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REVIEW: COORDINATE TRANSFORMATION OF ACOUSTIC AND MAXWELL'S EQUATIONS: THE MAKING OF ANISOTROPIC MASS DENSITY AND PERMITTIVITY

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ABSTRACT

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It is a remarkable fact that Maxwell's equations under any coordinate transformation can be written in an identical mathematical form as the ones in Cartesian coordinates. However, in some particular coordinate transformations, like the cylindrical coordinate transformations, the physical properties become anisotropic, even if they are isotropic in the Cartesian coordinates. Even the permittivity can be anisotropic. The remarkable invariance of Maxwell's equations under coordinate transformation extend to acoustic wave equations. In other words, the acoustic wave equations are also invariant under any coordinate transformation. However, the mass density can become anisotropic. We here review these fundamental results.

KEY WORDS: Maxwell equations, coordinate transformations, acoustic wave equation, anisotropic mass density, cylindrical coordinates.

INTRODUCTION

Now that the controlled source electromagnetic (CSEM) acquisition technique has taken hold as an oil and gas exploration and production tool, there is a need to develop modeling and inversion methods for analysis CSEM data,

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and even to revamp classical petroleum seismology classes to include electromagnetic methods. These developments will greatly benefit from the significant progress made in the last four decades in seismic modeling and inversion and in the centuries of electromagnetic-wave studies. One important aspect of these developments is understanding the similarities of and differences between Maxwell's equations and elastic field equations are possible. In Ikelle (2010) we describe examples of these equivalences in Cartesian coordinates. The seismology studies are not limited to Cartesian coordinates. For example, in the study of sonic logging and of earthquake sources, we often considered the wave propagation in cylindrical coordinates and even spherical coordinates. We here examine the similarities and differences between Maxwell's equations and acoustic field equations for other coordinate systems. Our formulation is quite general and is valid for any transformation of Cartesian coordinate systems, including transformation from Cartesian coordinates to curvilinear coordinates.

BASIC EQUATIONS OF COORDINATE TRANSFORMATIONS

In this section, we recall some basic formulae of coordinate transformation. We consider two coordinate systems: an "old" system and a "new" system. The position in an old coordinate system is specified by

$$\mathbf{x} = [x_1, x_2, x_3]^T \quad . \tag{1}$$

The symbol T indicates a transpose. In our definitions of elastic and electromagnetic wave equations, the subscript notation for vectors and tensors as well as the Einstein summation convention (also known as a summation over repeated indices) will be used. Lowercase Latin subscripts are employed for this purpose (e.g., v_k, τ_{pq}); they are to be assigned the values 1, 2, and 3. Boldface symbols (e.g., $\mathbf{v}$, $\boldsymbol{\tau}$) will be used to indicate vectors or tensors. The position in the new coordinate system is specified by

$$\mathbf{x}' = [x_1', x_2', x_3']^T \quad , \tag{2}$$

with $\mathbf{x}' = \mathbf{x}'(\mathbf{x})$, or more explicitly, $x_i' = x_i'(x_j)$. We will use the prime and tilde symbols to indicate fields and physical properties in the new coordinate system (e.g., v_k', τ_{pq}', $\tilde{\mu}_0^{(ab)}$ or $\mathbf{v}'$, $\boldsymbol{\tau}'$, $\tilde{\boldsymbol{\mu}}_0$). We assume that the transformation from the old system to a new system [i.e., $\mathbf{x} = \mathbf{x}(\mathbf{x}')$, or more explicitly, $x_i = x_i(x_j')$], is uniquely defined.

