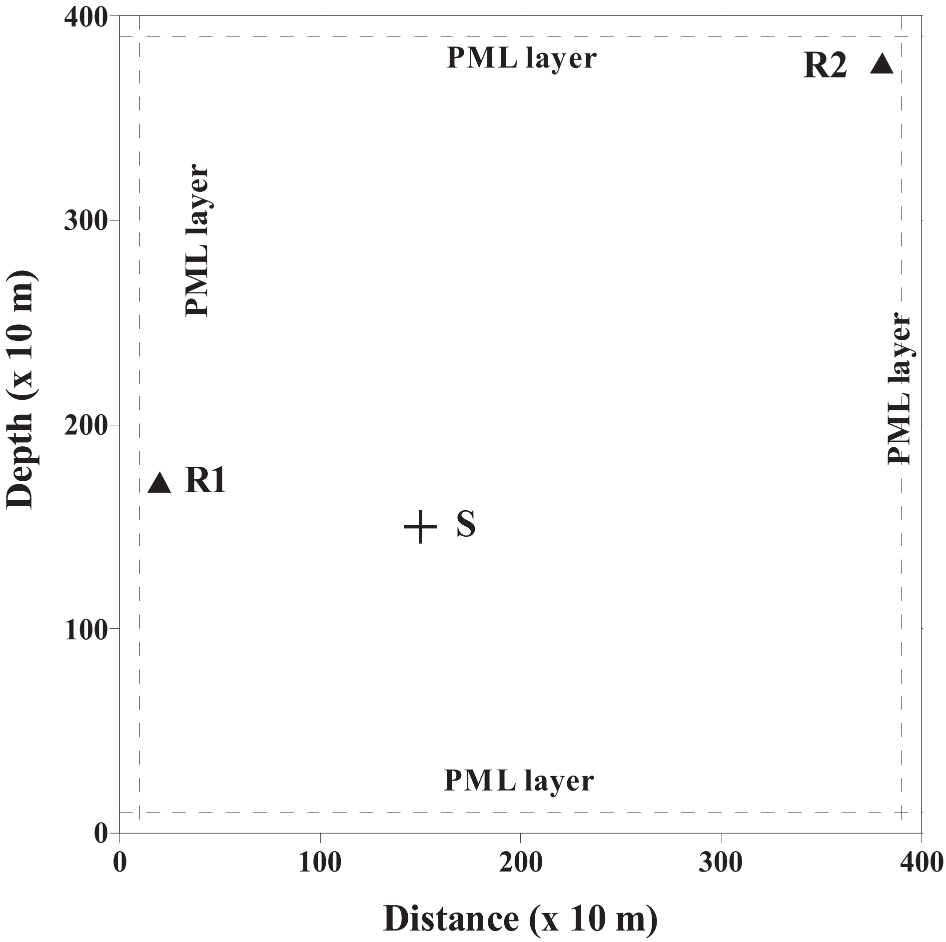

Fig. 1. The configuration of the 2D acoustic model of size 4000 × 4000 m. The black cross indicates source; the two black-filled triangles indicate receivers R_1 and R_2, respectively; the dashed lines represent PML layers.

Fig. 3 shows the same snapshots of the acoustic wave propagation with NPML as Fig. 2. No spurious waves of significant amplitude are visible along all snapshots in Fig. 2 and 3. These snapshots indicate that NPML has the same absorbing efficiency as CPML.

For further comparison, we study the decay of energy with time. Fig. 4a represents the time decay of total energy (0-1.6 s): $E = \rho|v|^2$ in the main domain without PML layers. The energy carried by the acoustic wave is gradually absorbed (theoretically beyond 0.42 s) in the PML layers. After approximately 1.07 s, theoretically there should remain no energy in the model

