JOINT INVERSION OF SEISMIC REFLECTION TRAVELTIMES AND WAVE POLARIZATIONS FOR ANISOTROPIC PARAMETERS USING SIMULATED ANNEALING: A MODELING STUDY

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ABSTRACT

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It is very important to utilize as much information as possible to constrain the solution due to the non-unique solutions of seismic inversion. In this paper, a joint inversion scheme of seismic reflection traveltime and wave polarization data is proposed to determine three of the five anisotropic parameters (vertical P-wave velocity v_{po} and Thomsen's parameters ε and δ) in transversely isotropic media with a vertical symmetry axis (VTI). The shooting ray tracing method is applied to the forward model to obtain the seismic reflection traveltimes and polarizations. The numerical tests demonstrate that the joint inversion method can provide a better constrain to the anisotropic parameters than other inversions using single dataset (traveltime or polarization).

KEY WORDS: P-wave traveltimes, polarization angles, joint inversion, anisotropy, simulated annealing.

INTRODUCTION

Various inversion methods have been proposed to estimate seismic velocities in isotropic media (Lutter and Nowack, 1990; Zelt and Smith, 1992; Hu and William, 1992; Hu et al., 1993; Pullammanappallil and Louie, 1993, 1994, 1997). Among the studies cited above, either traveltime or polarization of P-wave data are usually used. Over the last thirty years, seismic anisotropy

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has attracted increasing attention, and plays an important role in the inversion of exploration geophysics and seismology (Zhang et al., 2003; Al-Lazki, 2003). Joint inversion methods of traveltimes and polarizations have also been introduced (Horne and Leaney, 2000; Chen et al., 2006, 2009; Farra and Bégat, 1995), but the techniques have not attracted much more attention. Full-waveform inversions using genetic algorithm (GA) have been applied to seismic data (Ji et al., 2000; Yamanaka, 2001), but the techniques are not as fully exploited as they deserve to be. Yamanaka (2001) also provides the comparison between simulated annealing (SA) and GA, and finds the results from SA inversion have less error than those from GA inversion.

Chen et al. (2006, 2009) presented joint inversion methods of seismic reflection traveltimes and polarizations. However, only a flat-layered model was used. In this study, a joint inversion scheme is proposed, which uses both reflection traveltime and polarization of P-wave data to determine three of the five anisotropic parameters (vertical P-wave velocity v_{po} and Thomsen's parameters ε and δ) in dipping layered transversely isotropic media with a vertical symmetry axis (VTI). The other two parameters [vertical S-wave velocity v_{so} and Thomsen parameter γ (Thomsen, 1986)] require S-wave data (Tsvankin, 1996, 2001).

In our model, each layer interface is specified by an arbitrary number and spacing of boundary nodepoints connected by polynomial interpolation. It is assumed that the layer interfaces must cross the model from left to right side without crossing other boundaries. In the cases where seismic ray cannot reach the observation point due to the limitation of shooting ray tracing (Landa et al., 1988; Kim et al., 2006), the cubic spline interpolation is used to obtain reflection traveltime and polarization data at observation stations. The proposed inversion scheme is based on the layer-stripping simulated annealing (SA) process - a Monte Carlo which is a nonlinear global optimization scheme (Kirkpatrick et al., 1983; Rothman, 1985) to solve inverse problems. Prior to determining the unknown parameters in the k-th layer, all parameters above the k-th layer should have been determined (k = 1,...,N, N is the total layers number). The application of this scheme to synthetic data shows the proposed joint inversion can provide reliable results comparing with other inversion methods based on only traveltime or polarization data.

FORWARD COMPUTATION

The real crust is assumed to be adequately modeled as a series of anisotropic homogeneous layers separated by interfaces. Let X be the horizontal distance along the earth's surface, Z the depth, and N the number of layers (Fig. 1). The vertical P- and SV-wave velocities v_{po} and v_{so}, and Thomsen parameters ε and δ are assumed to be constant in each layer and the interfaces