Let us define the Jacobian matrix for the coordinate transformation from the old coordinate system to the new one. We denote this Jacobian matrix as $\mathbf{A}$, and its elements are defined as follows:

$$A_{ij} = \partial x_i' / \partial x_j \quad . \tag{3}$$

We assume that the Jacobian matrix is non-singular. The Jacobian matrix of the reciprocal transform is denoted $\mathbf{A}'$, and its elements are

$$A'_{ij} = \partial x_i/\partial x'_j = \partial \ln(\alpha)/\partial A_{ij} \quad . \tag{4}$$

where

$$\alpha = \det(\mathbf{A}) = \epsilon_{ijk}(\partial x'_1/\partial x_i)(\partial x'_2/\partial x_j)(\partial x'_3/\partial x_k) = \epsilon_{ijk}A_{1i}A_{2j}A_{3k} \quad , \tag{5}$$

and where ϵ_{jnr} is the Levi-Civita symbol ($\epsilon_{ijk} = 1$ if ijk is an even permutation, $\epsilon_{ijk} = -1$ if ijk is an odd permutation, and $\epsilon_{ijk} = 0$ otherwise). We also have the classical identities

$$\partial/\partial x_i = (\partial x'_j/\partial x_i)(\partial/\partial x'_j) \quad , \tag{6}$$

$$v_i = (\partial x'_j/\partial x_i)v'_j = A_{ji}v'_j \quad , \tag{7}$$

where v_i and v'_j are components of the vectors $\mathbf{v}$ and $\mathbf{v}'$, respectively.

To add more concreteness to our definitions of Jacobian matrices, let us consider the particular case of a transformation from Cartesian coordinates to cylindrical coordinates. This transformation is defined as follows:

$$\begin{cases} x_1 = r\cos\theta \\ x_2 = r\sin\theta \quad , \\ x_3 = z \end{cases} \tag{8}$$

where x_1, x_2 and x_3 represent the old coordinate system and r, θ and z represent the new coordinate system. The Jacobian matrices $\mathbf{A}$ and $\mathbf{A}'$ for this transformation are

$$\mathbf{A} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -(\sin\theta)/r & (\cos\theta)/r & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{A}' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{9}$$

The determinants of these Jacobian matrices are

$$\alpha = \det(\mathbf{A}) = 1/r \text{ and } \det(\mathbf{A}') = 1/\alpha = r \quad . \tag{10}$$

COORDINATE TRANSFORMATION IN ELECTROMAGNETISM

Let **E** and **B** be the electric field and magnetic field vectors, respectively. We can define them as follows:

$$E_n \Leftrightarrow \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad \text{and} \quad H_k \Leftrightarrow \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} . \tag{11}$$

Using these fields, we can write the Maxwell's equations as follows [e.g., de Hoop (1995)]:

$$-\epsilon_{ijk}[\partial H_k(\mathbf{x},t,\mathbf{x}_s)/\partial x_j)] + \epsilon_0^{(il)}(\mathbf{x})[\partial E_l(\mathbf{x},t,\mathbf{x}_s)/\partial t)] = -J_i(\mathbf{x},t,\mathbf{x}_s) , \tag{12}$$

$$\epsilon_{nmp}[\partial E_p(\mathbf{x},t,\mathbf{x}_s)/\partial x_m)] + \mu_0^{(nr)}(\mathbf{x})[\partial H_r(\mathbf{x},t,\mathbf{x}_s)/\partial t)] = -K_n(\mathbf{x},t,\mathbf{x}_s) , \tag{13}$$

where $\epsilon_0^{(il)}(\mathbf{x})$ and $\mu_0^{(nr)}(\mathbf{x})$ are the permittivity and permeability tensors, respectively. These equations are quite general because we have considered that the permittivity and permeability can be anisotropic by describing them as second-rank tensors. The quantities **J** and **K** are the volume density of the material electric current and the volume density of the material magnetic current, respectively. In a vacuum domain, **J** and **K** are zero. The position of these sources is specified by $\mathbf{x}_s$.

