APPLICATIONS OF EMPIRICAL MODE DECOMPOSITION IN RANDOM NOISE ATTENUATION OF SEISMIC DATA

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ABSTRACT

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In this paper, we give an exclusive introduction about the applications of empirical mode decomposition (EMD) to random noise attenuation of seismic data. EMD can be used to denoise each 1D signal from the 2D seismic profile in time-space (t-x) domain either along the time direction or space direction. However, because of the mode-mixing problem, t-x domain EMD along the time direction will cause some damage to a useful seismic signal. A better way is to apply EMD along the space direction and remove the highly oscillating components. The frequency-space (f-x) domain EMD can help obtain faster implementation and even better performances. In order to deal with complex seismic profiles, a hybrid denoising approach based on f-x EMD is also introduced. The hybrid denoising approach can also be inserted into an iterative blending noise attenuation framework, and can help obtain better results. We use both synthetic and field data examples to demonstrate the proposed applications of EMD.

KEY WORDS: empirical mode decomposition, random noise attenuation, t-x EMD, f-x EMD, iterative blending noise attenuation, shaping regularization.

INTRODUCTION

Empirical mode decomposition (EMD) is a new signal processing method (Huang et al., 1998), which was proposed to prepare a stable input for the Hilbert Transform. The essence of EMD is to stabilize a non-stationary signal. That is, to decompose a signal into a series of intrinsic mode functions (IMF).

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Each IMF has a relatively local-constant frequency. The frequency of each IMF decreases according to the separation sequence of each IMF. EMD is a breakthrough in the analysis of linear and stable spectra. It adaptively separates nonlinear and non-stationary signals, which are features of seismic data, into different frequency ranges.

EMD has found successful application in the signal-processing field. However, the application of EMD in exploration geophysics community is still unexploited. In this paper, we give an exclusive introduction about the applications of EMD to random noise attenuation of seismic data. We intend to provide an alternative to the existing random noise attenuation approaches, such as those prediction based approaches (Abma and Claerbout, 1995). We first give a short review of the 1D EMD theory, and introduce the basic idea for using the 1D EMD to a denoising 1D signal. Then, we introduced two ways to attenuate random noise in the time-space t-x domain. One is along the time direction, where the problem of mode-mixing is severe. The other way is to apply EMD along the space direction, which can fully utilize the spatially stationary property of post-stack and NMO-corrected profile to achieve a successful performance. Next, we introduce another domain for performing EMD, which is the frequency-space (f-x) domain, in which the computational cost will be reduced a great deal because of the conjugate symmetric property of FFT for real traces and band-limited property of seismic data. In order to deal with the complex seismic profile, we introduced a hybrid denoising approach based on f-x EMD. We also applied the proposed hybrid approach to the iterative blending noise attenuation process arising from the recent development of simultaneous-source acquisition. Finally, three synthetic examples and one field data example demonstrate the performances of the introduced methods.

THEORY

1D EMD

The aim of empirical mode decomposition (EMD) is to empirically decompose a nonstationary signal into a finite set of sub-signals, which are termed intrinsic mode functions (IMF) and are considered to be stable. The IMFs satisfy two conditions: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero (Huang et al., 1998).

Provided that s(t), $c_n(t)$, r(t), and N denote the original non-stationary signal, the separated IMFs, the residual and the number of IMFs, respectively, the mathematical principle of EMD can be expressed as:

$$s(t) = \sum_{n=1}^{N} c_n(t) \quad . \tag{1}$$

For a non-stationary signal s(t), using eq. (6), we get a finite set of sub-signals $c_n(t)$, (n = 1, 2, ..., N). The last component is the residual such that:

$$c_N(t) = r \quad , \tag{2}$$

where r denotes the residual after the EMD procedures.

A special property of EMD is that the IMFs represent different oscillations embedded in the data, where the oscillating frequency for each sub-signal $c_n(t)$ decreases as the sequence number of the IMF becomes larger.

Due to the low pass filtering effects, EMD has been used outside geophysics for noise attenuation (Mao and Que, 2007; Kopsinis, 2009). Since random noise represents mainly the highly oscillating components, by removing the IMFs with the highest frequency, we can attenuate this type of noise.