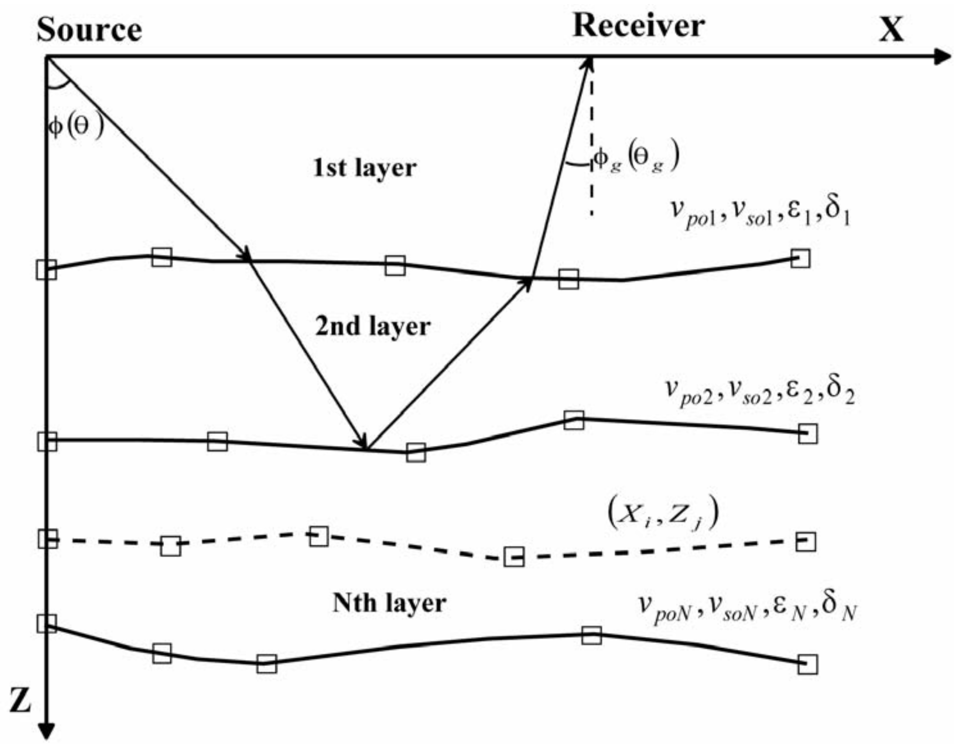

Fig. 1. The parameterization of the model with N layers.

are represented by the polynomials which are determined by M_o nodepoints $Z_n(X_1), Z_n(X_2), \ldots, Z_n(X_{Mo})$, n = 1,...,N. For simplicity in the following discussion, the number of nodepoints is the same for each layer although this is not essential.

The shooting ray tracing method is used to compute seismic reflection traveltimes and polarization angles. Chen et al. (2009) gives the details for the case of flat interfaces. When the seismic ray hits the reflection interface, the seismic ray incidence, reflection and transmission obey Snell's law (Fig. 2).

$$[\sin(\theta_1 \pm \alpha)]/v_{p1}(\theta_1) = [\sin(\theta_2 \pm \alpha)]/v_{p2}(\theta_2)$$

$$= [\sin(\theta_3 \mp \alpha)]/v_{p1}(\theta_3) = p \quad . \tag{1}$$

θ_1, θ_2, and θ_3 are P-wave phase angles of the incidence, transmission and reflection at the interface, respectively. v_{p1} and v_{p2} are the P-wave phase velocities of the upper and lower layers separated by the interface. α denotes the dip angle of the interface.

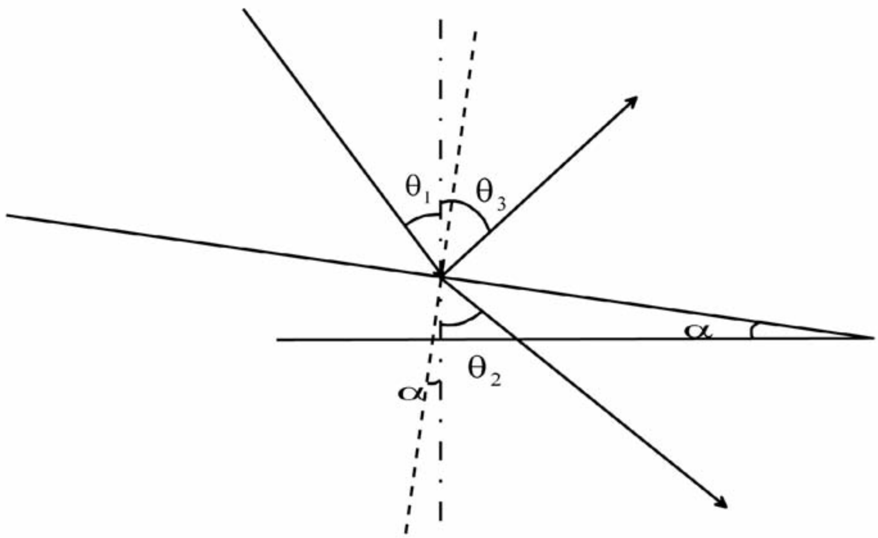

Fig. 2. The schematic illustration of the Snell's law at the dipping interface.

