NEARLY PERFECTLY MATCHED LAYER BOUNDARY CONDITION FOR SECOND-ORDER ANISOTROPIC ACOUSTIC WAVE EQUATIONS

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ABSTRACT

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During the numerical simulation of seismic wave propagation, the artificial layers are used at the computational boundaries to truncate the unbounded media which cause the unwanted reflections. In this study, the validity of the nearly perfectly matched layer as an absorbing layer, which has proven to be very efficient for first-order acoustic and elastic wave equations in stress and velocity, is detailed investigated to suppress those spurious reflections for second-order anisotropic acoustic wave equations. The numerical test results show that the nearly perfectly matched layer has a significant performance to absorb the outgoing waves at the model edges.

KEY WORDS: nearly perfectly matched layer, second-order, anisotropic, acoustic equations, numerical modeling.

INTRODUCTION

Seismic numerical modeling in anisotropic media, which has become an increasingly important part in the exploration geophysics and seismology, enables us to better understand the behavior of seismic wave propagation than numerical modeling in isotropic media (Helbig, 1983; Thomsen, 1986; Tsvankin

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et al., 2010). Due to the large computational cost and complexity in seismic anisotropic modeling, some authors have derived second-order pseudo-acousticwave (quasi-P) equations in transverse isotropy with a vertical symmetry axis (VTI) media (Alkhalifah, 2000; Zhou et al., 2006; Du et al., 2008).

During the seismic numerical simulation for wave propagation, unwanted artificial reflections will occur at the edges of the computational domain due to the truncation treatment for the media. The perfectly matched layer (PML) which was first proposed by Bérenger (1994) in electromagnetic (EM) media shows better performance to avoid the spurious reflections than other classical absorbing boundary conditions (Bérenger, 1994; Hastings et al., 1996; Collino and Tsogka, 2001; Zeng et al., 2001). After then, a modified PML technique named as the nearly perfectly matched layer (NPML) was presented by Cummer (2003) in EM media and further applied into seismic modeling in acoustic and elastic media (Hu et al., 2007; Chen and Zhao, 2011; Chen, 2012). NPML does not change the form of original differential equations and is easy to implement in linear media (Cummer, 2003; Hu et al., 2007). However, those applications of NPML are related to first-order equation formulation in velocity and stress. As Komatitsch and Tromp (2003) presented the PML formulation for the second-order wave equations, McGarry and Moghaddam (2009) derived NPML formulation for the second-order VTI pseudo-acoustic equations in their SEG extended abstract. Here, we will detailed discuss the absorbing performance of NPML for second-order VTI pseudo-acoustic equations.

In this paper, we first introduce second-order acoustic wave equations in VTI media and NPML implementation. Finite difference operators (Kelly et al. 1976; Alkhalifah, 2000) are used to replace spatial and time derivatives in the equations for the grid-based model. In a homogeneous model test, snapshots of wave propagation and seismogram are investigated in detail for checking the absorbing capability of NPML. Finally, we place the source in the upper layer (isotropic layer) of a two-layer model and present the snapshots of pure quasi-P wave propagation.

THEORY

Acoustic wave equations for VTI media

Through the assumption of free shear wave velocity along the symmetry axis, Alkhalifah (2000) presented a pseudo-acoustic wave approximation formulation in VTI media that reduces the computational cost. Subsequently, Zhou et al. (2006) and Du et al. (2008) proposed the modified pseudo-acoustic wave equations to simulate the quasi-P wave propagation. According to the study from Du et al. (2008) and McGarry and Moghaddam (2009), the 2D VTI pseudo-acoustic wave equations are expressed as:

$$V_v^{-2}\partial_{tt}P = (1 + 2\varepsilon)\partial_{xx}P + \partial_{zz}Q \ , \tag{1a}$$

$$V_v^{-2}\partial_{tt}Q = (1 + 2\delta)\partial_{xx}P + \partial_{zz}Q \ , \tag{1b}$$

where P(x,z,t) is the acoustic pressure wavefield, Q(x,z,t) is an auxiliary field; V_v is the vertical velocity along the symmetry axis, ε and δ are the well-known Thomsen parameters for anisotropic media (Thomsen, 1986).