Random noise attenuation by EMD in the t-x domain

A naive extension from the signal-processing field to the geophysics community is to apply the 1D EMD onto each trace of seismic profile and by removing the first several IMFs, we can remove the highly oscillating components in the time direction, which mainly correspond to random noise. The mathematical formulation can be expressed as:

$$\hat{s}(m,t) = \sum_{n_t=2}^{N_t} d_{n_t}(m,t) \quad , \tag{3}$$

where $d_{n_t}(m,t)$ is n_t-th decomposed signal (along the time direction) such that

$$d(m,t) = \sum_{n_t=1}^{N_t} d_{n_t}(m,t) \quad , \tag{4}$$

where d(m,t) is the 2D seismic data. m and t denote the indices in space and time directions, respectively.

However, in exploration geophysics, applying EMD to time traces is not effective because of the mode-mixing problem. Kopecky (2010) defined mode mixing as any IMF consisting of frequencies of dramatically disparate scales. When mode mixing exists, the first one or two IMFs contain a great deal of useful reflection energy. Extensions to EMD, such as ensemble empirical mode decomposition (EEMD) (Wu and Huang, 2009) and complete ensemble empirical mode decomposition (CEEMD) (Torres et al., 2011) have been proposed to solve the mode-mixing problem in signal processing. However, the computational cost of EEMD and CEEMD can be much larger than the conventional EMD, which impedes the practical application of EEMD and CEEMD.

An alternative way to remove random noise in the time-space domain is to apply EMD to space traces. For post-stack or NMO-corrected seismic profiles, the events are generally flat, which indicates a spatially stationary properties of the seismic data. We can remove the first several IMFs to remove the spatially non-stationary components, which are mainly corresponding to random noise. This scheme can be formulated as:

$$\hat{s}(m,t) = \sum_{n_m=R}^{N_m} d_{n_m}(m,t) \ ,$$
$$d(m,t) = \sum_{n_m=1}^{N_m} d_{n_m}(m,t) \ , \qquad (5)$$

where R denotes the minimum index of the preserved IMFs.

Radon noise attenuation by EMD in the f-x domain

Considering that the space-direction EMD should be applied to each space traces, which is time-consuming when the number of temporal samples is large, we can turn the t-x domain to f-x domain. We can apply EMD onto each frequency slices along the space direction, and remove the first several IMFs in order to remove the random noise. Considering the conjugate symmetric property of FFT for real traces, the computational cost can be decreased a lot, especially when the number of temporal samples are around an order of 2 (due to the basic FFT property). Note that the computation cost for the additional FFT is much less compared with that of EMD. Due to the fact that most seismic signals are band-limited, we can only process on a few frequency slices, which helps in saving more computational cost. The f-x domain approach can be summarized as:

$$\hat{s}(m,t) = \mathscr{F}^{-1}[\sum_{n_m=R}^{N_m} C_{n_m}(m,w)] \ , \quad (w_l \le w \le w_h)$$
$$\mathscr{F}d(m,t) = \sum_{n_m=1}^{N_m} C_{n_m}(m,w) \ , \quad (0 \le w \le w_{nyq}) \qquad (6)$$

where $\hat{s}(m,t)$ and $d(m,t)$ denote the estimated signal and acquired noisy signal, respectively. $\mathscr{F}$ and $\mathscr{F}^{-1}$ denote the forward and inverse Fourier transforms along the time axis, respectively. $C_{n_m}(m,w)$ denotes the n_m-th EMD decomposed component in the f-x domain. w_l and w_h denote the lowest and highest frequency for processing. w_{nyq} denotes the Nyquist (or folding) frequency.

Radon noise attenuation by hybrid f-x domain EMD

A problem occurs when applying f-x EMD, because the dipping events will also be removed. This problem occurs because, for many data sets, the random noise and any steeply dipping coherent energy make a significantly larger contribution to the high-wavenumber energy in the f-x domain than any desired signal (Bekara and van der Baan, 2009).

Provided that the noisy data **d** is composed of the clean data **s** and noise **n**, f-x EMD can get a denoised section with all the horizontal events $\hat{s}_h$, while leaving the dipping events in the noise section:

$$\hat{\mathbf{s}}_h \approx \mathbf{E}[\mathbf{d}] \ , \qquad \mathbf{s}_d + \mathbf{n} \approx \mathbf{d} - \mathbf{E}[\mathbf{d}] \ . \tag{7}$$

Here, **E** denotes the noise attenuation operator by f-x EMD, $\mathbf{s}_d$ denotes the true dipping events, and **n** denotes random noise in the original seismic section.

