CURVELET DOMAIN ADAPTIVE LEAST-SQUARES SUBTRACTION OF INTERNAL MULTIPLES

YANHUA YUAN, YIBO WANG, YIKE LIU and XU CHANG

Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing, P.R. China.

(Received January 30, 2011; revised version accepted June 10, 2011)

ABSTRACT

Yuan, Y., Wang, Y., Liu, Y. and Chang, X., 2011. Curvelet domain adaptive least-squares subtraction of internal multiples. *Journal of Seismic Exploration*, 20: 273-288.

The elimination of internal multiples has always been a challenge in seismic processing. Compared to surface-related multiples, the amplitudes of internal multiples are more complicated and have a wider range due to different geological interfaces, and these complexities make the conventional prediction-subtraction algorithm not easy to be implemented. In this paper, a new method has been proposed to reduce the negative influence of energy diversity in internal multiples subtraction based on curvelet transform. First, we apply multi-resolution and multi-directional analysis to the seismic record with internal multiples, to map seismic events with different spectral and directional features into different curvelet domains. Then, internal multiples can be estimated by minimizing the misfit between the curvelet coefficients of the real seismic data and components of the predicted multiples under least-squares sense in curvelet sub-domains. A simple experimental data with three crossed events and a synthetic seismic record with complex internal multiples are used to validate the effectiveness of the proposed method. Results indicate that our approach is effective in suppressing internal multiples, preserving geological signals and avoiding distortion of primary events even when intersection or coincidence occurs.

KEY WORDS: internal multiples, curvelet transform, least-squares, prediction-subtraction.

INTRODUCTION

The removal of multiples from seismic records forms an essential part in both land-based and ocean-based seismic data processing. Multiple reflected events can usually be divided into two classes based on where the downward reflections in their ray-paths take place. Surface related multiples are events that

0963-0651/11/$5.00

have at least one downward reflection initiated at the surface. Internal multiples, on the other hand, have all of their downward reflections initiated below the surface.

Strong impedance contrast at internal reflectors such as top of salt and coal bed, disconformities, etc. (Weglein et al., 1997; Verschuur and Berkhout, 2005; Jin et al., 2008; Liu et al., 2005), would lead to complicated sets of internal multiples that can easily disturb primary reflections. Even though surface reflected multiples usually exhibit stronger amplitudes than internal multiples, it is much more complicated to eliminate internal multiples. First of all, internal multiples might be related to several subsurface reflectors that are underdetermined, thus the situation for internal multiples is more complex compared to free-surface multiples which are effected by properties of surface. Secondly, amplitudes of internal multiples are highly diversified from layer to layer, while energies of free-surface multiples are relatively stable. The amplitude diversity makes it hard to implement conventional subtraction operator in the total seismic record. Finally, vertical and lateral variation of interfaces will further add to the complexity of internal multiples subtraction.

Prediction-subtraction algorithm (Verschuur and Prein, 1999) for internal multiples elimination includes prediction and adaptive subtraction procedures. Methods for internal multiples prediction can be classified into two generic categories: model-driven methods that make use of statistical assumptions and a priori information about the subsurface velocity distribution, and data-driven algorithm that is based on wave theoretical principles in which multiples are estimated from the measured data itself as a predictive operator. An improved inverse scattering series (ISS) approach that has been proposed to estimate internal multiples, is both effective and velocity-independent (Weglein et al., 1997; Jin et al., 2008).

The second implement of the prediction-subtraction method to remove internal multiples is, however, much more crucial in noise elimination, and many efforts have been made in the field subtraction algorithms. L_1-norm adaptive subtraction (Guitton and Verschuur, 2004; Li et al., 2010) and L_2-norm adaptive subtraction (Wiener, 1949; Verschuur et al., 1992; Verschuur and Kelamis, 1997) have been commonly applied in temporal-spatial space. Wang et al. (2009) introduced curvelet domain hybrid L_1/L_2-norm subtraction algorithm to suppress surface interferometric predicting surface waves by minimizing a curvelet domain hybrid L_1- and L_2-norm objective function. Liu proposed multiple subtraction using statistically estimated inverse wavelets (Liu et al., 2010). Herrmann and Verschuur (2004) applied a block-coordinate relaxation method that seeks the sparsest set for weighted curvelet coefficients of multiples in the process of subtraction. Fomel (2009) developed a nonstationary regression algorithm of matched filtering for adaptive multiple suppression. Li et al. (2010) used least-squares matching approach in time and space overlapping windows for surface-related multiple subtraction in 2D seismic data, where the residual energy between original data and undesirable noise is minimized.

An incorrect matching subtraction will lead to residual multiple energies in the result or may lead to distortion of the primaries. However, internal multiple subtraction is still a challenging and significant problem required to be solved. In this paper, we propose a new and effective method for internal multiple subtraction in curvelet domain. Due to curvelet transform's superior characteristics of multi-scale, multi-direction and localization, primaries and multiples could have minimum overlap in each curvelet sub-domain, which represents a sub temporal-spatial seismic record with specific spectral band and directional range of the total seismic data, and in which conventional adaptive least-squares matched filtering algorithm could be applied separately. Given a first estimate for internal multiples, we first make proper scale-angular analysis of the original data with multiples to separate internal multiples and primary reflections into different curvelet series. Then the predicted multiple records are decomposed in the same style for the convenience of matched filtering. Finally, we try to adaptively subtract multiple reflections by minimizing the difference between the curvelet components of estimated multiples with the true coefficients of the multiples under the least-squares criterion.