During the computation of the seismic reflection traveltimes and polarization angles, the group (energy) velocity of the seismic waves (Thomsen, 1986) arriving from the k-th layer is described by V_{pk},

$$V_{pk}^2[\phi_k(\theta_k)] = v_{pk}^2(\theta_k) + (dv_{pk}/d\theta)^2 \ , \tag{2}$$

$$v_{pk}(\theta_k) = v_{pok}[1 + \delta_k \sin^2\theta_k + (\varepsilon_k - \delta_k)\sin^4\theta_k] \ , \tag{3}$$

$$\tan[\phi_k(\theta_k)] = [\tan\theta_k + (1/v_{pk})(dv_{pk}/d\theta_k)] \\ / \ [1 - (\tan\theta_k/v_{pk})(dv_{pk}/d\theta_k)] \ , \tag{4}$$

where V_{pk}, ϕ_k and θ_k are the P-wave group velocity, the group angle and the phase angle associated to the ray arriving from the k-th layer, respectively; v_{pk} is the P-wave phase velocity associated to the k-th layer; v_{pok} is the vertical P-wave velocity in the k-th layer; ε_k and δ_k are the Thomsen's parameters for the k-th layer.

The expression of the P-wave polarization angle ψ_{pk} (Rommel, 1994) associated with phase angle θ_k is given by

$$\psi_{pk} = \theta_k + B[\delta_k + 2(\varepsilon_k - \delta_k)\sin^2\theta_k]\sin 2\theta_k \ , \tag{5}$$

where $B = 1/[2(1 - v_{sok}^2/v_{pok}^2)]$, and v_{sok} is the vertical SV-wave velocity associated to the k-th layer.

The shooting ray tracing method introduced in Chen et al. (2009) is implemented to compute the P-wave reflection traveltimes and polarization angles at the i-th receiver with offset x_i. The cubic spline interpolation is used to obtain reflection traveltime and polarization data at observation stations where seismic ray cannot reach (Landa et al., 1988; Kim et al., 2006). Fig. 3 is an example of the interpolation approach for reflection traveltimes from the second reflector in a two-layered model (Revised from Landa et al., 1988). In Fig. 3a, the dashed lines represent corresponding traveltime curves; the crosses '+' represent the exact solution obtained by shooting ray tracing, and the black-filled triangles are the value obtained from a cubic spline interpolation. The black-filled circle and the triangle in Fig. 3b represent the source and the observation station, respectively.

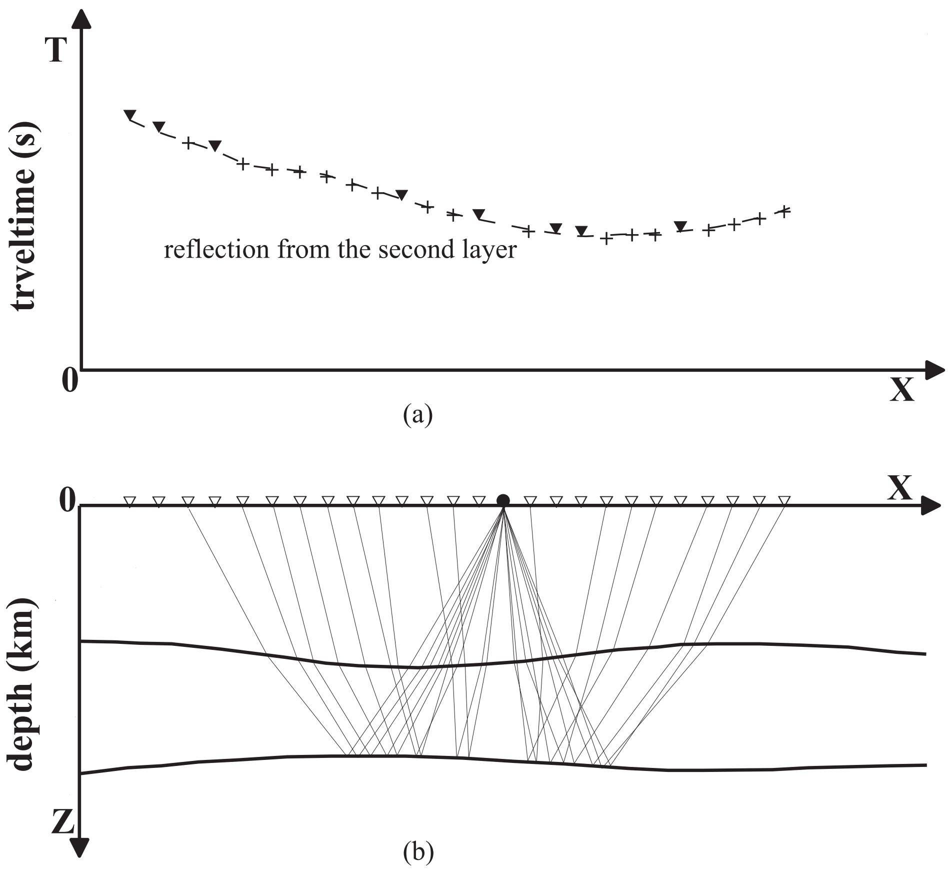

Fig. 3. (a) Reflection traveltime curves (dashed line) from the second reflector. The crosses denote the exact solution, and the black-filled triangles represent the solution obtained from the interpolation solution. (b) Two-layer model and ray paths from the second layer. The black-filled circle denotes the sources, and the triangles denote receivers. (Revised from Landa et al., 1988).