We can retrieve the useful dipping events by applying another denoising operator onto the noise section,

$$\hat{\mathbf{s}}_d \approx \mathbf{P}[\mathbf{d} - \mathbf{E}[\mathbf{d}]] \ , \tag{8}$$

where **P** denotes a denoising operator which estimate the lost dipping events from the initial noise section, and $\hat{\mathbf{s}}_d$ denotes the estimated dipping events. The final denoised section $\hat{\mathbf{s}}$ is given by the summation of the horizontal and dipping signal section:

$$\hat{\mathbf{s}} = \hat{\mathbf{s}}_h + \hat{\mathbf{s}}_d \approx \mathbf{E}[\mathbf{d}] + \mathbf{P}[\mathbf{d} - \mathbf{E}[\mathbf{d}]] \ . \tag{9}$$

The denoising operator **P** in eq. (8) can be chosen as f-x predictive filtering (Chen and Ma, 2014), wavelet-domain thresholding (Chen et al., 2012), or curvelet-domain thresholding (Dong et al., 2013). Thus, eq. (8) becomes a general framework for all those f-x EMD based random noise attenuation approaches.

Iterative blending noise attenuation by EMD

As the development of simultaneous-source technique, blending noise attenuation is becoming more and more important in seismic data processing. The blending noise differs from the conventional random noise in two aspects. On the one hand, the blending noise appears as spike-like random noise, and on the other hand, the blending noise can be modeled, because it's caused by another simultaneous source with a pre-defined time shift. Thus, we can use inversion approach to remove the blending noise. In this paper, we propose to use EMD based denoising operator to iteratively remove the blending noise.

Let us first consider the classic modeling equation:

$$\mathbf{Fm} = \mathbf{d} \quad , \tag{10}$$

where $\mathbf{F}$ denotes the forward blending operator (Chen et al., 2014), $\mathbf{m}$ denotes the unblended clean data and $\mathbf{d}$ denotes the noisy blended data.

In order to solve $\mathbf{m}$ efficiently, we utilize shaping regularization framework (Chen et al., 2014):

$$\mathbf{m}_{n+1} = \mathbf{S}[\mathbf{m}_n + \lambda(\mathbf{d} - \mathbf{Fm})] \quad , \tag{11}$$

where λ denotes the length of each update step, $\mathbf{S}$ denotes the shaping operator which aims to shape each model to its admissible model space iteratively. In this paper, we use two-source blending, and thus the best selection for λ is 0.5 (Chen et al., 2014), and $\mathbf{S}$ is chosen as a hybrid f-x EMD approach.

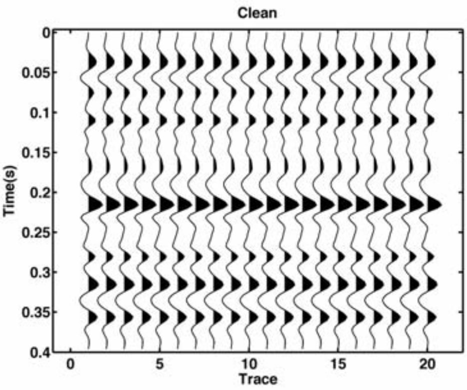
EXAMPLES

Synthetic examples

In this first synthetic example, we use a simple flat-events profile. The clean and noisy data are shown in Figs. 1a and 1b, respectively. We applied three different approaches to remove the random Gaussian white noise. Figs. 2a and 2b show the denoised data using t-x domain EMD along the time direction and the corresponding noise section. Figs. 2c and 2d show the denoised data using t-x domain EMD along the space direction and the corresponding noise section. Figs. 2e and 2f show the denoised data using f-x domain EMD along the space direction and the corresponding noise section. In order to numerically test the effectiveness of the three approaches, we define the signal-to-noise ratio as:

$$\mathrm{SNR} = 10\log_{10}[\|\mathbf{s}\|_2^2/\|\mathbf{s} - \hat{\mathbf{s}}\|_2^2] \quad , \tag{12}$$

where s is the noise-free signal and $\hat{s}$ is the denoised signal. In Table 1, we listed the comparison of SNRs and CPU cost. It's obvious that the f-x EMD can get better results (higher SNR) than both t-x domain approaches by less CPU cost.