NPML formulation

Based on some modification introduced in McGarry and Moghaddom (2009), the second-order VTI acoustic wave eqs. (1a)-(1b) with NPML implementation are written as:

$$V_v^{-2}\partial_{tt}P = (1 + 2\varepsilon)\partial_{xx}\hat{P}^x + \partial_{zz}\hat{Q} + (1 + 2\varepsilon)\partial_x(\hat{P}^x_x d_x) + \partial_z(\hat{q}d_z) \ , \tag{2a}$$

$$V_v^{-2}\partial_{tt}Q = (1 + 2\delta)\partial_{xx}\hat{P}^x + \partial_{zz}\hat{Q} + (1 + 2\delta)\partial_x(\hat{P}^x_x d_x) + \partial_z(\hat{q}d_z) \ , \tag{2b}$$

$$\partial_t\hat{P}^x + \hat{P}^x d_x = \partial_t P \ , \tag{3}$$

$$\partial_t\hat{Q} + \hat{Q}d_z = \partial_t Q \ , \tag{4}$$

$$\partial_t\hat{P}^x_x + \hat{P}^x_x d_x = -\partial_x\hat{P}^x \ , \tag{5}$$

and

$$\partial_t\hat{q} + \hat{q}d_z = -\partial_z\hat{Q} \ , \tag{6}$$

where $\hat{P}^x(x,z,t)$, $\hat{Q}(x,z,t)$, $\hat{P}^x_x(x,z,t)$, and $\hat{q}(x,z,t)$ are auxiliary functions, d_x and d_z are damping factors along x- and z-coordinates. d_x is expressed as $d_x = -3V_v\log(R_t)/(2\delta)\cdot(x_d/\delta)^2$. R_t is the theoretical reflection coefficient ($R_t = 0.0001$). δ is the thickness of the NPML layer, x_d is the horizontal distances to the inner boundary within the NPML layer. d_z has the similar expression as d_x. The damping factors d_x and d_z are zero in the interior of computational domain.

Finite difference operator for the grid-based model

In this study, second-order time derivatives and first-order spatial derivatives in eqs. (2)-(6) will be respectively replaced by second-order and first-order centered-finite difference approximations (Kelly et al. 1976; Alkhalifah, 2000). In addition, first-order time derivatives are replaced by forward-finite difference approximations. The finite difference implementation for the discrete NPML model can be expressed as follows:

$$
\begin{aligned}
&[1/V_v^2(i,j)]\{[P(i,j,k+1) - 2P(i,j,k) + P(i,j,k-1)]/\Delta t^2\} \\
&= [1 + 2\varepsilon(i,j)]\{[\hat{P}^x(i+1,j,k) - 2\hat{P}^x(i,j,k) + \hat{P}^x(i-1,j,k)]/\Delta x^2\} \\
&\quad + \{[\hat{Q}(i,j+1,k) - 2\hat{Q}(i,j,k) + \hat{Q}(i,j-1,k)]/\Delta z^2\} \\
&\quad + [1 + 2\varepsilon(i,j)]\hat{P}_x^x(i,j,k)[d_x(i+1) - d_x(i-1)]/2\Delta x \\
&\quad + [1 + 2\varepsilon(i,j)]d_x(i)[\hat{P}_z^z(i+1,j,k) - \hat{P}_z^z(i-1,j,k)]/2\Delta x \\
&\quad + \hat{q}(i,j,k)[d_z(j+1) - d_z(j-1)]/2\Delta z \\
&\quad + d_z(j)[\hat{q}(i,j+1,k) - \hat{q}(i,j-1,k)]/2\Delta z \quad , \qquad (7a)
\end{aligned}
$$

$$
\begin{aligned}
&[1/V_v^2(i,j)]\{[Q(i,j,k+1) - 2Q(i,j,k) + Q(i,j,k-1)]/\Delta t^2\} \\
&= [1 + 2\delta(i,j)]\{[\hat{P}^x(i+1,j,k) - 2\hat{P}^x(i,j,k) + \hat{P}^x(i-1,j,k)]/\Delta x^2\} \\
&\quad + \{[\hat{Q}(i,j+1,k) - 2\hat{Q}(i,j,k) + \hat{Q}(i,j-1,k)]/\Delta z^2\} \\
&\quad + [1 + 2\delta(i,j)]\hat{P}_x^x(i,j,k)[d_x(i+1) - d_x(i-1)]/2\Delta x \\
&\quad + [1 + 2\delta(i,j)]d_x(i)[\hat{P}_z^z(i+1,j,k) - \hat{P}_z^z(i-1,j,k)]/2\Delta x \\
&\quad + \hat{q}(i,j,k)[d_z(j+1) - d_z(j-1)]/2\Delta z \\
&\quad + d_z(j)[\hat{q}(i,j+1,k) - \hat{q}(i,j-1,k)]/2\Delta z \quad , \qquad (7b)
\end{aligned}
$$