Let us now show that eqs. (12) and (13) are invariant under coordinate transformation. We will start by rewriting (12) in the new coordinate system by using the definition in (7) for vectors; i.e.,

$$-\epsilon_{ijk}(\partial/\partial x_j)[(\partial x_b'/\partial x_k)H_b'(\mathbf{x}',t,\mathbf{x}_s)] + \epsilon_0^{(il)}(\mathbf{x})[(\partial x_d'/\partial x_l)E_d'(\mathbf{x}',t,\mathbf{x}_s)/\partial t]$$

$$= -J_i(\mathbf{x},t,\mathbf{x}_s) . \tag{14}$$

After expanding the first term on the lefthand side of (14), we arrive at

$$-\epsilon_{ijk}(\partial^2 x_b'/\partial x_j \partial x_k)H_b'(\mathbf{x}',t,\mathbf{x}_s) - \epsilon_{ijk}(\partial x_b'/\partial x_k)(\partial x_c'/\partial x_j)H_b'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_c'$$

$$+ \epsilon_0^{(il)}(\mathbf{x})[(\partial x_d'/\partial x_l)E_d'(\mathbf{x}',t,\mathbf{x}_s)/\partial t] = -J_i(\mathbf{x},t,\mathbf{x}_s) . \tag{15}$$

Notice that the first term on the lefthand side of (15) is zero. By multiplying the remaining expression by $\partial x_a'/\partial x_i$, we arrive at

$$-\epsilon_{ijk}(\partial x_a'/\partial x_i)(\partial x_b'/\partial x_k)(\partial x_c'/\partial x_j)\partial H_b'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_c'$$

$$+\ [(\partial x_a'/\partial x_i)\epsilon_0^{(il)}(\mathbf{x})(\partial x_d'/\partial x_l)]\partial E_d'(\mathbf{x}',t,\mathbf{x}_s)/\partial t$$

$$= -(\partial x_a'/\partial x_i)J_i(\mathbf{x},t,\mathbf{x}_s)\ . \tag{16}$$

By using the definition of the determinant of the Jacobian matrix given in (5), we can verify that

$$\epsilon_{ijk}(\partial x_a'/\partial x_i)(\partial x_b'/\partial x_k)(\partial x_c'/\partial x_j) = \alpha\epsilon_{abc}\ , \tag{17}$$

By substituting (17) into (16), we arrive at the same form of eq. (12); that is,

$$\epsilon_{bpa}[\partial H_p'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_a' + \tilde{\epsilon}_0^{(ad)}(\mathbf{x}')[\partial E_d'(\mathbf{x}',t,\mathbf{x}_s)/\partial t]$$

$$= -J_a'(\mathbf{x}',t,\mathbf{x}_s)\ , \tag{18}$$

where

$$\tilde{\epsilon}_0^{(ad)}(\mathbf{x}') = (1/\alpha)[(\partial x_a'/\partial x_i)\epsilon_0^{(il)}(\mathbf{x})(\partial x_d'/\partial x_l)]$$

and

$$J_a'(\mathbf{x}',t,\mathbf{x}_s) = (1/\alpha)(\partial x_a'/\partial x_i)J_i(\mathbf{x},t,\mathbf{x}_s)\ . \tag{19}$$

Thus, we see that we can interpret Ampere's law in arbitrary coordinates as the usual equation in Euclidean coordinates, as long as we use the new permittivity tensor and the new source term in (19).

By using identical derivations, one can also show that equation (13) can be written in the transformed coordinates as follows:

$$\epsilon_{uvw}[\partial E_w'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_v')] + \tilde{\mu}_0^{(ue)}(\mathbf{x}')[\partial H_e'(\mathbf{x}',t,\mathbf{x}_s)/\partial t)] = -K_u'(\mathbf{x}',t,\mathbf{x}_s)\ , \tag{20}$$

where

$$\tilde{\mu}_0^{(ue)}(\mathbf{x}') = (1/\alpha)[(\partial x_u'/\partial x_n)\mu_0^{(nr)}(\mathbf{x})(\partial x_e'/\partial x_r)]$$

and

$$K_u'(\mathbf{x}',t,\mathbf{x}_s) = (1/\alpha)(\partial x_u'/\partial x_n)K_n(\mathbf{x},t,\mathbf{x}_s)\ . \tag{21}$$