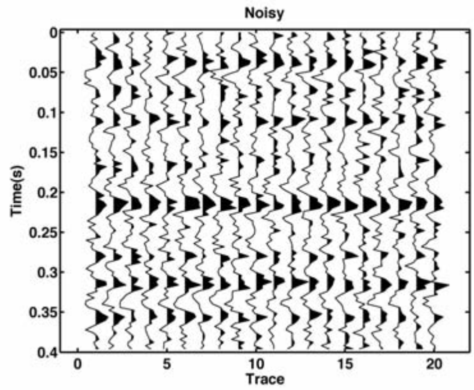


Fig. 1. Synthetic flat-events profile. (a) Clean data. (b) Noisy data.

The second synthetic example is a complex profile, containing two dipping events (Fig. 3). In this example, we compare the results using four different approaches. The denoised results are all shown in Fig. 4. As can be seen from the caption, we use f-x EMD, combined f-x EMD and auto regression (AR) (Canales, 1984), combined f-x EMD and Cadzow filtering (Oropeza and Sacchi, 2011) and combined f-x EMD and curvelet-domain thresholding (Neelamani et al., 2008). Fig. 5 shows the corresponding noise sections.

Table 1. Comparison of SNR and CPU cost using different approaches (flat-events profile).

Method	SNR (dB)	CPU cost (s)
Original	0.648	-
t-x EMD-T	3.926	1.356
t-x EMD-X	7.746	8.278
f-x EMD-X	10.848	4.446

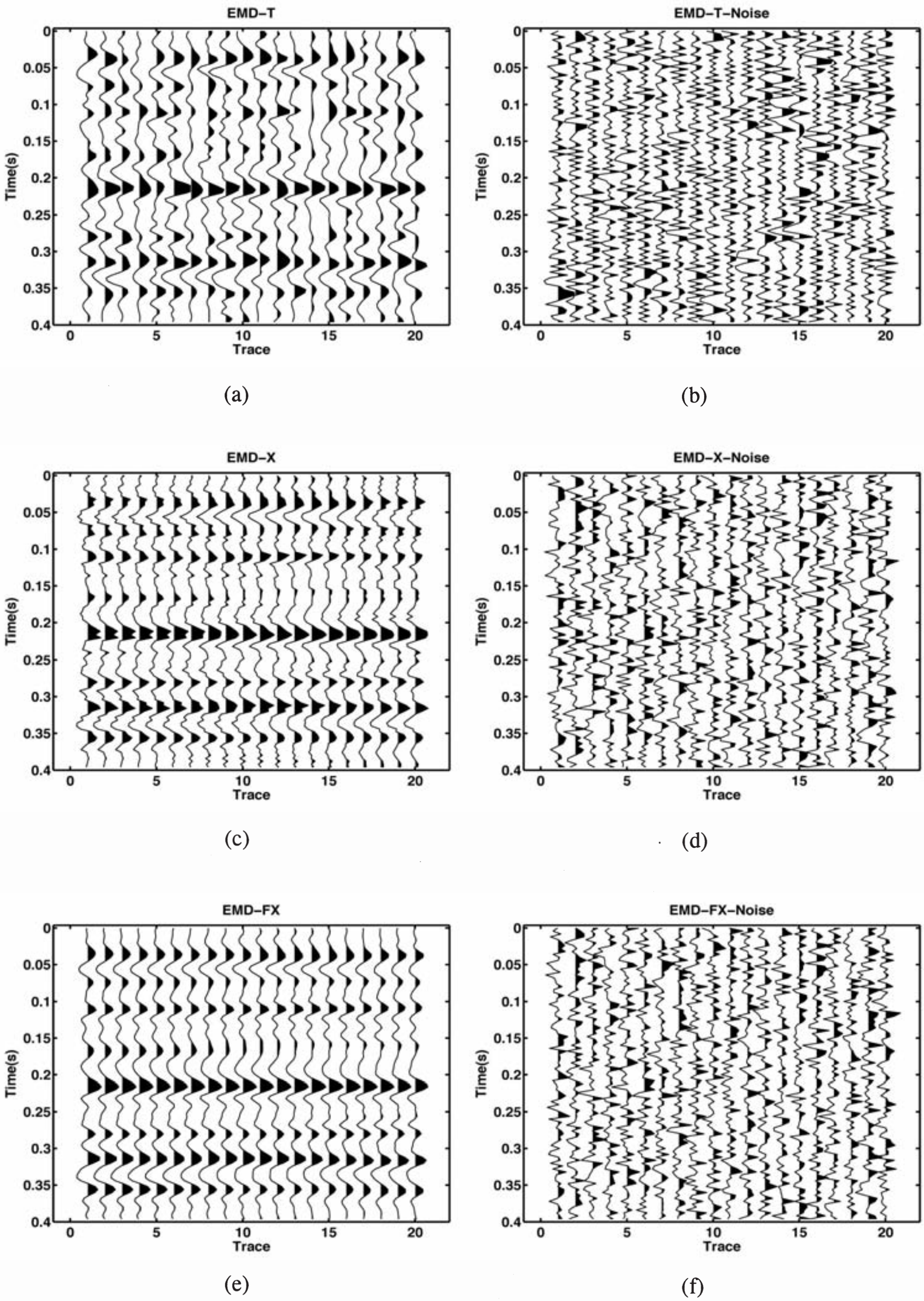
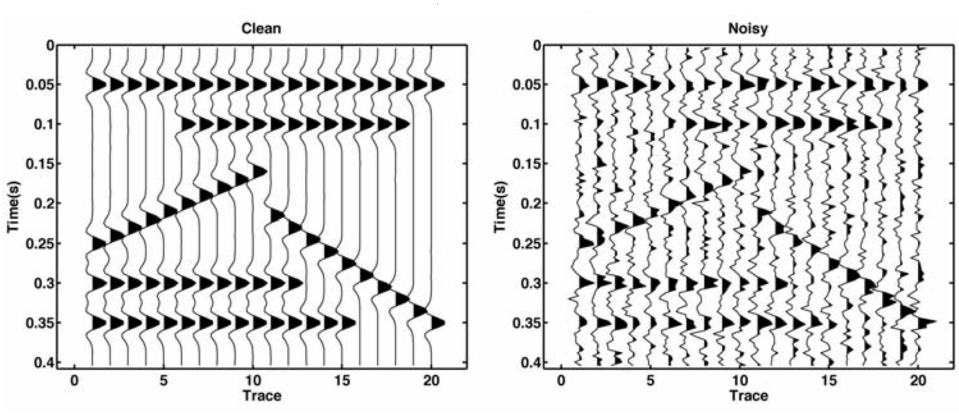