SIMULATED ANNEALING INVERSION

There are M receivers on the free surface and the recorded seismic signals are from layered interfaces beneath the surface (Fig. 1). The seismic reflection traveltime and polarization from the j-th reflector, recorded by the i-th receiver are denoted by $t_{i,j}$ and $\psi_{i,j}$, respectively. The number of the measurements for the joint inversion of traveltimes and polarizations will therefore be $M \times N$. The objective of the joint inversion is to find the optimization solutions to obtain vertical P-wave velocity, and anisotropic parameters until calculated reflection traveltimes and polarizations match the measured values. Our goal is to minimize the following objective function

$$E_j = \sum_{i=1}^{M} [(t_{i,j}^{obs} - t_{i,j}^{cal})/t_{i,j}^{obs}]^2 + \sum_{i=1}^{M} [(\psi_{i,j}^{obs} - \psi_{i,j}^{cal})/\psi_{i,j}^{obs}]^2 \quad , \tag{6}$$

where i denotes i-th receiver, j represents j-th reflector, t^{obs} and t^{cal} are the observed and calculated reflection traveltimes, ψ^{obs} and ψ^{cal} are the observed and calculated polarizations.

The standard simulated annealing (SA) process is used to search unknown parameters for the global optimization (Rothman, 1985). The SA is described in details in above mentioned paper. Here, only the decreasing rate of the temperature and stopping criterion are introduced (Chen et al., 2006).

DECREASING RATE OF TEMPERATURE

During the inversion procedure, the maximum number of iterations is set to 2000, and the objective function is computed at each initial temperature T_k. The same iteration process is calculated at a new temperature T_{k+1} with the following simple rule:

$$T_{k+1} = 0.5 \times T_k \quad , \quad k = 0,1,2,\ldots \ . \tag{7}$$

STOPPING CRITERION

When the difference in the least-square error between successive models and the probability of accepting new models becomes very small (e.g., 0.00001), or the number of iterations reach a given large number of iterations (e.g., 100,000), the simulated annealing procedure stops.

TEST WITH SYNTHETIC DATA

A crustal model with four homogeneous VTI layers with a size of 40×100 km is introduced in the model tests. The seismic source at the surface is located at the origin of the coordinate system and the receivers are regularly spaced from 2 to 100 km with a spacing of 2 km. The parameters (nodes) of the interface structure and the physical variables (i.e., vertical P- and SV-wave velocities and Thomsen anisotropic parameters, ε and δ) in the model are demonstrated by Table 1 and Table 2, respectively. For simplicity, only six boundary nodepoints (node1-node6) with 20 km space starting from zero are used to describe each interface which is connected by polynomial interpolation.

Table 1. The boundary nodepoints of the true model with four layers.

Layer number	node1(km)	node2(km)	node3(km)	node4(km)	node5(km)	node6(km)
1st layer	10.0	11.0	10.2	9.2	10.0	10.8
2nd layer	20.0	19.5	20.2	20.5	19.5	19.7
3rd layer	30.0	30.5	30.2	29.2	30.5	31.0
4th layer	40.0	40.0	40.0	40.0	40.0	40.0

Table 2. The physical parameters of the model used in Table 1.

Layer number	v_{po} (km/s)	v_{so} (km/s)	ε	δ
1st layer	3.368	1.829	0.110	-0.035
2nd layer	4.100	2.412	-0.003	0.020
3rd layer	5.200	3.074	0.089	0.040
4th layer	6.500	3.752	-0.026	-0.033

Notations: ε, δ, Thomsen anisotropic parameters.

After the forward computation using the shooting ray tracing technique, the reflection traveltimes and polarization angles as a function of the offset were obtained. Fig. 4 (a) and (b) represent the seismic reflection traveltimes and polarization angles curves for four layers, and Fig. 4(c) denotes the ray paths.

For the implementation of joint inversion, the reflection traveltime and polarization data are first reconstructed by adding Gaussian noise with the standard deviation of ± 0.1 s and $\pm 0.1°$, respectively (case 1). All layers are initially assumed to be isotropic, i.e., $\varepsilon_0 = \delta_0 = 0$, and the initial temperature