$$
\begin{aligned}
&[\hat{P}^x(i,j,k) - \hat{P}^x(i,j,k-1)]/\Delta t + d_x(i)[\hat{P}^x(i,j,k) + \hat{P}^x(i,j,k-1)]/2 \\
&= [P(i,j,k) - P(i,j,k-1)]/\Delta t \quad , \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
&[\hat{Q}(i,j,k) - \hat{Q}(i,j,k-1)]/\Delta t + d_z(j)[\hat{Q}(i,j,k) + \hat{Q}(i,j,k-1)]/2 \\
&= [Q(i,j,k) - Q(i,j,k-1)]/\Delta t \quad , \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
&[\hat{P}_x^x(i,j,k) - \hat{P}_x^x(i,j,k-1)]/\Delta t + d_x(i)[\hat{P}_x^x(i,j,k) + \hat{P}_x^x(i,j,k-1)]/2 \\
&= -[\hat{P}^x(i+1,j,k) - \hat{P}^x(i-1,j,k)]/2\Delta x \quad , \qquad (10)
\end{aligned}
$$

and

$$
\begin{aligned}
&[\hat{q}(i,j,k) - \hat{q}(i,j,k-1)]/\Delta t + d_z(j)[\hat{q}(i,j,k) + \hat{q}(i,j,k-1)]/2 \\
&= -[\hat{Q}(i,j+1,k) - \hat{Q}(i,j-1,k)]/2\Delta z \quad , \qquad (11)
\end{aligned}
$$

where i and j are position increment indexes along x- and z-coordinates, whereas k is a time increment index; Δx, Δz and Δt are grid spacing and time sampling.

NUMERICAL TEST

Case 1: Homogenous model

The absorbing ability of NPML is tested using a 2D homogeneous VTI model named model-1 (Fig. 1). The model size is 6,000 × 6,000 m. The vertical velocity and Thomsen anisotropy parameters (ε and δ) are 3000 m/s, 0.22 and 0.04, respectively. The model grid spacing is 10 m. Time step is 1 ms that obeys the Courant-Friedrichs-Lewy stability condition (Courant et al., 1928). Two receivers marked R_1 and R_2 are placed at the grid points (575, 300) and (575, 50) where close to the inner boundary of right-side NPML layer (15-grid width). The Ricker wavelet source with dominant frequency 10 Hz is located at the center of the model. The wavelet peak occurs at 0.12 s.

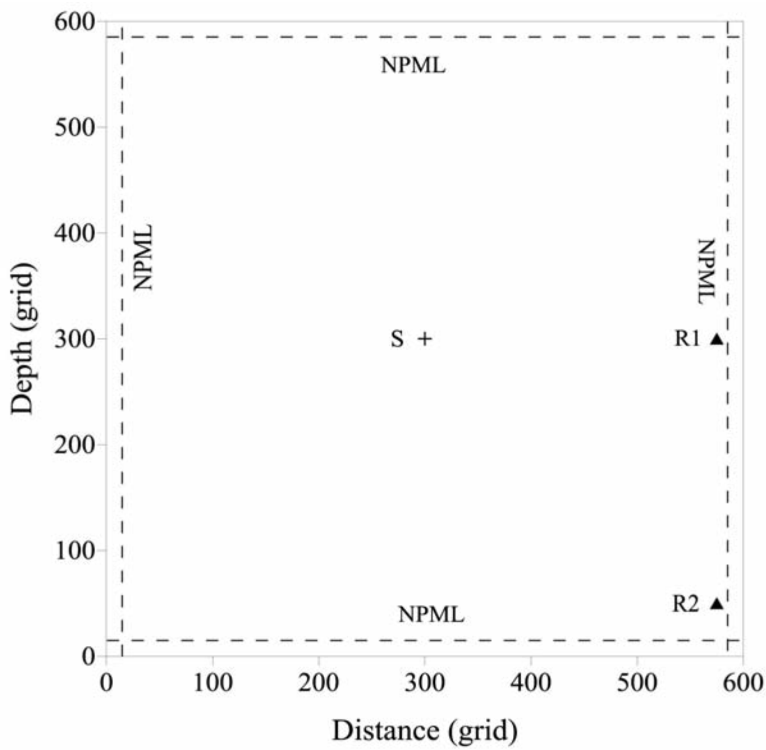

Fig. 1. Illustration of a 2D homogeneous VTI model (model-1). The sources (cross) is located at the center of the model. Two receivers R_1 and R_2 (black-filled triangles) are placed close to the inner boundary of the right-side NPML layers (dashed lines).