Fig. 2. (a) t-x domain EMD along the t-direction. (b) Noise corresponding to (a). (c) t-x domain EMD along the x-direction. (d) Noise corresponding to (c). (e) f-x domain EMD along the x-direction. (f) Noise corresponding to (e).

Table 2 shows the comparison of SNRs and CPU cost for the four approaches. We can conclude from the comparison both visually and numerically that the combined f-x EMD and AR can obtain the best denoised result. The CPU cost for the four approaches are nearly the same. So there should not be any computational bias for practical applications.

Table 2. Comparison of SNR and CPU cost using different approaches (Complex profile). CF denotes Cadzow filtering and CT denotes curvelet-domain thresholding.

Method	SNR (dB)	CPU cost (s)
Original	1.267	-
f-x EMD	3.917	9.191
f-x EMD and AR	4.069	9.174
f-x EMD and CF	3.164	8.944
f-x EMD and CT	2.719	8.843



(a) (b)

Fig. 3. Synthetic complex profile. (a) Clean data. (b) Noisy data.

The third synthetic example is a hyperbolic-event profile (Fig. 6), used to test the iterative deblending performance of the f-x EMD based denoising approaches. In this case, we compare the combined f-x EMD and AR with the conventional AR, and show their deblending result after 20 iterations in Figs. 7a and 7b. The blending noise sections are shown in Figs. 7c and 7d. The convergence diagram is shown in Fig. 8. It is clear that the combined f-x EMD and AR can get an obvious better result than conventional AR.