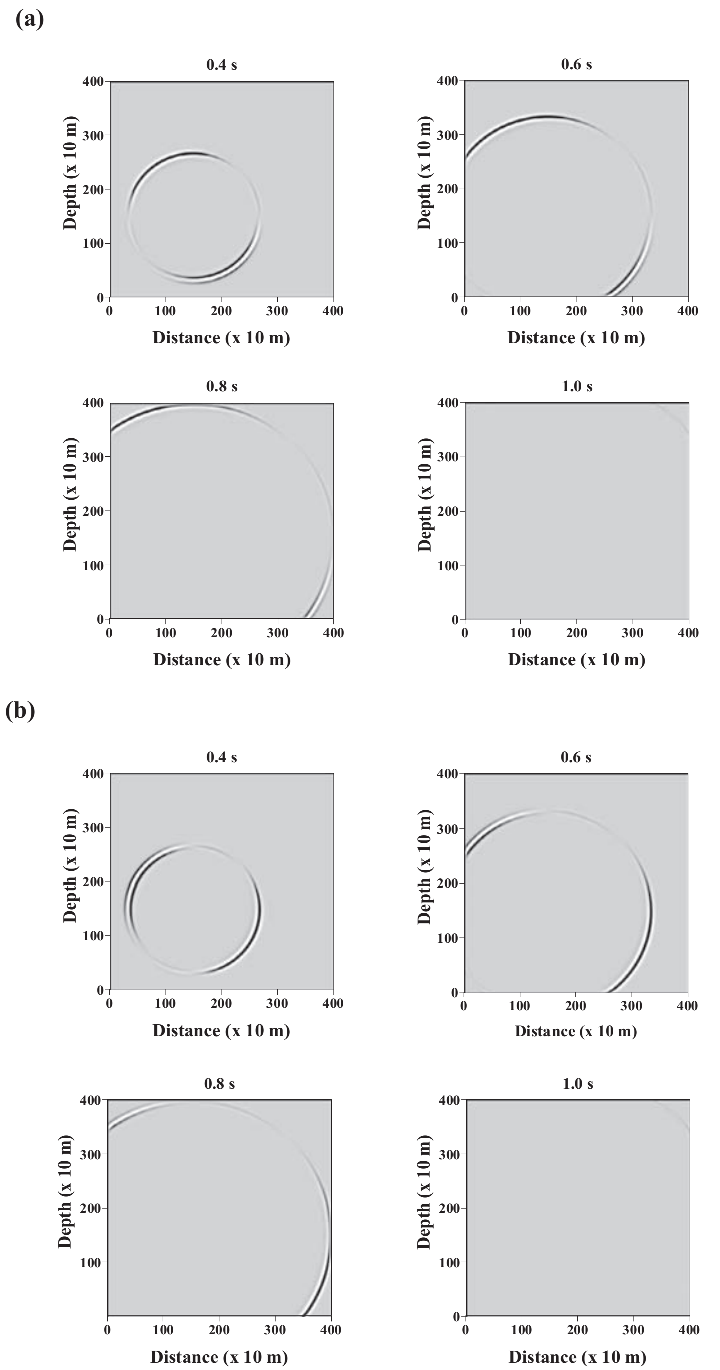

Fig. 2. The snapshots of the acoustic wave propagation with CPML at 0.4 s, 0.6 s, 0.8 s and 1.0 s. (a) indicates the x-component of the velocity vectors v_x; (b) indicates the z-component of the velocity vectors v_z.