Fig. 2 shows the snapshots of seismic wave propagation at 0.4 s, 1.2 s, 4.0 s and 10.0 s. One can observe that NPML has significant performance to absorb the outgoing waves at the inner edges of NPML layers. We also observe that NPML not only effectively absorb the unwanted quasi-P reflection wave (elliptical shape), but strongly suppress the artificial SV wave (quasi-SV, diamond shape). Fig. 3 shows the snapshots of seismic wave propagation at 1.2 s, 1.6 s, 4.0 s and 10.0 s. Strong artificial reflections from the edges of the model severely contaminate the seismic wavefield that will have serious impact on seismic signals.

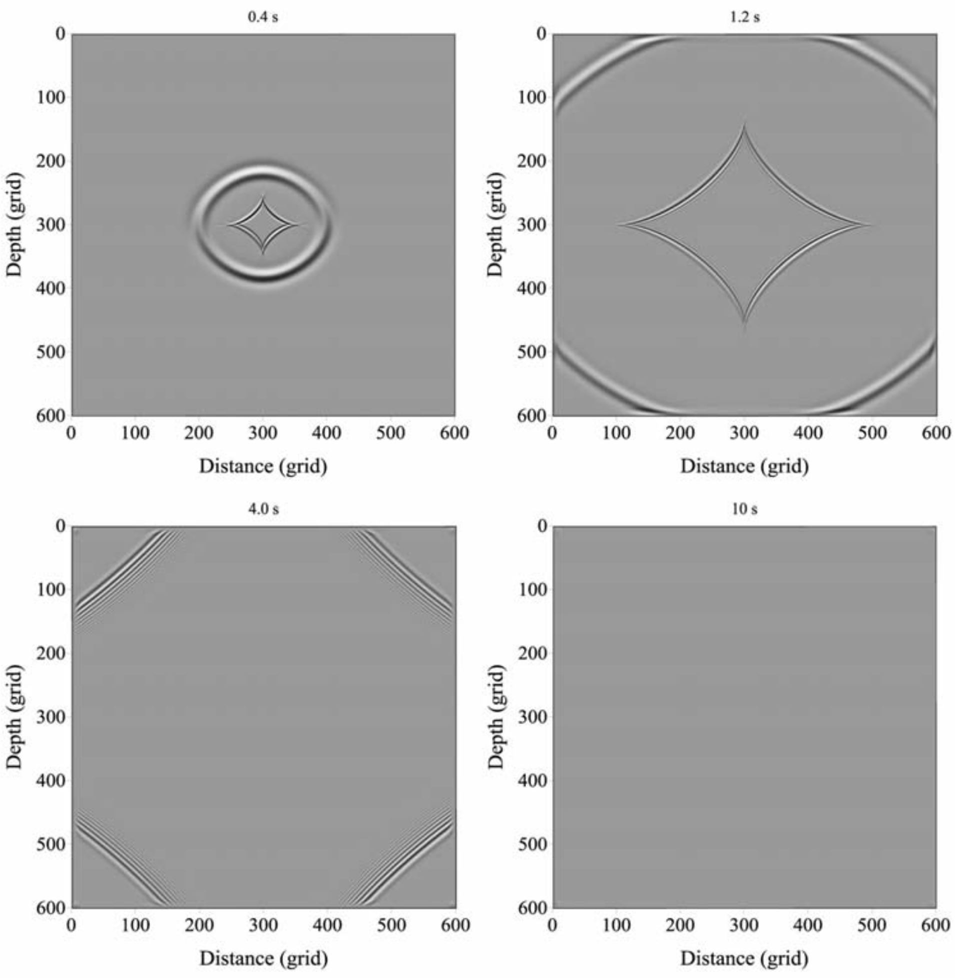

Fig. 2. Snapshots of quasi-P wave field at 0.4 s, 1.2 s, 4.0 s and 10.0 s with NPML application. NPML is able to absorb both quasi-P and quasi-SV waves.