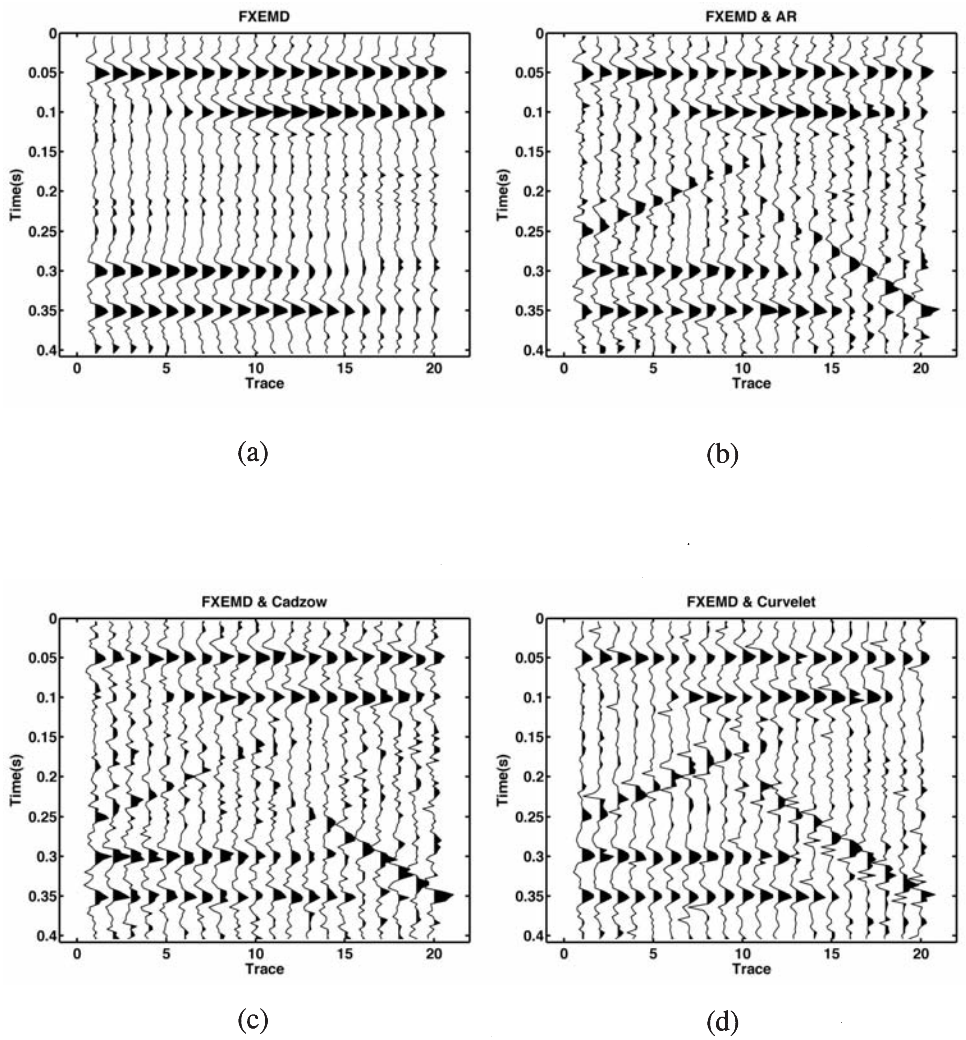
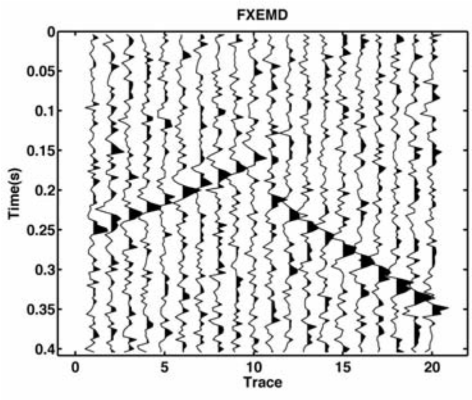
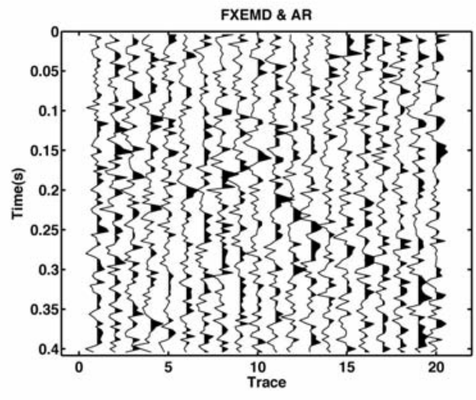


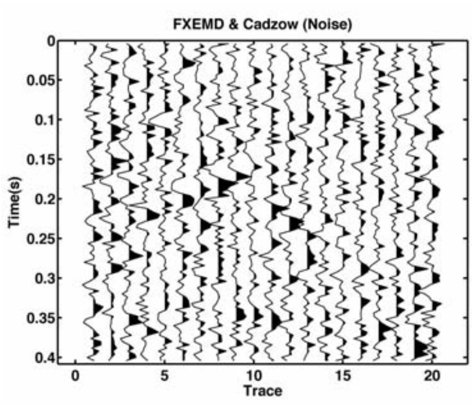
Fig. 4. Denoised results. (a) Using f-x EMD. (b) Using f-x EMD and AR. (c) Using f-x EMD and Cadzow filtering. (d) Using f-x EMD and curvelet-domain thresholding.