There are mainly two advantages of such a separation-matching approach. One is to reduce the distortion of primaries and the leakage of multiple reflected energies in the process of suppressing multiples since primary events and multiples have minimum overlap with appropriate analysis. This primaries-multiples separation property is a key trait that guarantees the stability of adaptive matched filtering and that enables effective removal of the multiples components. Another advantage lies in the efficiency of such method: the least-squares algorithm is implemented in the whole curvelet coefficients space with specific scale and dip parameter, and sliding windowing along time and space axis (Dragoset, 1999; Wildrow and Sterns, 1985; Jin et al., 2008; Lin et al., 2004) can be avoided.

CURVELET TRANSFORM

Curvelet transform is a localized directional decomposition process in the harmonic scale and represents the wave-front of seismic events more precisely with the needle-shaped basis elements, which have super directional sensitivity ability and smooth contours capturing efficiency than wavelet transform does. In this paper, we apply the second generation discrete curvelet transform, which is implemented in four steps: 1) computing 2D fast Fourier transform for the seismic record; 2) creating the wedge window by product of the scale and angle windows; 3) wrapping the product around the center point of the wedge;

4) applying the 2D inverse fast Fourier transform to the wrapped data. Fig. 1(a) shows the way of dyadic scale and angular partition of the frequency-wavenumber plane, and Fig. 1(b) presents the way of visualization of curvelet coefficients for the convenience of making analogy, in which the x-axis indexes direction and the y-axis indicates scale.

We first define a series of functions: $\varphi(j,l,k)$, where j indexes scale (increase from coarsest to finest), l represents angle or direction, and $k = (k_1,k_2)$ is the t-x location parameter. These functions, known as curvelets, are used to decompose the original data into a series of local components defined by different scales and dips. Assume f[t,x] is the seismic record in the t-x domain, then the Curvelet transform can be defined as the inner product of the data f[t,x] and the curvelet functions (Candès and Donoho, 2004; Candès and Donoho, 2006),

$$c(j,l,k) = \langle f,\varphi_{j,l,k}\rangle \sum_{t=1}^{M} \sum_{x=1}^{N} f[t,x]\overline{\varphi_{j,l,k}}[t,x] \quad , \tag{1}$$

where $\overline{\varphi_{j,l,k}}$ is the conjugate transpose of curvelet function $\varphi(j,l,k)$.

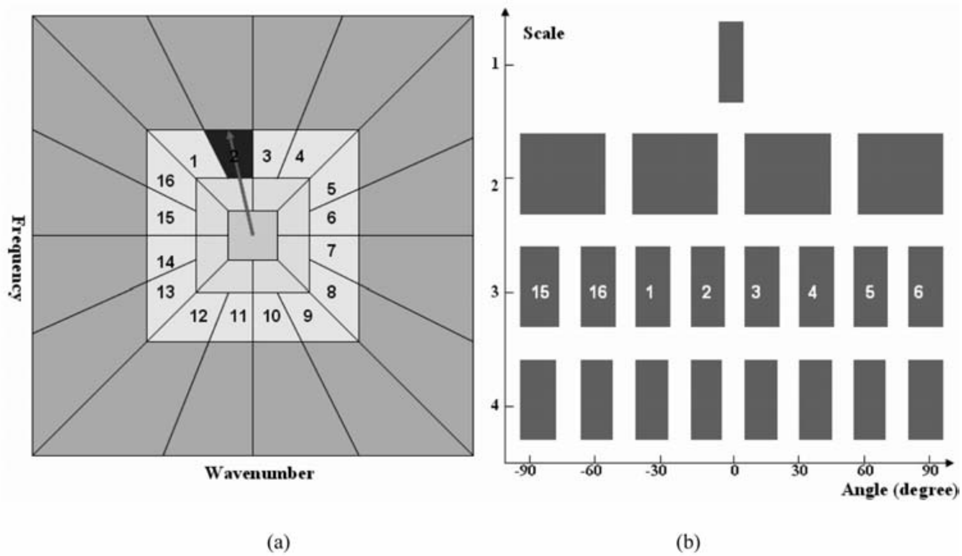


Fig. 1. The second generation discrete curvelet transform.(a) is multi-scale and multi-directional partitioning of the f-k plane. (b) is visualization of the corresponding curvelet coefficients (according to the conjugate symmetric property, only half of the curvelet coefficients are displayed). The horizontal axis indicates angles from $-90° \sim 90°$ and the vertical axis represents scale. The blue wedge in scale 3 and angle index 2 smoothly localizes the Fourier transform, confined by upper and lower spectral limits, and by left and right directional ranges. The red arrow points to the direction of the wedge.

Not only curvelet transform has optimal non-linear approximate rates, but also all curvelet functions can constitute compact frames, and the data is reconstructed with a moderate redundancy. The reconstruction formula is:

$$f[t,x] = \sum_{j,l,k} <f,\varphi_{j,l,k}> \varphi_{j,l,k} = \sum_{j} \sum_{l} \sum_{k} c(j,l,k)\varphi_{j,l,k}[t,x] \quad . \tag{2}$$

From the equation we can see that curvelet transform represents a seismic data as a weighted sum of a series of curvelet functions with different scales, directions and positions, and the weights are commonly referred to as the curvelet components of the record, which represent the contribution of each point in seismic record in the time-spatial domain to individual wedge window in the frequency-wavenumber domain.