The efficiency of NPML can be also checked through comparing with other absorbing boundary conditions (Chen, 2012) or reference model (Metin et al., 2013). Here, we choose a 2D reference model with size 15,000 × 15,000 m that is used to simulate seismic wave propagation in unbounded media (Fig. 4). It has the same model parameters as model-1. The source is set at the grid points (750, 750) and two receivers are located at the grid points (1025, 750) and (1025, 500). We keep the same source-receiver layout in reference model as the geometry used in the model-1.

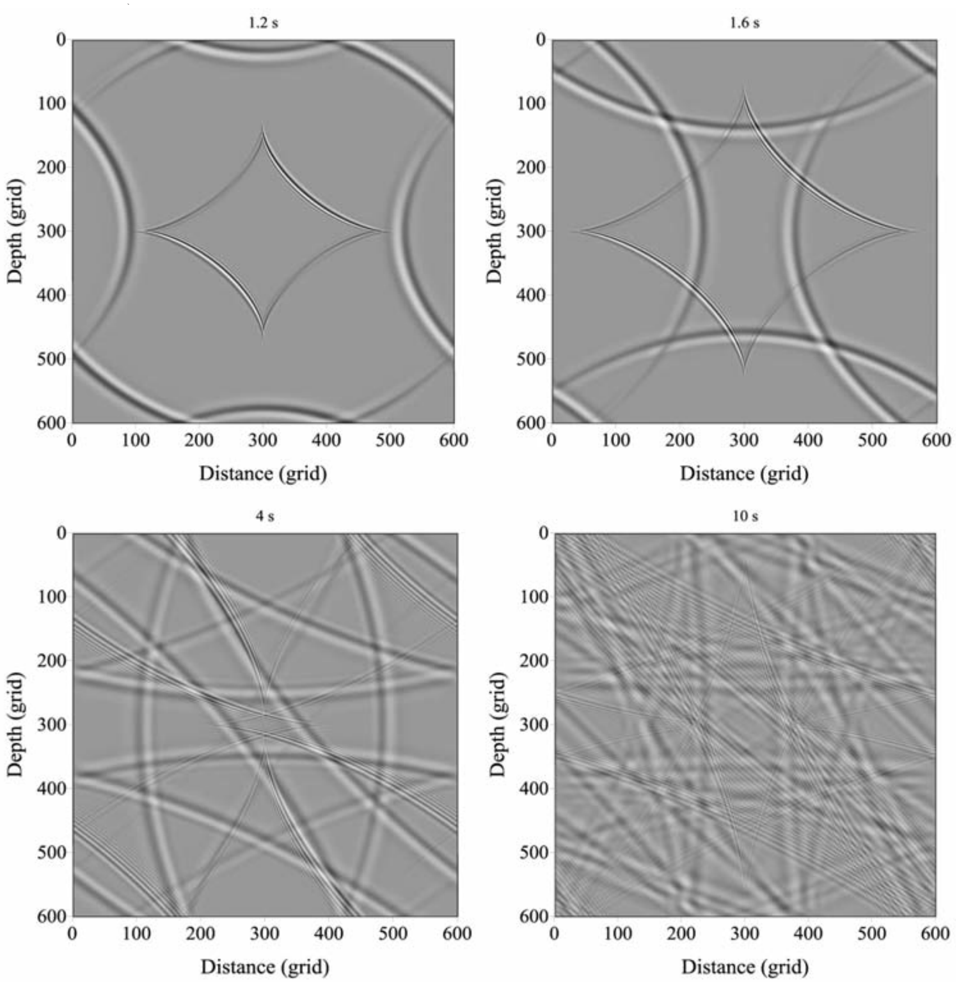

Fig. 3. Snapshots of quasi-P wave field at 1.2 s, 1.6 s, 4.0 s and 10.0 s without NPML application. Strong artificial reflections are observed.