The results in (18) and (20) are simply remarkable. Variants of these equations have appeared often in the literature, such as a book on the geometry of electromagnetism by Post (1962). These equations say that we can use the same set of Maxwell's equations for the numerical simulation of electromagnetism data, for example, irrespective of the coordinate system. We simply have to redefine the permittivity and permeability in accordance with (19) and (21). To add more concreteness to this observation, let us consider Maxwell's equations for a homogeneous isotropic medium defined by ϵ_0 and μ_0 in the Cartesian coordinate system (old system). We can use the same Maxwell's equations in a cylindrical coordinate system (new system) as long as we replace the homogeneous isotropic medium by a heterogeneous anisotropic medium defined by the following diagonal tensors:

$$\epsilon_0^{(ad)}(r) = \epsilon_0 \begin{bmatrix} r & 0 & 0 \\ 0 & 1/r & 0 \\ 0 & 0 & r \end{bmatrix} ,$$

$$\tilde{\mu}_0^{(ue)}(r) = \mu_0 \begin{bmatrix} r & 0 & 0 \\ 0 & 1/r & 0 \\ 0 & 0 & 0 \end{bmatrix} . \tag{22}$$

x_1', x_2', and x_3' are labeled by r, θ and z, respectively. We arrive at this description of the heterogeneous anisotropic medium by substituting the elements of the Jacobian matrix in (9) into (19) and (21).

Maxwell's equations can be written in an alternative form involving the electric potential (also called scalar potential) and the magnetic potential (also called vector potential). For example, the governing Maxwell's equation for the scalar potential can be written as follows:

$$(\partial/\partial x_i)[\eta_{ij}(\mathbf{x})\partial V(\mathbf{x},t,\mathbf{x}_s)/\partial x_j] = \zeta(\mathbf{x},t,\mathbf{x}_s) \ , \tag{23}$$

where $\eta_{ij}(\mathbf{x})$ represents the elements of the conductivity tensor, $V(\mathbf{x},t,\mathbf{x}_s)$ is the potential, and $\zeta(\mathbf{x},t,\mathbf{x}_s)$ is the source term. Post (1962) pointed out that this equation is also invariant with respect to coordinate transformation. We are here interested by the invariance of (23) because the invariance of the acoustic wave equation that we will consider later can be deduced from that of (23). So it is important here to show that (23) is indeed invariant with respect to the coordinate transformation, as our notations are here quite different from those of Post (1962), and our formulation is much simpler than that of Post (1962); we do not invoke the notions of pseudo tensors, tensor density, contraction, alternation, etc., in our formulation, whereas Post (1962) does.

Let us start the proof of the invariance of (23) by rewriting in the new coordinates as follows:

$$(\partial_a'/\partial x_i)(\partial/\partial_a')[(\partial_b'/\partial x_j)\eta_{ij}(\mathbf{x})\partial V'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_b'] = \zeta(\mathbf{x},t,\mathbf{x}_s) \ . \tag{24}$$

Using the elements of the Jacobian matrix of the reciprocal transform (3) and the results of the coordinate of second-rank tensors in (19), the conductivity tensor can be written as follows:

$$\eta_{ij}(\mathbf{x}) = \alpha(\partial x_i/\partial x_p')(\partial x_j/\partial x_q')\eta_{pq}'(\mathbf{x}') \ . \tag{25}$$

where $\eta_{pq}'(\mathbf{x}')$ represents the elements of the conductivity tensor in the new coordinates. By substituting (25) in (24), we obtain an expression of Maxwell's potential equation in the new coordinate system; i.e.,

$$(\partial_a'/\partial x_i)(\partial/\partial_a')[\alpha(\partial x_i/\partial x_p')\eta_{pb}'(\mathbf{x}')\partial V'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_b'] = \zeta(\mathbf{x},t,\mathbf{x}_s) \ . \tag{26}$$