Next we mainly discuss the sparsity behavior of curvelet transform. Curvelets satisfies the parabolic scale relation which determines that curvelet transform has the capacity to represent data with edges on piece-wise smooth curves ($f \in C^2$ with a finite number of jumps to be precise) in an optimal sparse way. In approximate theory, assume 2D signal f[t,x] can be transformed into a series of curvelet coefficients, and $f_m[t,x]$ is the m-term approximate (constructed data with the first m curvelet coefficients). If the 2D signal f[t,x] is piece-wise smooth along the edges and the non-linear approximation rate satisfies:

$$\| f - f_m \|^2_{L^2} \propto m^{-2} \quad , \qquad m \to \infty .$$

In the case of wavelet transform, however, the reconstruction error is:

$$\| f - f_m \|^2_{L^2} \propto m^{-1} \quad , \qquad m \to \infty .$$

In the case of Fourier transform, in sharp contrast, the non-linear approximation rate follows:

$$\| f - f_m \|^2_{L^2} \propto m^{-1/2} \quad , \qquad m \to \infty .$$

Consequently, compared to wavelet and Fourier transform, curvelet transform has near optimal non-linear approximate rates. Fig. 2 gives non-linear approximate of wavelet and curvelet, respectively, which shows that curvelet transform can approximate piece-wise smooth curves (seismic events in this case) better than wavelet transform, a fact that indicates curvelet transform would play a vital role in seismic data processing, especially in noise attenuation.

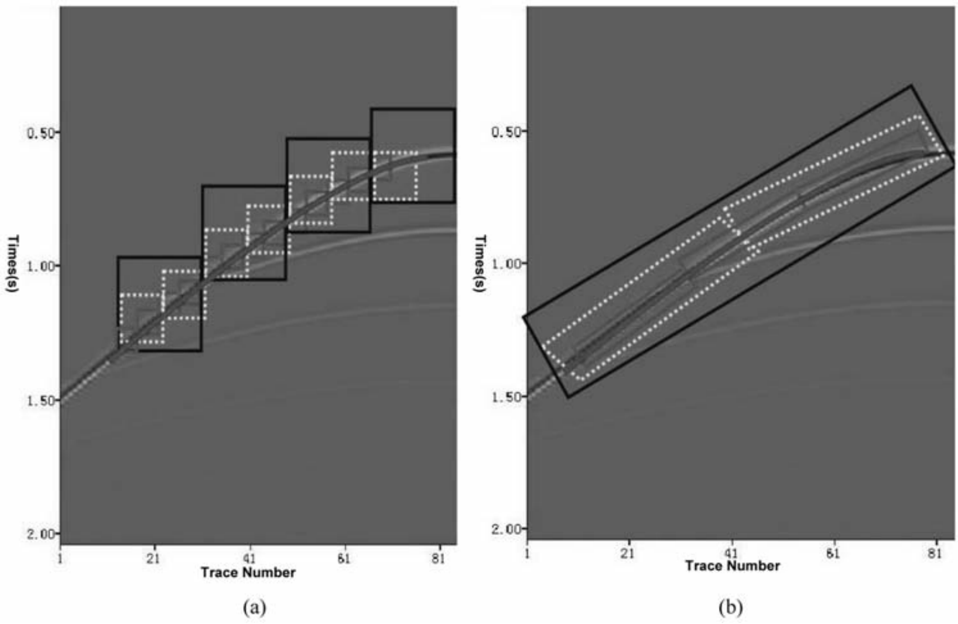


Fig. 2. Comparison of non-linear approximation rates of a seismic hyperbolic event by wavelets (a) and curvelets (b).

ADAPTIVE LEAST-SQUARES MATCHING IN CURVELET SUB-DOMAIN

Any linear problems in seismic processing and imaging can be seen as a special case of the following generic problems: how to obtain s from real seismic record d in the presence of noise m. In our case, m represents internal multiples.

$$d = s + m \quad . \tag{3}$$

Conventional matching approach fits the predicted multiples $\check{m}$ through a correction operator G to the true multiples present in the total data d, which consists of the sum of primaries s and multiples m.

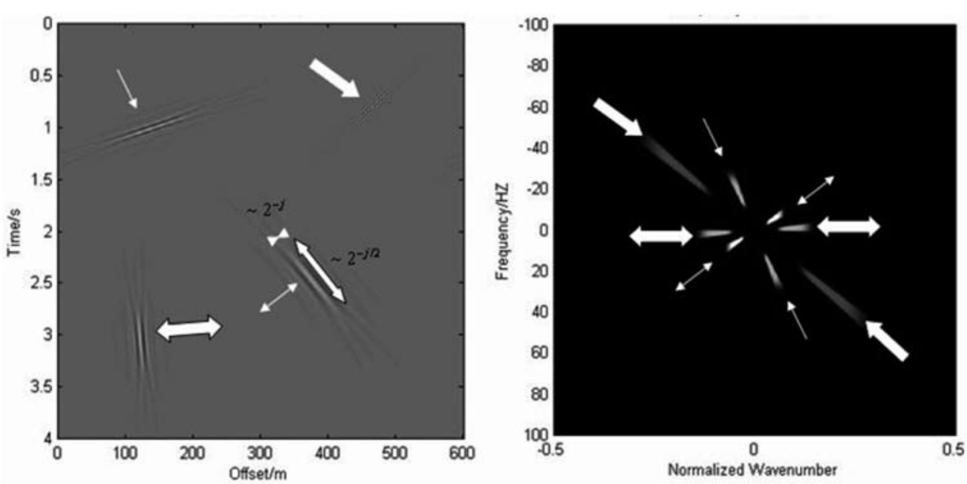
$$m = G\check{m} \quad . \tag{4}$$

As the true multiples are unknown, the least-squares method minimizes the energy mismatch between the total data and the predicted multiples:

$$\min_{G} = \| d - G\check{m} \|_2^2 \quad . \tag{5}$$

There are two main assumptions underlying the least-squares matching method: orthogonality and max energy conditions (Levinson, 1947; Jin et al., 2008). In the case of multiple subtraction, the multiples and primaries are assumed to be orthogonal and have no overlap in the total record, and the energy of multiples cannot be too weak for the security of sufficient matching. Unfortunately, this matching approach fails with real data because the underlying assumptions are seriously violated. To address the issues, minimizing data misfit between the predicted multiples and true seismic record under L_2-norm is implemented in sliding windows along time and space axes (Jin et al., 2008; Lin et al., 2004). Even though the windowed least-squares matching method has improved the attenuation of surface-related multiples, this approach continues to suffer from distortions of primaries and sub-subtraction of multiple energy in terms of internal multiples, especially in the case of large amplitude difference. We now turn our attention to adaptive least-squares internal multiple subtraction in curvelet domain.