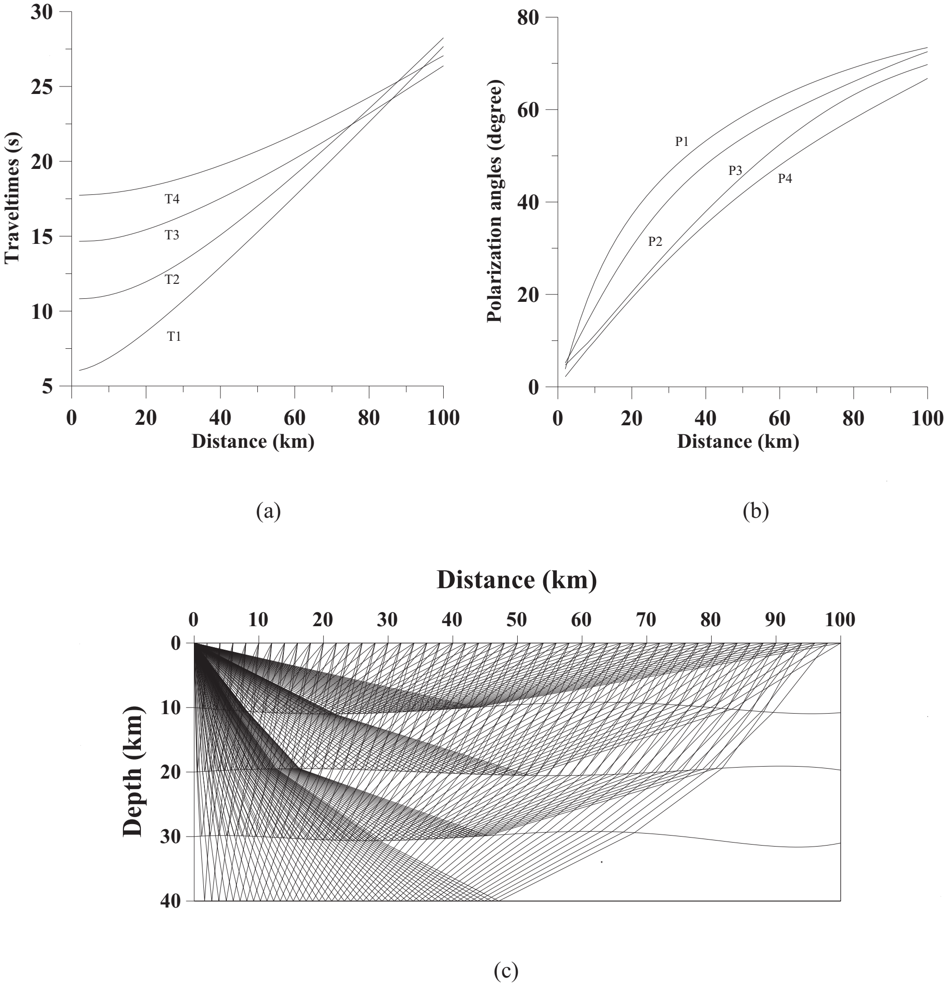

Fig. 4. (a) and (b) are reflection traveltimes and polarization angles curves with four layers as a function of the offset, respectively, (c) denotes seismic ray paths after forward computation.

$T_0 = 15°$. The average relative errors of the vertical P-wave velocities between the initial model and real model are 10%. The largest relative error of the vertical P-wave velocities is 12% for the 3rd layer. At the initial temperature, the number of iterations was set at 2000, then decreased the temperature using the rule given in eq. (7) and calculated the objective functions for a total of 2000 iterations at each temperature. The temperature-decreasing curve with number of iterations is shown in Fig. 5. When the difference in the least-square error between successive models and the probability of accepting new models becomes very small, the simulated annealing process stops. Fig. 6 shows the optimal objective function (equation (6)) varies with temperature in the first layer, indicating that the solutions become stable when the temperature decreases to 7. Fig. 7 illustrates the inversion results of vertical P-wave velocity, Thomsen's parameters (ε and δ) using the proposed joint inversion and other single inversions (anisotropy traveltime, polarization), respectively. The results indicate that the joint inversion of traveltimes and polarizations can provide much better constrain to the anisotropic parameters than other two inversion methods.

To further test our joint inversion scheme, the reflection traveltime and polarization data are reconstructed again by adding Gaussian noise with the standard deviation of ± 0.2 s and $\pm 0.2°$, respectively (case 2). The initial isotropic model, initial temperature and number of iterations are selected same values as those in case 1. Fig. 8 shows the optimal objective function varies with temperature in the first layer. Fig. 9 compares the inversion results of vertical P-wave velocity, Thomsen's parameters (ε and δ) using the proposed joint inversion with other single inversions (anisotropy traveltime, polarization).