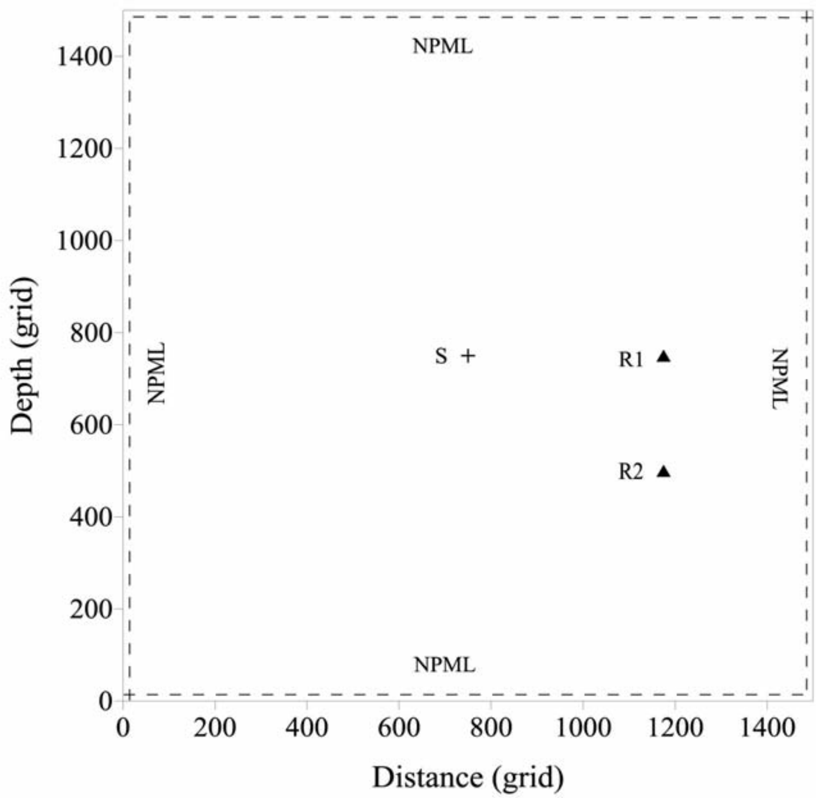

Fig. 4. The frame of the reference model that is assumed as an unbounded medium. It has the same source-receiver geometry as model-1 has.

Figs. 5 and 6 show the seismic signals (seismograms) of total 10 s length recorded at receivers R_1 and R_2. The solid gray lines and dashed black lines respectively represent seismograms obtained in model-1 and reference model. The results show that the seismograms received from model-1 with NPML layer and reference model have great agreement. We also reach a conclusion that the NPML algorithm has significant stability for long time simulation through observing Figs. 2, 5 and 6.

Case 2: Two-layer model

As shown in case 1, one can observe the weaker quasi-SV wave (diamond shape), which is considered as noise, follows quasi-P wave propagation in the numerical modeling. Alkhalifah (2000) suggests locating the seismic source in the isotropic media ($\varepsilon = \delta = 0$) to eliminate the quasi-SV effect. Here, seismic wave propagation is simulated in a two-layer model with size 6,000 $\times$ 6,000 m

(Fig. 7). The upper and lower layers are respectively assumed as isotropic and anisotropic media. The thickness of the upper layer is 200 grids. The vertical velocity and Thomsen anisotropy parameters (ε, δ) for two layers are listed as parameter groups (2500 m/s, 0, 0) and (4000 m/s, 0.22, 0.04). The grid spacing is 10 m and time step is 1 ms. Moreover, the width of the NPML is 10 grids. The exploration source is located at the grid point (300, 25). The same source parameters are selected as those used in case 1.

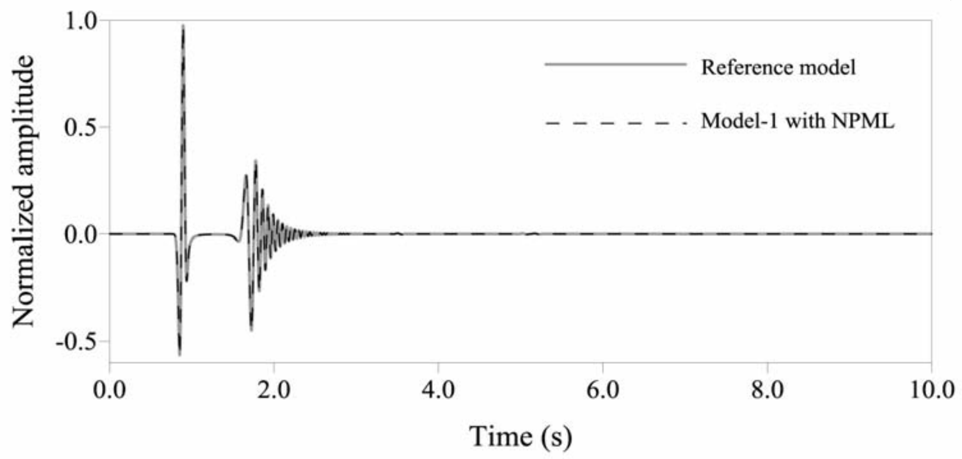

Fig. 5. Seismograms of the quasi-P wave field recorded at receiver R_1. NPML has effective absorbing ability which can simulate seismic wave propagation in unbounded media.