From the definition of curvelets, we can conclude that curvelet transform is both a sparse representation (discussed in the previous section), and a local, multi-resolution, multi-directional analysis process (as is displayed in Fig. 3). Considering multiples and primaries have locally different time-spatial, spectral



(a) (b)

Fig. 3. Four curvelets in time-space domain (a) and frequency-wavenumber space(b), which indicates that curvelets are multi-scale, multi-directional (confined in the wedge of the f-k domain and the direction of curvelets in the t-x space is perpendicular to the principal direction of the f-k wedge), and localized both in space- and frequency-domain.

and dip/directional behaviors, they can be mapped into different areas in the curvelet domain, thus multiple reflected energy and primary events can be separated to have minimum overlap and applying least-squares matching operation in each curvelet sub-domain would reduce the negative effect on primaries after appropriate scale-angular decomposition. Due to sparsity property of curvelets, internal multiples with different characteristics of directions and frequencies are distributed or concentrated in different curvelet domains with few coefficients, the energy diversity can be properly solved by performing curvelet transformation. Thus the underlying assumptions of the least-squares criterion can be well met in the curvelet domain, and spatial and temporal sliding windowing can be avoided.

The introduction of a least-squares sense in the curvelet domain is also theoretically sound. Curvelet transform can be regarded as a weighted sum of curvelet basic functions with different scales, dips and positions, and the weights are curvelet coefficients which represents the contribution of each point f[t,x] in the t-x domain to a specific wedge in the f-k domain. The curvelets are spatially localized. Thus through curvelet transform, real seismic data is decomposed into a series of sub records with different behaviors of spectral bands and directional ranges, which are also regarded as data in the time-space domain. Therefore, the conventional least-squares matching idea that is conducted in t-x records can be equally feasible in the curvelet domain.

By posing the least-squares matching subtraction process in the curvelet domain, we first represent total seismic data with internal multiples in terms of curvelet coefficients. This would allow us to separate internal multiples and primaries into different curvelet sets. Since the curvelet transform is linear, curvelet coefficients of the total data is the sum of corresponding coefficients of the primaries and the multiples.

$$d_c = s_c + m_c \quad , \tag{6}$$

where d_c, s_c, m_c are curvelet coefficients of the total data, primaries and multiples respectively.

The predicted multiples are decomposed by curvelets in the same scale-angular analysis way as the total seismic data for the sake of matching implementation. Thus,

$$\breve{m} = C^T \breve{m}_c \quad , \tag{7}$$

in which $\breve{m}_c$ is the curvelet coefficients of the predicated internal multiples $\breve{m}$ and C^T is the adjoint operator of curvelet transform C, which is equal to the pseudo-inverse operator of C.

The least-squares misfit minimization algorithm in each curvelet sub-domain can be expressed as follows,

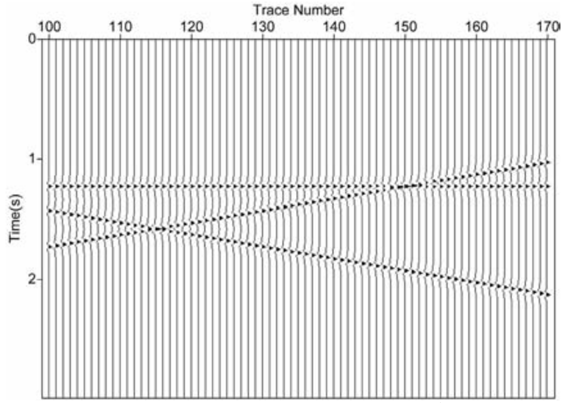
$$\min_{G} = \| d_c - G\check{m}_c \|_2^2 \quad . \tag{8}$$

The optimization of the above objective function can be solved by conjugate-gradient (CG): Fletcher and Reeves (1964), Levinson algorithm (Levinson, 1947; Robinson and Treitel, 1980), or Newton methods assuming that the first derivative of the objective function is continuous. By implementing the least-squares criterion in all sub-domains of curvelets, we find an optimal shaping filter G or curvelet coefficients m_c in each curvelet domain that could minimize the mismatching energy between curvelet coefficients of the predicted multiples and the primaries in that space, and use all of these filtered coefficients m_c to reconstruct the multiple data via the curvelet adjoint operator $m = C^T m_c$.

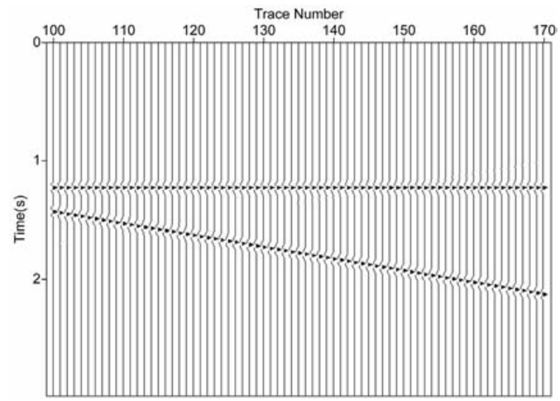
NUMERICAL RESULTS

In this section, we discuss the effectiveness of the curvelet-based approach in internal multiples subtraction. First, we take a simple model [in Fig. 4(a)] to remove the coherent event with negative dip which intersects two other events. The initial estimation model is created through shifting the event by half wavelength and halving its amplitudes. By applying least-squares matching to the data, the other two events could be seriously corrupted at intersections because of the non-orthogonality of the signals and the undesired noise. After operating curvelet transformation, however, the three events with different dips are mapped into different curvelet domains according to the multi-directional property of curvelets. Then we analyze and locate the curvelet domains where the event we want to suppress is distributed before operating the L_2-norm matching algorithm. The results after adaptive matching [Fig. 4(c)] and subtraction [Fig. 4(b)] show that the undesired event is properly filtered and amplitudes of the other two signals are well preserved with only slight interruption when intersection occurs.