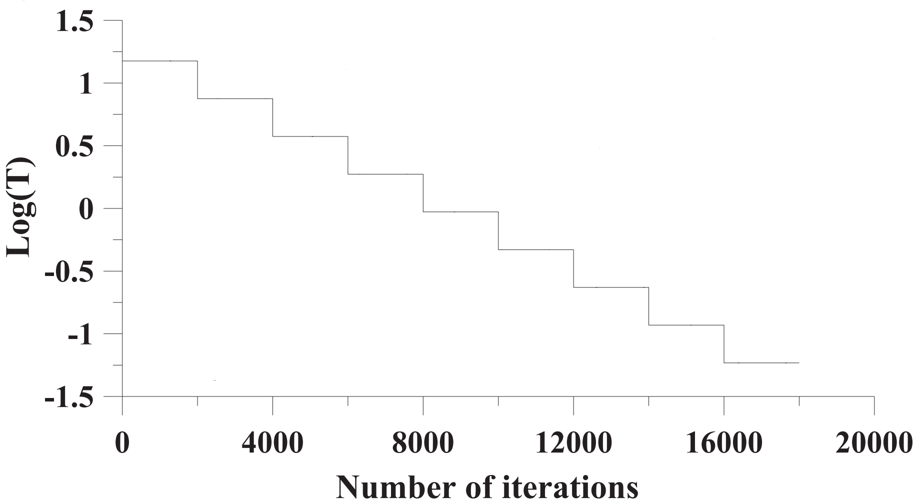

Fig. 5. Variation of annealing temperature with number of iterations. The initial temperature is set to be 15°.

Table 3 shows the comparison between the final model obtained from case 2 and the true model. The results again indicate that the joint inversion can provide a favorable constrain to the anisotropic parameters, also are demonstrated our inversion algorithm is quite effective and robust even at high noise level.

Table 3. Comparison of true and final models for case 2.

Layer number	v_{po} (km/s)			ε			δ		
	True value	Initial value	Final value	True value	Initial value	Final value	True value	Initial value	Final value
1st layer	3.368	3.000	3.375	0.110	0.000	0.111	-0.035	0.000	-0.034
2nd layer	4.100	4.500	4.108	-0.003	0.000	-0.001	0.020	0.000	0.023
3rd layer	5.200	5.800	5.206	0.089	0.000	0.087	0.040	0.000	0.039
4th layer	6.500	6.000	6.512	-0.026	0.000	-0.022	-0.033	0.000	-0.032

Notations: ε , δ , Thomsen anisotropic parameters.

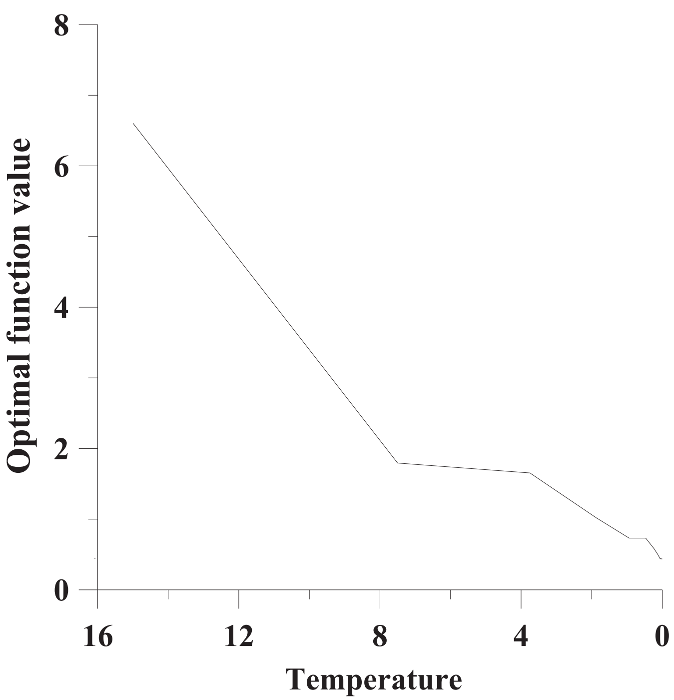

Fig. 6. Variation of the optimal objective function with temperature (case 1) in the first layer.

CONCLUSIONS

A method of joint inversion of seismic reflection traveltimes and wave polarizations for determining 2D anisotropic parameters is presented here. The nonlinear working scheme is implemented layer by layer by applying standard simulated annealing (SA) technique. Based on the successful modeling study, application to real seismic data will be founded in a further study.