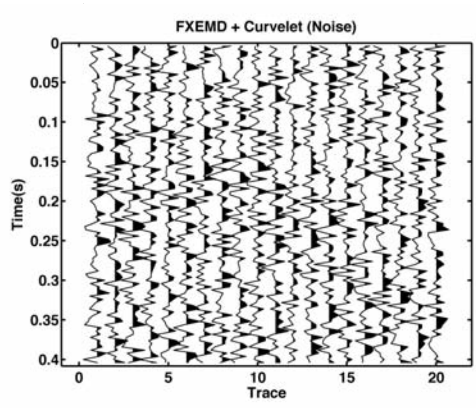
(a)



(b)



(c)



(d)

Fig. 5. Noise sections. (a) Using f-x EMD. (b) Using f-x EMD and AR. (c) Using f-x EMD and Cadzow filtering. (d) Using f-x EMD and curvelet domain thresholding.

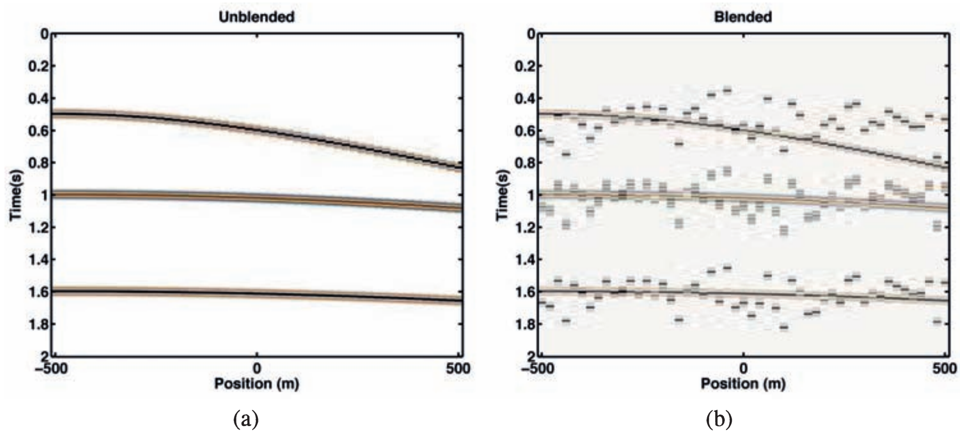


Fig. 6. (a) Unblended data. (b) Blended data.