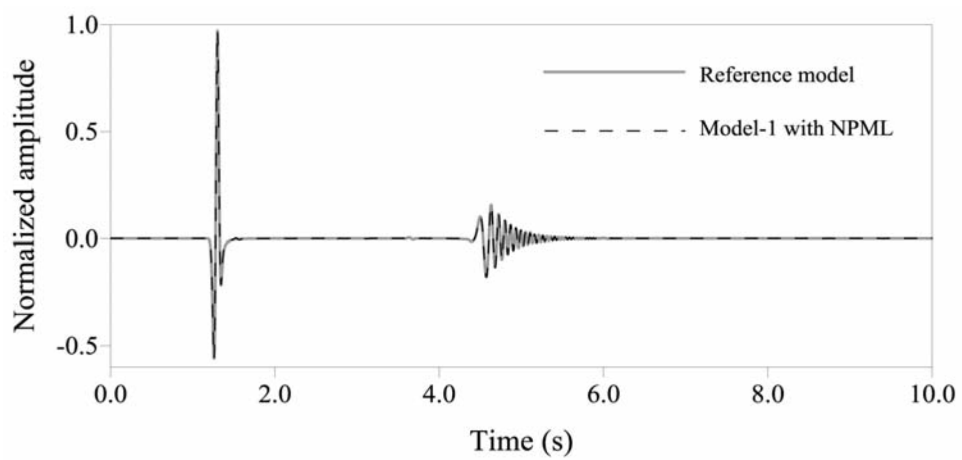

Fig. 6. Seismograms of quasi-P wave field recorded at receiver R_2. The agreement between seismic signals from model-1 with NPML and reference model is great.

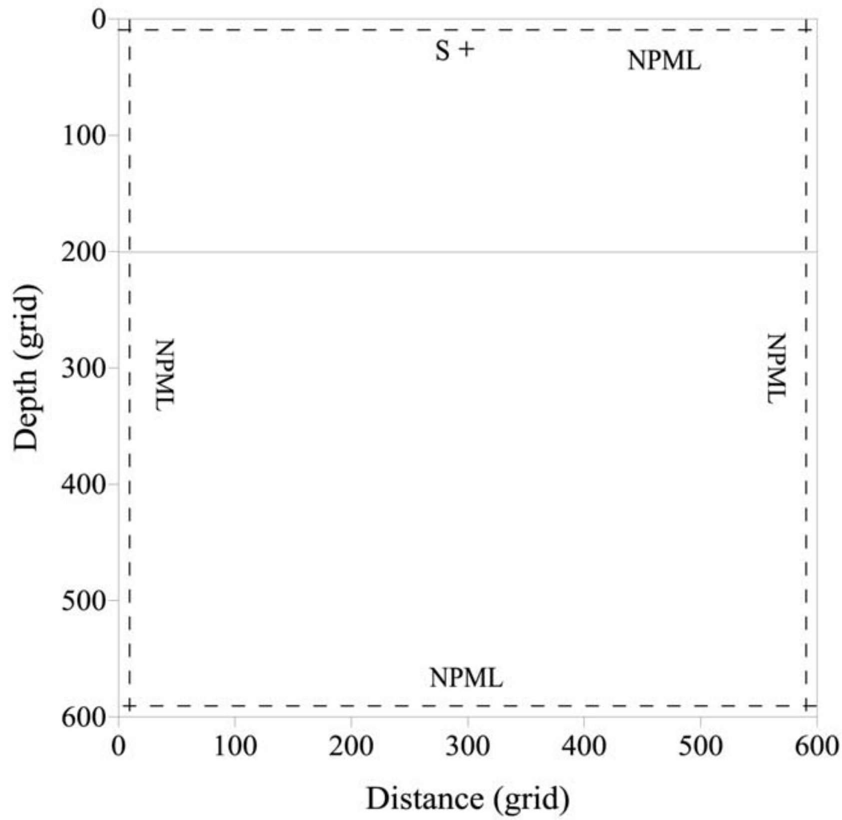

Fig. 7. Illustration of a 2D two-layer model. The sources (cross) is placed in the isotropic media (upper layer).