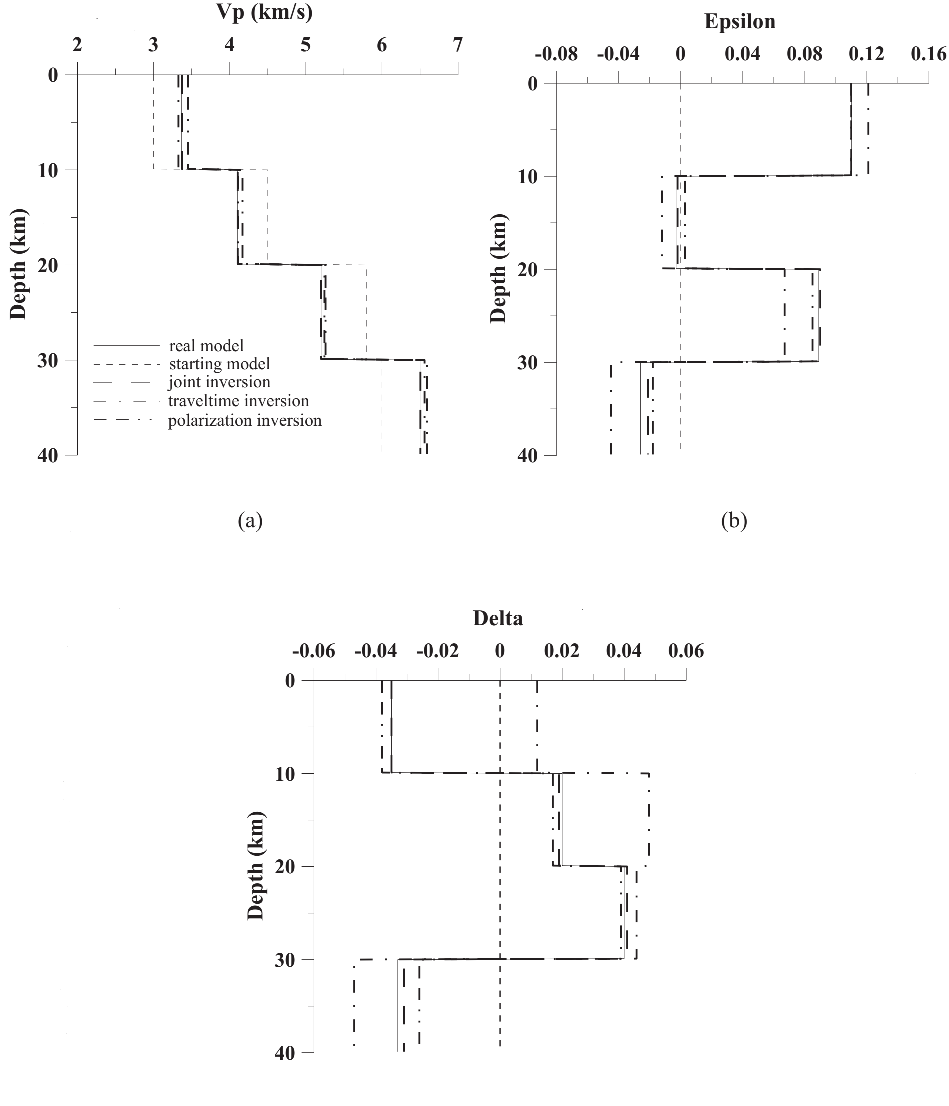

Fig. 7. The vertical P-wave velocity (a), Thomsen parameters (b) and (c) as derived by joint inversion and other single dataset inversions (traveltime, polarization) (case 1).