Fig. 6(a) gives another synthetic shot gather that we have used to examine the effectiveness of our adaptive subtraction algorithm in internal multiples elimination. The synthetic seismic record is generated using an elastic finite-difference forwarding method with a complicated velocity model shown in Fig. 5(a), which consists of both velocity reverse layers and lateral variations in two interfaces. The generation of free-surface multiples can be avoided in the process of forward modeling if an absorbing boundary is used at the surface. The synthetic seismic data contains five primaries corresponding to five reflectors in the velocity model and a series of internal multiple events with a)



b)



c)

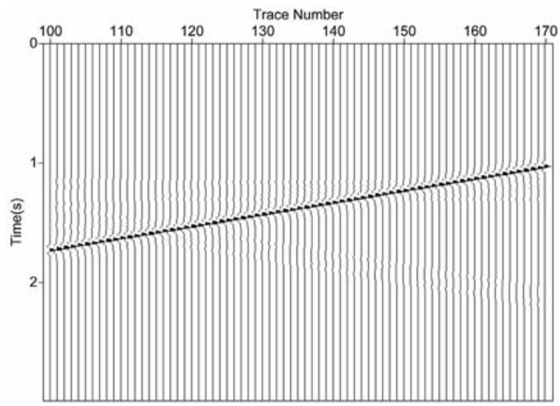


Fig. 4. A simple model with three crossed events to test the effectiveness of our proposed curvelet-based prediction-subtraction algorithm. (a) Model with three crossed events. The event with negative dip is designed to eliminate. (b) Data after L_2-norm matching in curvelet domain. (c) The estimated event after L_2-norm matching in curvelet domain.

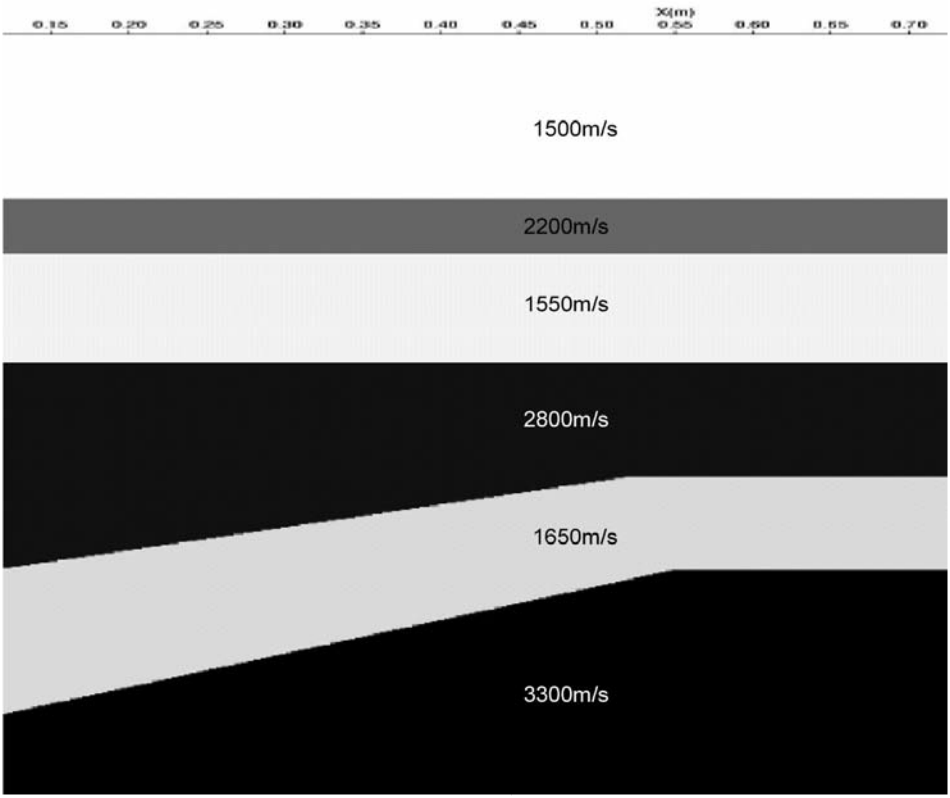
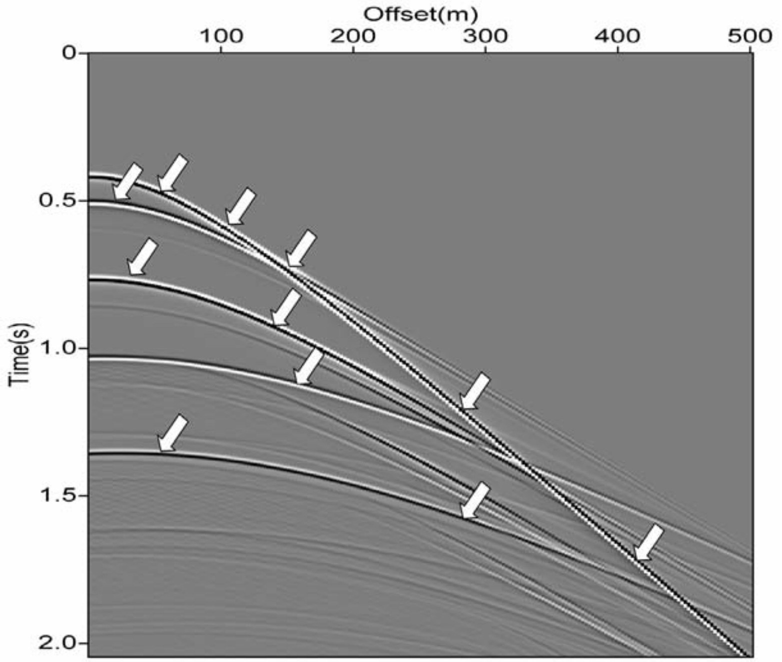