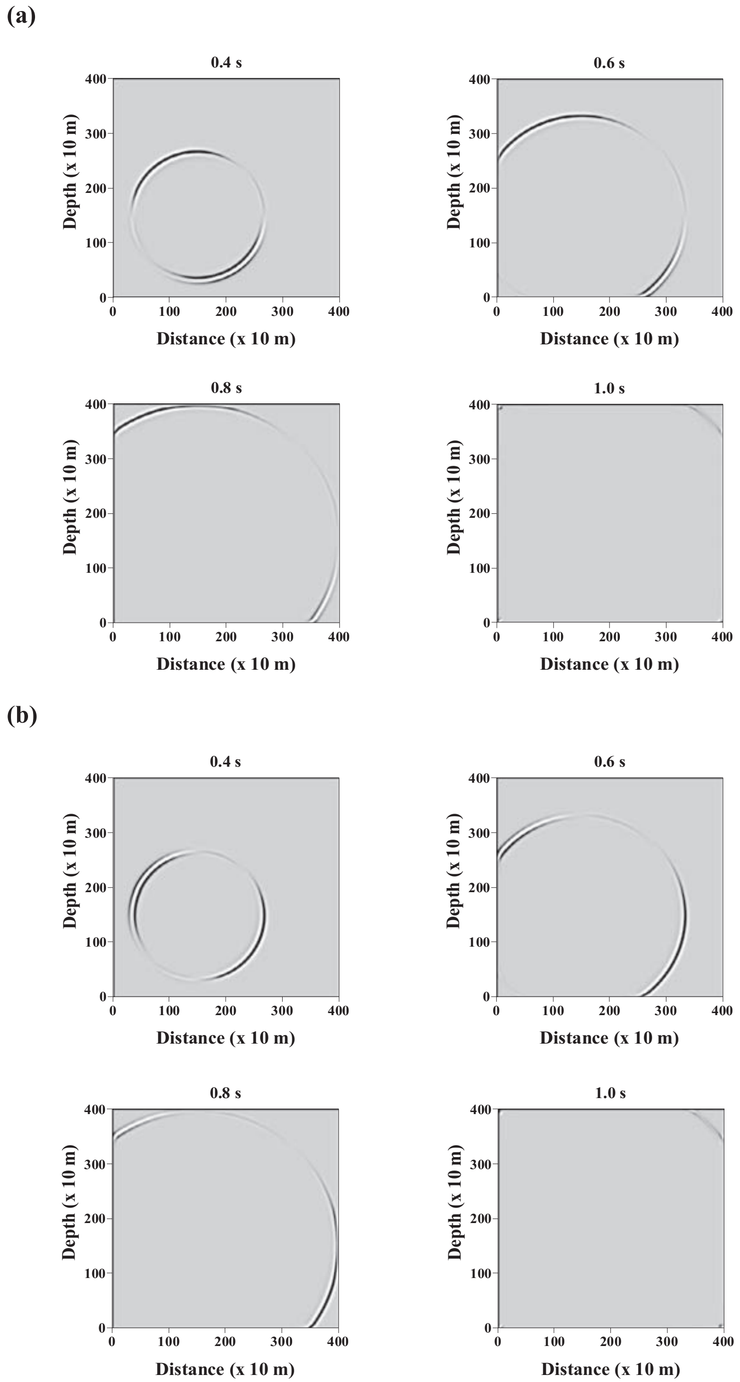

Fig. 3. The snapshots of the acoustic wave propagation with NPML at 0.4 s, 0.6 s, 0.8 s and 1.0 s. (a) indicates the x-component of the velocity vectors v_x; (b) indicates the z-component of the velocity vectors v_z.

because the acoustic wave has left the main domain. All the energy that remains is therefore spurious. We can observe that the total energy computed with CPML and NPML is close to zero beyond 1.07 s in Fig. 4a. One important observation is that the decay of total energy computed with CPML and NPML is amazingly agreement, we almost cannot find any difference between them. However, we can observe that the absorbing performance of NPML is better than CPML through the zoomed-in Fig. 4b, the value of the total energy with NPML is smaller than the value with CPML beyond 0.9 s. We must recognize the difference of the decay of the total energy between CPML and NPML might be ignored. These algorithms use floating point arithmetic and the scale of the number ranges for these relatively quiescent computations could create numerical discontinuities. These small differences could also be a by-product of the algorithm implemention. These numerical concerns need further study to find the root causes.

Fig. 5 illustrates the seismograms of v_x and v_z components of velocity vectors at receiver R_1 (a) and receiver R_2 (b), respectively. The agreement between CPML and NPML is also excellent satisfactory, where receivers are both placed close to PML layers. However, from the two seismograms of v_z-components, we can observe that the NPML absorbing performance (dashed lines) is better than CPML (solid lines). There exists the spurious energy in the v_z-seismograms with CPML. These algorithms use floating point arithmetic and the scale of the number ranges for these relatively quiescent computations could create numerical discontinuities. These small differences could also be a by-product of the algorithm implementation. These numerical concerns need further study to find the root causes. In considering our simple 2D acoustic model, further analyses of the absorbing abilities of CPML and NPML in complex media are needed.

CONCLUSIONS

In this paper, we provide the numerical comparison between CPML and NPML techniques through 2D acoustic modeling study. The testing results indicate there remains excellent agreement with their absorbing performances in removing the reflections from model boundaries. Given the success of this comparison we will pursue in complex media analysis in future efforts.