After taking the derivative of the terms in the square brackets and using the fact that

$$(\partial/\partial x_i)[\alpha(\partial x_i/\partial x_p')] = 0 \ , \tag{27}$$

we arrive at the following equation:

$$\eta_{ab}'(\mathbf{x}')\partial^2 V'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_a'\partial x_b'] + [\partial\eta_{ab}'(\mathbf{x}')/\partial x_a'][\partial V'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_b']$$

$$= (1/\alpha)\zeta(\mathbf{x},t,\mathbf{x}_s) \ , \tag{28}$$

which can be reorganized as follows:

$$(\partial/\partial x_a')[\eta_{ab}'(\mathbf{x}')\partial V'(\mathbf{x}',t,\mathbf{x}_s)/\partial x_b'] = \zeta'(\mathbf{x}',t,\mathbf{x}_s) \ , \tag{29}$$

where

$$\eta_{ab}'(\mathbf{x}') = (1/\alpha)[(\partial x_a'/\partial x_i)\eta_{ij}(\mathbf{x})(\partial x_b'/\partial x_j)] \ ,$$

and (30)

$$\zeta'(\mathbf{x}',t,\mathbf{x}_s) = (1/\alpha)\zeta(\mathbf{x},t,\mathbf{x}_s) \ .$$

Eq. (29) confirms that the Maxwell's potential equations are also invariant with respect to the coordinate system changes.

COORDINATE TRANSFORMATION IN ACOUSTICS

In a domain occupied by water or any other nonviscous fluid, the wavefield can be characterized by the acoustic pressure, denoted here by $p = p(\mathbf{x},t,\mathbf{x}_s)$; and the particle velocity is denoted by $v_r = v_r(\mathbf{x},t,\mathbf{x}_s)$, for a shot point located at $\mathbf{x}_s$ and for a generic point $\mathbf{x}$. For each shot point $\mathbf{x}_s$, we can predict the pressure and the particle velocity at any point $\mathbf{x}$ by solving a system of two first-order differential equations. These equations are (i) the equation of wave motion,

$$\partial p(\mathbf{x},t,\mathbf{x}_s)/\partial x_k + \rho_{kr}(\mathbf{x})\partial v_r(\mathbf{x},t,\mathbf{x}_s)/\partial t = 0 \ , \tag{31}$$

and (ii) the constitutive equation,

$$\partial v_r(\mathbf{x},t,\mathbf{x}_s)/\partial x_r + \kappa(\mathbf{x})\partial p(\mathbf{x},t,\mathbf{x}_s)/\partial t = q(\mathbf{x},t,\mathbf{x}_s) \ , \tag{32}$$

in which $\rho_{kr} = \rho_{kr}(\mathbf{x})$ represents the elements of the fluid-volume density tensor and $\kappa = \kappa(\mathbf{x})$ is the fluid compressibility (the reciprocal of the bulk modulus). The source term $q = q(\mathbf{x},t,\mathbf{x}_s)$ is the fluid-volume source density of the injection rate. By taking the derivative of (31) with respect to x_k and then substituting the equation of the wave motion (31) into the constitutive eq. (32), we obtain the following second-order differential equation:

$$(\partial/\partial x_r)[\sigma_{rk}(\mathbf{x})\partial p(\mathbf{x},t,\mathbf{x}_s)/\partial x_k] = \kappa(\mathbf{x})\partial^2 p(\mathbf{x},t,\mathbf{x}_s)/\partial t^2 + \partial q(\mathbf{x},t,\mathbf{x}_s)/\partial t \ , \tag{33}$$

where $\sigma_{rk} = \sigma_{rk}(\mathbf{x})$ is the specific volume (the reciprocal of the density tensor), i.e.,

$$\rho_{ka}\sigma_{ar} = \delta_{kr} \ . \tag{34}$$

By comparing (23) and (33) we can notice the following equivalence between the Maxwell's potential equation and the acoustic wave equation (Cummer and Schurig, 2007; Diatta et al., 2010):

$$[V,\eta,\zeta] \Leftrightarrow [p,\sigma,\kappa(\partial^2 p/\partial t^2) + (\partial q/\partial t)] \ . \tag{35}$$