Fig. 5. Velocity model.

conflicting dips and serious amplitude differences (Fig. 6a), making the L_2-norm operator unsuitable for internal multiples subtraction. The initial internal multiple model [as is seen in Fig. 6(b)] is estimated making use of the inverse scattering series algorithm, which is presented in detail by Weglein et al. (1997). Due to the convolution operation in the process of predication, the involved amplitudes are summed up. Thus, the amplitudes of internal multiples are much larger than those of the true multiples, which might lead to leakage of primaries since part of the energy of the internal multiples is matched with the primaries in the process of least-squares filtering. Because of the curvelet's sparsity property (as is discussed in the section on Curvelet Transforms) and parameterization by position, scale and dip (as is displayed in Fig. 3), primaries a)



b)

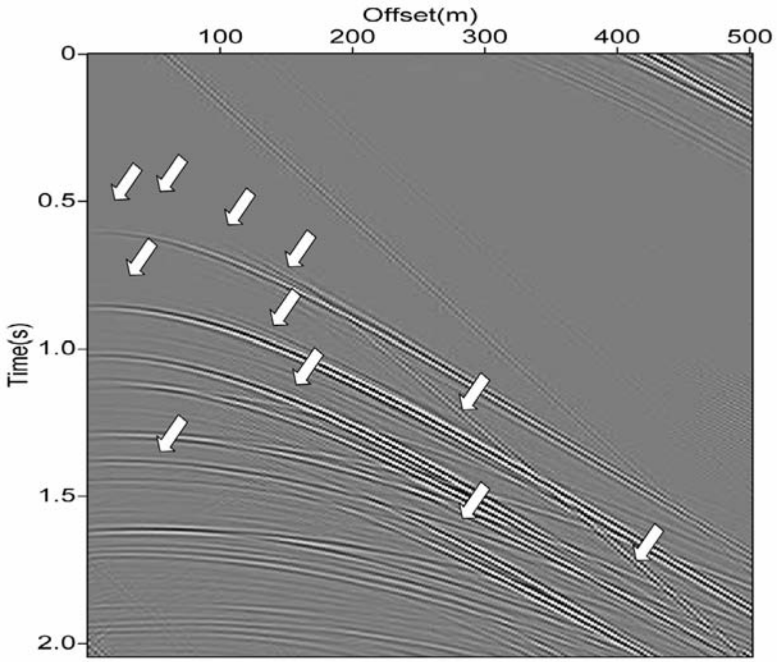
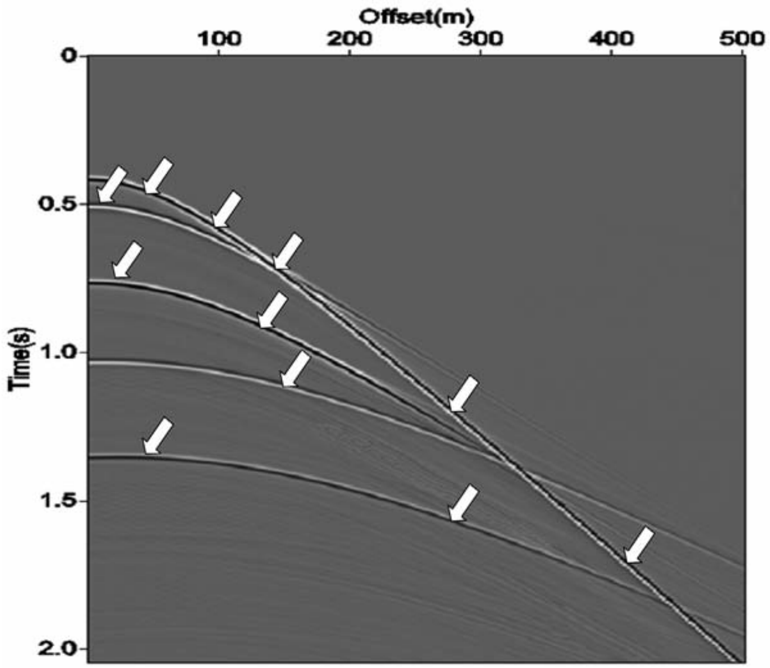


Fig. 6. Internal multiples attenuation results. (a) The synthetic seismic record with internal multiples; (b) the predicted internal multiples.

c)



d)