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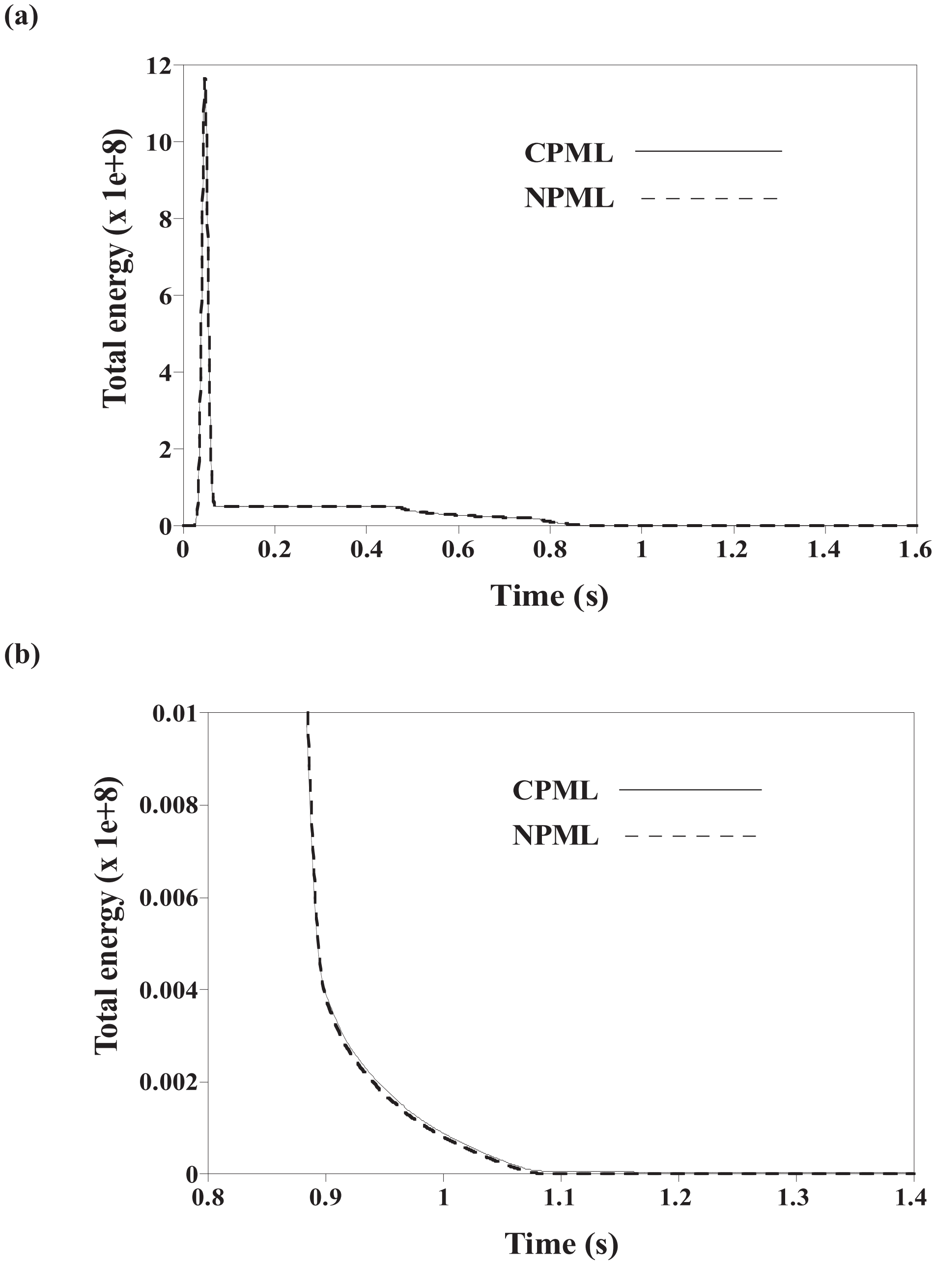

Fig. 4. Decay of the total energy with time in the domain without the PML layers. (a) the decay of the total energy from 0 to 1.6 s; (b) the zoomed-in observation from 0.8 s to 1.4 s. The solid lines indicate the energy computation with CPML, and the dashed lines indicate the energy computation with NPML.