In other words, because the Maxwell's potential equation is invariant with coordinate systems, the acoustic wave equation is also invariant with coordinate systems. Hence, using the procedure described above for mapping $\mathbf{x}$ and $\mathbf{x}'$ for the Maxwell's potential equation, we can also map $\mathbf{x}$ and $\mathbf{x}'$ for acoustic media, as follows:

$$(\partial/\partial x'_a)[\sigma'_{ab}(\mathbf{x}')\partial p'(\mathbf{x}',t,\mathbf{x}_s)/\partial x'_b]$$

$$= \kappa'(\mathbf{x}')\partial^2 p'(\mathbf{x}',t,\mathbf{x}_s)/\partial t^2 + \partial q'(\mathbf{x}',t,\mathbf{x}_s)/\partial t \ , \tag{36}$$

where

$$\sigma'_{ab}(\mathbf{x}') = (1/\alpha)[(\partial x'_a/\partial x_i)\sigma_{ij}(\mathbf{x})(\partial x'_b/\partial x_j)] \ , \quad \kappa'(\mathbf{x}') = (1/\alpha)\kappa(\mathbf{x}) \ ,$$

$$p'(\mathbf{x}',t,\mathbf{x}_s) = p(\mathbf{x},t,\mathbf{x}_s) \ , \ \text{and} \quad q'(\mathbf{x}',t,\mathbf{x}_s) = (1/\alpha)q(\mathbf{x},t,\mathbf{x}_s) \ . \tag{37}$$

We can see that the acoustic wave equation in transformed coordinates has the same form as in Cartesian space. However, the definitions of the specific volume and bulk modulus are different, as shown in eq. (37). Notice that the specific volume in acoustics has the same form as the permittivity and the permeability for the same transformation in electromagnetics.

Consider an acoustic medium characterized by compressibily $\kappa(\mathbf{x})$ and an isotropic specific volume $\sigma_0(\mathbf{x})$ [i.e., $\sigma_{ij}(\mathbf{x}) = \sigma_0(\mathbf{x})\delta_{ij}$)]. We can use the Jacobian matrix in (9) and the determinant in (10) to show that eq. (36) for cylindrical coordinates becomes

$$(1/r)(\partial/\partial r)[r\sigma_0(r,\theta,z)\partial p'(r,\theta,z,t,\mathbf{x}_s)/\partial r]$$

$$+ (1/r^2)(\partial/\partial\theta)[\sigma_0(r,\theta,z)\partial p'(r,\theta,z,t,\mathbf{x}_s)/\partial\theta]$$

$$+ (\partial/\partial z)[\sigma_0(r,\theta,z)\partial p'(r,\theta,z,t,\mathbf{x}_s)/\partial z]$$

$$= \tilde{\kappa}(r,\theta,z)[\partial^2 p'(r,\theta,z,t,\mathbf{x}_s)/\partial t^2] + \partial q(r,\theta,z,t,\mathbf{x}_s)/\partial t \ . \qquad (38)$$

This formula is consistent with the classic formula of the acoustic wave equation given in the literature [e.g., Aki and Richards (1981), Bath and Berkhout (1984), and Skudrzyk (1984).

CONCLUSIONS

In this review, I have discussed the invariance of Maxwell's equations and elastodynamic equations under coordinate transformations. Proof was presented that the mathematical forms of Maxwell's equations and acoustic wave equations are invariant with coordinates transformations.

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