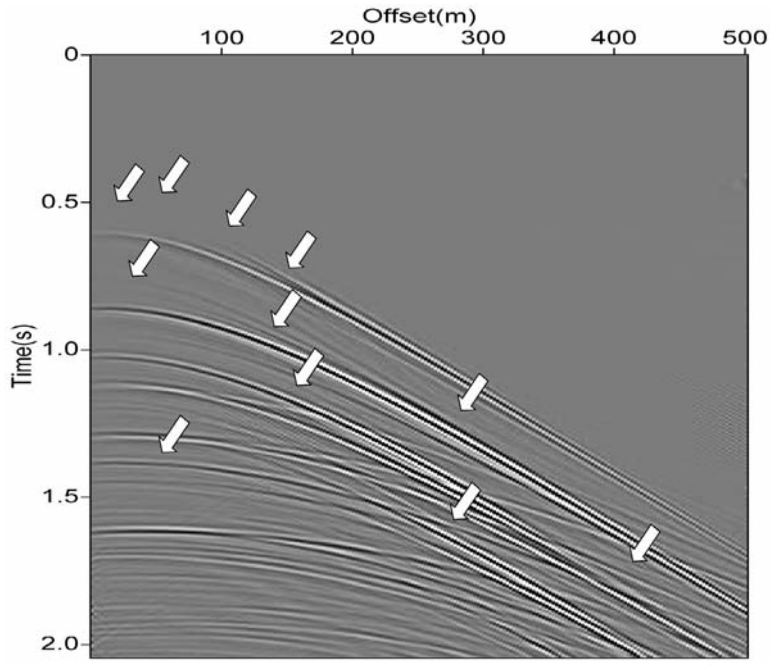


Fig. 6. Internal multiples attenuation results. (c) The seismic record after internal multiples subtraction and (d) the internal multiples after matching.

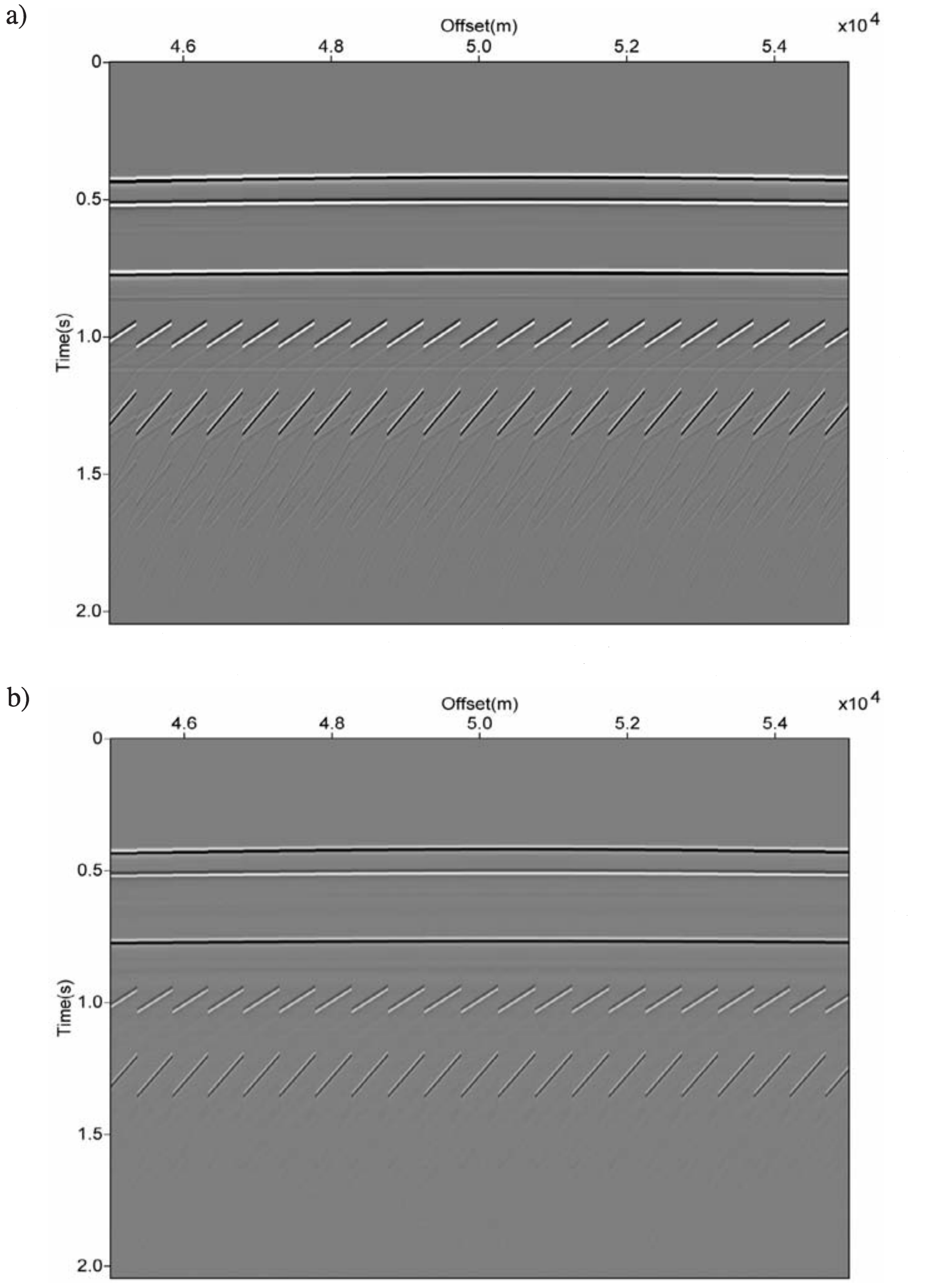


Fig. 7. Results of internal multiples attenuation in common offset gather (a) is common offset record contaminated with internal multiples; (b) is seismic record after internal multiples subtraction.

and multiples could be preliminarily separated after performing curvelet transformation. And problems with L_2-norm performance: the non-orthogonality of primaries and multiples, and energy diversity of internal multiple model could be alleviated by multi-resolution and multi-directional curvelet analysis. From the filtered internal multiples [Fig. 6(d)] and the result of seismic data after adaptive subtraction of multiples [Fig. 6(c)], we can conclude that by implementing least-squares subtraction algorithm in curvelet sub-domain, internal multiples can be appropriately suppressed while the energies of primary events being effectively protected.