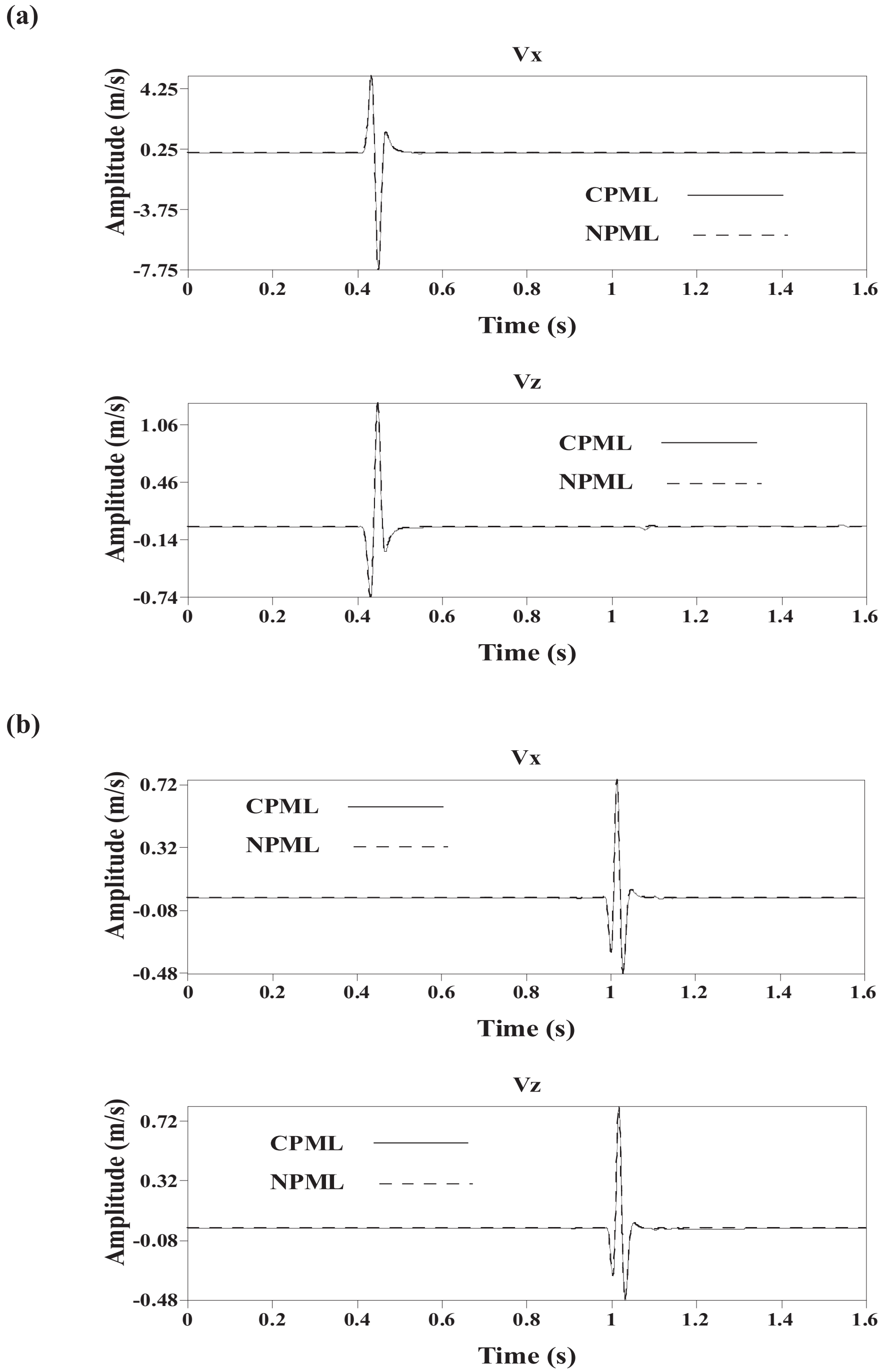

Fig. 5. The seismograms of v_x and v_z at the two receivers R_1 and R_2. The solid lines indicate the seismograms recorded with CPML, and the dashed lines indicate the seismograms recorded with NPML.

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