For simplicity, we only show snapshots of pure quasi-P wave propagation at 0.4 s, 0.8 s, 1.2 s and 1.8 s to check NPML absorbing performance (Fig. 8). Clearly, NPML works well for suppressing the unwanted reflections.

CONCLUSIONS

In this paper, the absorbing performance of the nearly perfectly matched layer boundary condition was investigated for second-order anisotropic acoustic wave equations. We illustrated that NPML is capable of not only suppressing artificial reflections of quasi-P wave, but also absorbing the weaker quasi-SV wave at the edges of the model through study of snapshot of wave propagation and seismogram within the computational domain. Further study is needed to explore the absorbing ability of the nearly perfectly matched layer for other second-order wave equations.

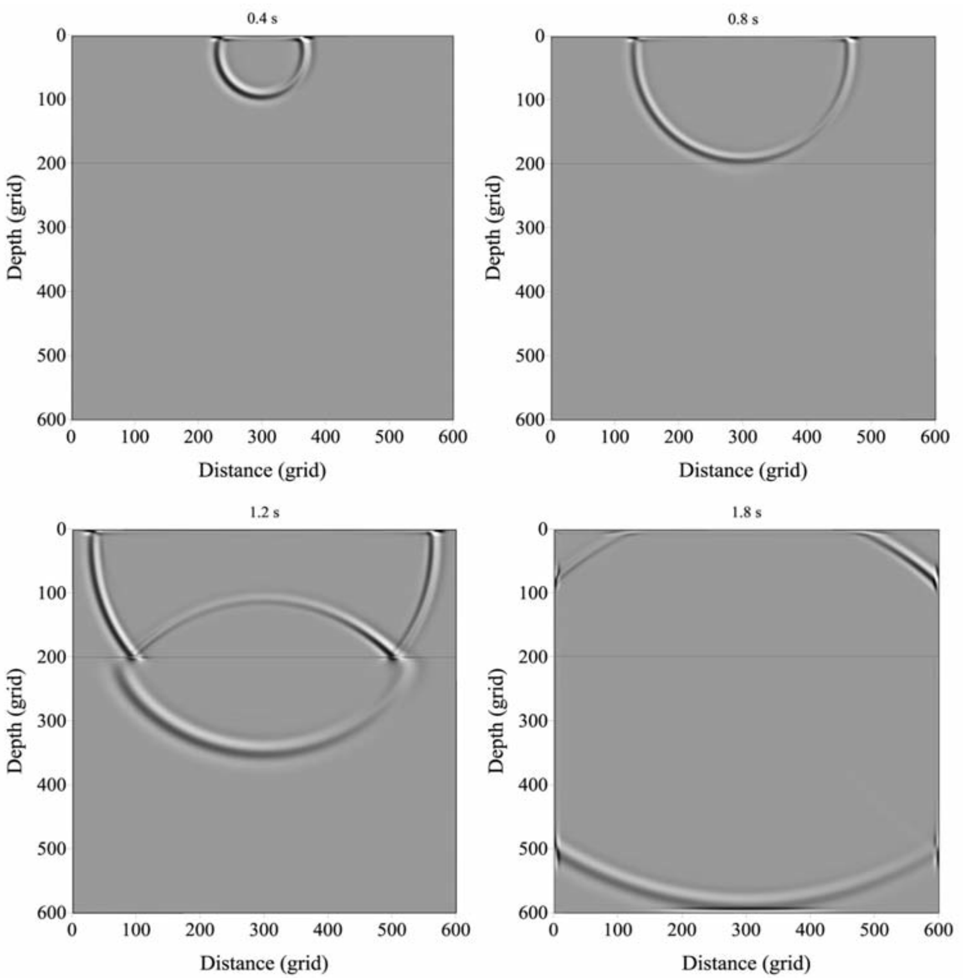

Fig. 8. Snapshots of pure quasi-P wave propagation at 0.4 s, 0.8 s, 1.2 s and 1.8 s using NPML layer.

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