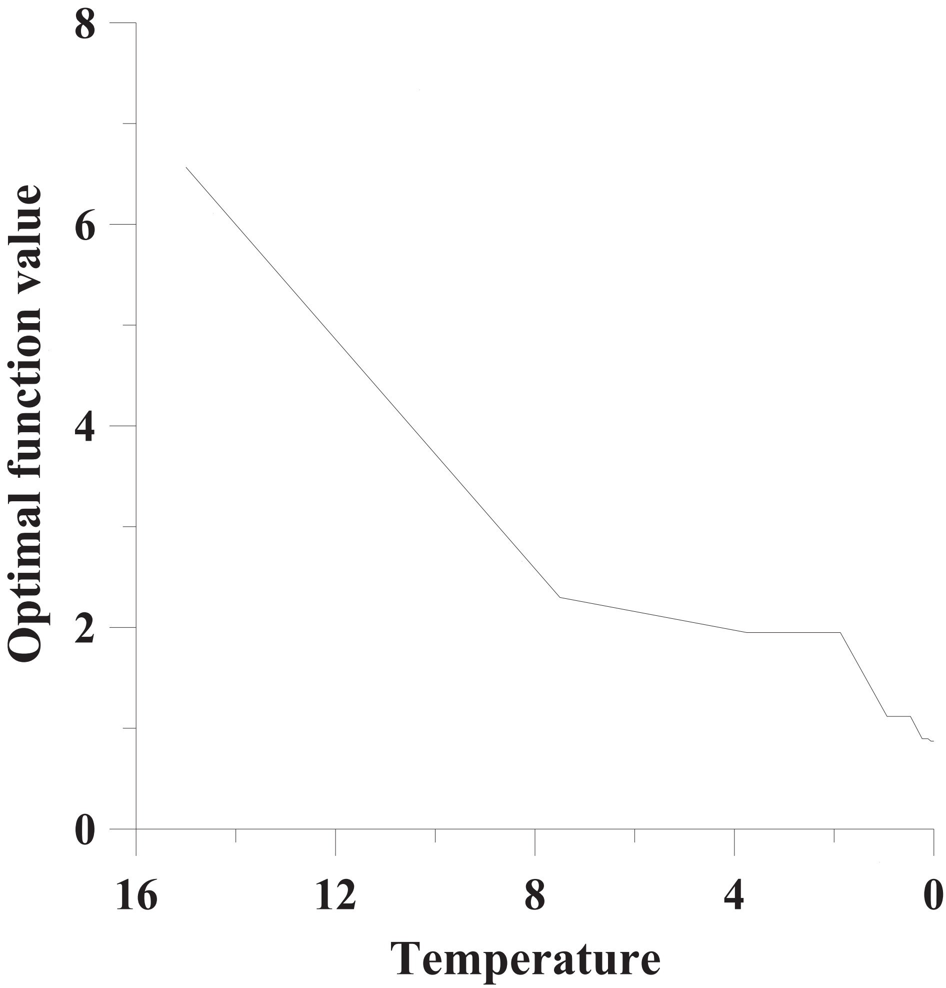

Fig. 8. Variation of the optimal objective function with temperature (case 2) in the first layer.

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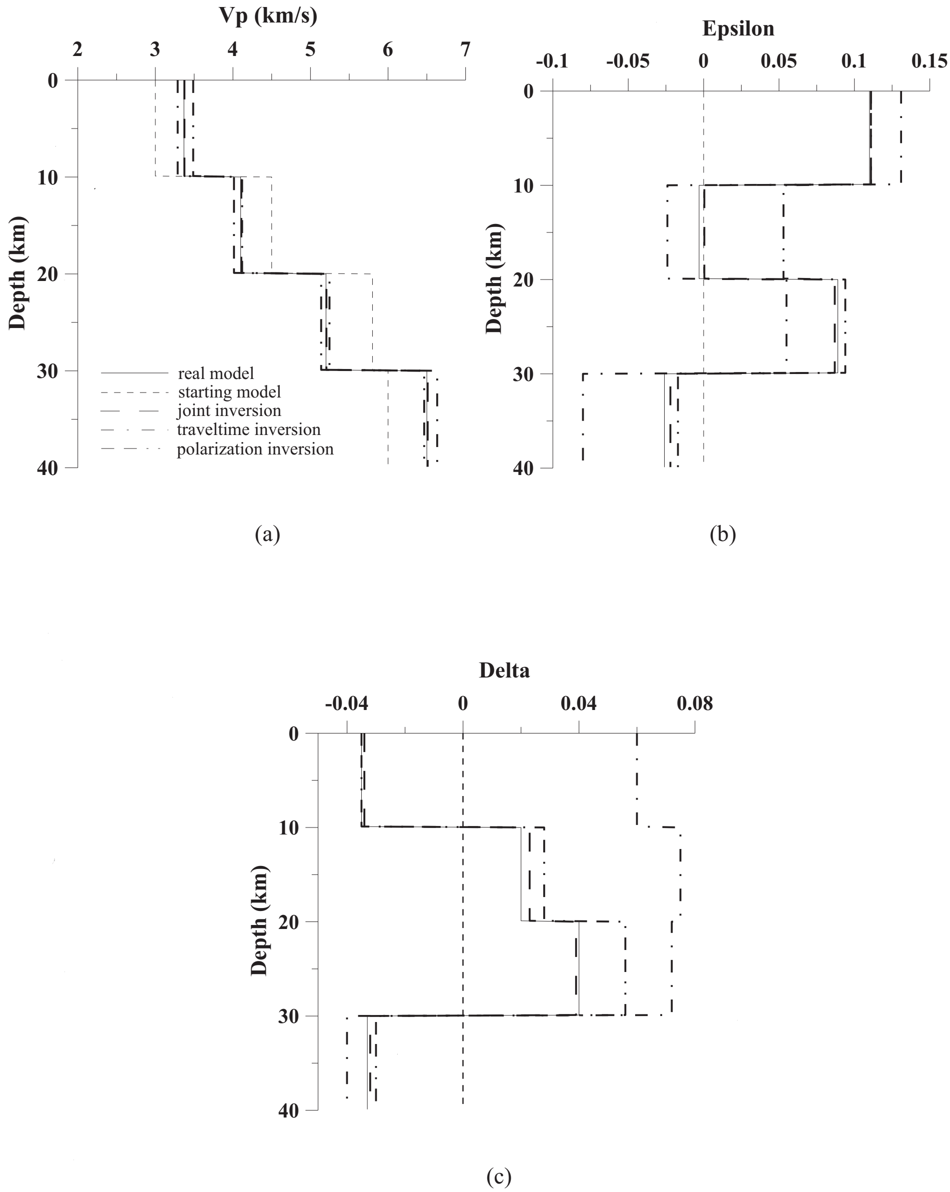

Fig. 9. The vertical P-wave velocity (a), Thomsen parameters (b) and (c) as derived by the joint inversion and other single dataset inversions (traveltime, polarization) (case 2).

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