In the following, we remove the internal multiples mentioned above in the common offset domain based on curvelets. Fig. 7(a) shows the seismic record with internal multiples for the common offset gather. The estimated primaries after adaptive subtraction in curvelet sub-domains clearly indicate that the proposed approach is sound in preserving amplitudes of geologically implicate events and suppressing internal multiples.

CONCLUSIONS

In this paper, we have developed a new and effective method to suppress internal multiples in the curvelet domain. The curvelet transform is an extraordinary time-frequency analysis process that is both multi-resolution, directional selective, and localized in spatial and spectral space. Our method uses the predicated multiples to suppress the curvelet components related to internal multiple energy in the real seismic data. There are two basic steps in our curvelet-based adaptive matching approach: primaries and multiples are first multi-scale and multi-direction analyzed, and then adaptively matched. The success of such algorithm depends largely on the transform style, which is assumed to map multiples and primaries into different curvelet areas. From the simple model and synthetic seismic record, it can be concluded that the adaptive least-squares matching methods of internal multiple suppression in curvelet sub-domains can perform very well in internal subtraction and signal protection.

ACKNOWLEDGEMENTS

The research was financially supported by the National Natural Foundation of China (Grant No. 40830422 and No. 40930421) and the National 973 Program of China (Grant No. 2009CB219405). The authors thank Emmanuel Candès and David Donoho for public access to their curvelet code. We also would like to thank the anonymous reviewers for many constructive comments.

REFERENCES

Candès, E.J. and Donoho, D.L., 1999. Curvelets: A Surprisingly Effective Nonadaptive Representation of Objects with Edges. Vanderbilt University Press, New York.

Candès, E.J. and Donoho, D.L., 2002a. Continuous Curvelet Transform: I. Resolution of the Wavefront Set. Manuscript, Department of Statistics, Stanford University, Stanford.

Candès, E.J. and Donoho, D.L., 2002b. Continuous Curvelet Transform: II. Discretization into Frames. Manuscript, Department of Statistics, Stanford University, Stanford.

Candès, E.J. and Donoho, D.L., 2004. New tight frames of curvelets and optimal representations of objects with piecewise C^2 singularities. Communic. Pure Appl. Mathemat., 57: 219-266.

Candès, E.J. and Donoho, D.L., 2006. Fast discrete curvelet transforms. Multiscale Model. Simulat., 5: 861-899.

Dragoset, W.H., 1999. A practical approach to surface multiple attenuation. The Leading Edge, 18: 104-108.

Fletcher, R.P., Fowler, P.J. and Kitchenside, P., 2005. Suppressing artifacts in prestack reverse time migration. Expanded Abstr., 75th Ann. Internat. SEG Mtg., Houston: 2049-2051.

Fomel, S., 2009. Adaptive multiple subtraction using regularized nonstationary regression. Geophysics, 74: V25.

Guitton, A. and Verschuur, D.J., 2004. Adaptive subtraction of multiples using L_1-norm. Geophys. Prosp., 52: 27-38.

Herrmann, F.J. and Verschuur, D.J., 2004. Curvelet-domain multiple elimination with sparseness constraints. Expanded Abstr., 74th Ann. Internat. SEG Mtg., Denver: 1333-1336.

Levinson, N., 1947. The Wiener rms (root mean square) error criterion in filter design and prediction. J. Mathemat. Phys., 25: 261-278.

Jin, D., Chang, X. and Liu, Y., 2008. Algorithm improvement and strategy of internal multiples prediction based on inverse scattering series method. Chin. J. Geophys., 51: 171-185.

Li, X., Liu, Y., Chang, X. and Li, P., 2010. The adaptive subtraction of multiple using the equipoise multichannel L_1-norm matching. Chin. J. Geophys., 53: 963-973.

Lin, D., Young, J. and Huang, Y., 2004. 3-D SRME application in the Gulf of Mexico. Expanded Abstr., 74th Ann. Internat. SEG Mtg., Denver: 1257-1260.

Liu, Y., Jin, D., Chang, X., Li, P. Sun, H. and Luo, Y., 2010. Multiple subtraction using statistically estimated inverse wavelets. Geophysics, 75: WB247.

Liu, Y., Sun, H. and Chang, X., 2005. Least-squares wave-path migration. Geophys. Prosp., 53: 811-816.

Robinson, E.A. and Treitel, S., 1980. Geophysical Signal Analysis. Prentice Hall Inc., Englewood Cliffs, New Jersey.

Verschuur, D.J., Berkhout, A.J. and Wapenaar, C.P.A., 1992. Adaptive surface-related multiple elimination. Geophysics, 57: 1166-1177.

Verschuur, D.J. and Kelamis P.G., 1997. Surface-related multiple elimination on 3-D land data. Expanded Abstr., 67th Ann. Internat. SEG Mtg., Dallas: 1383-1386.

Verschuur, D.J. and Prein, R.J., 1999. Multiple removal results from Delft University. The Leading Edge, 18: 86-91.

Verschuur, D.J., Berkhout, A.J., 2005. Removal of internal multiples with the common-focus-point (CFP) approach: Part 2 - Application strategies and data examples. Geophysics, 70: V61-V72.

Wang, Y., Dong, S. and Xue, Y., 2009. Surface waves suppression using interferometric prediction and curvelet domain hybrid L_1/L_2 norm subtraction. Expanded Abstr., 79th Ann. Internat. SEG Mtg., Houston: 3292.

Weglein, A.B., Carvalho, F.A. and Stolt, R.H., 1997. An inverse scattering series method for attenuating multiples in seismic reflection data. Geophysics, 62: 1975-1989.

Widrow, B. and Sterns, S.D., 1985. Adaptive Signal Processing. Prentice-Hall, New York.

Wiener, N., 1949. Exploration, Interpolation, and Smoothing of Stationary Time Series. J. Wiley, New York.