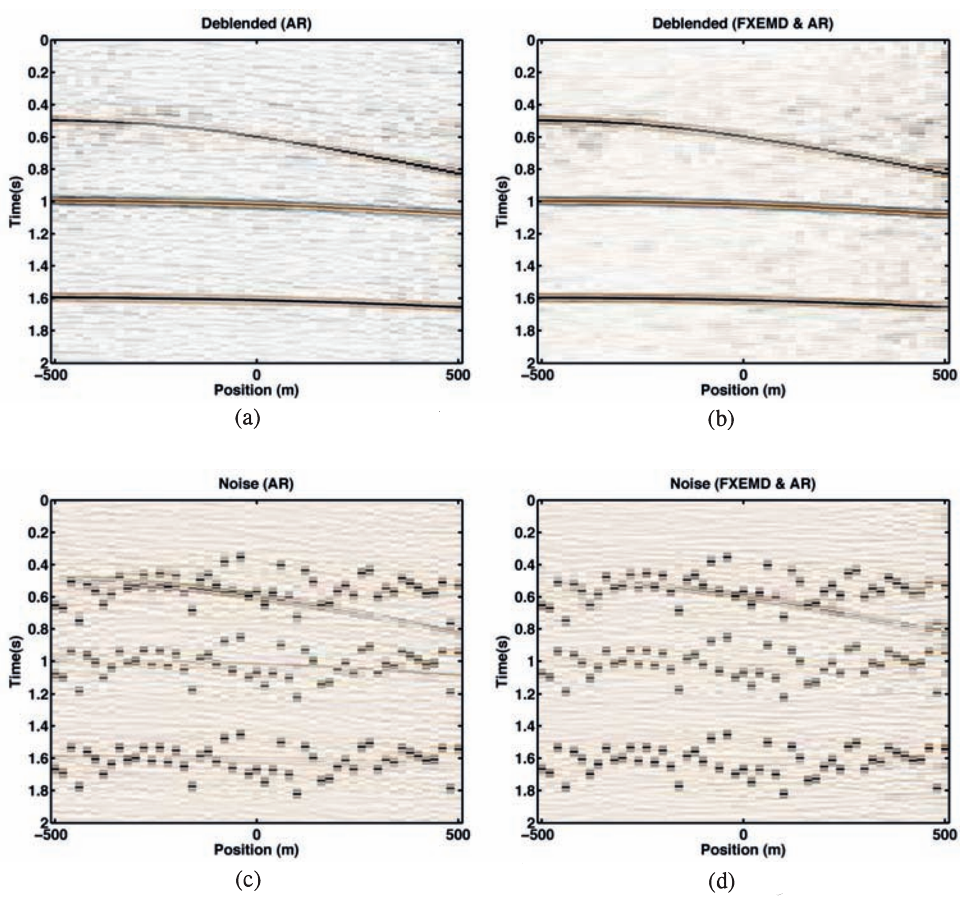


Fig. 7. (a) Deblended data using AR. (b) Deblended data using combined f-x EMD and AR. (c) Blending noise corresponding to (a). (d) Blending noise corresponding to (b).

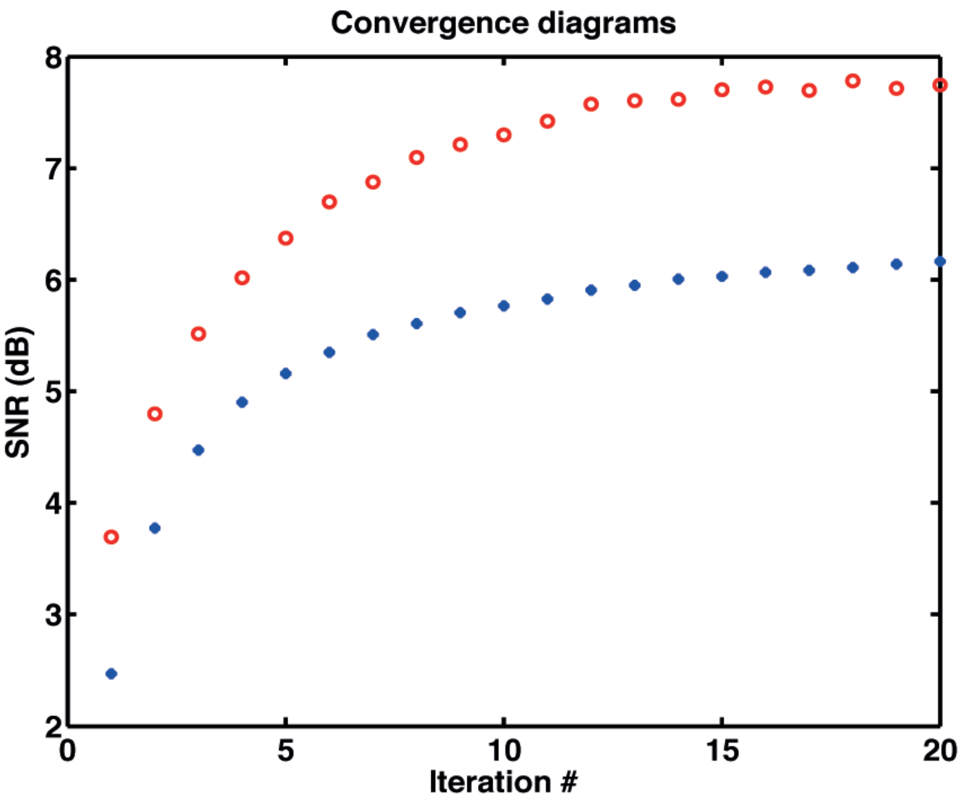


Fig. 8. Convergence diagram for AR and combined f-x EMD and AR. "•" denotes AR and "o" denotes the combined f-x EMD and AR.

Real data example

The field data comes from a part of a post-stack section from the South China Sea. We used f-x EMD and combined f-x EMD and AR, and combined f-x EMD and Cadzow filtering to remove the random ambient noise. The noisy field data and denoised results using different approaches are shown in Fig. 9. As the clean data for a real case is unknown, we can not calculate the SNR. However, from the denoised results, we can draw the conclusion that f-x EMD can get the cleanest image but harm most useful energy. The combined f-x EMD and AR can preserve the most useful small features.

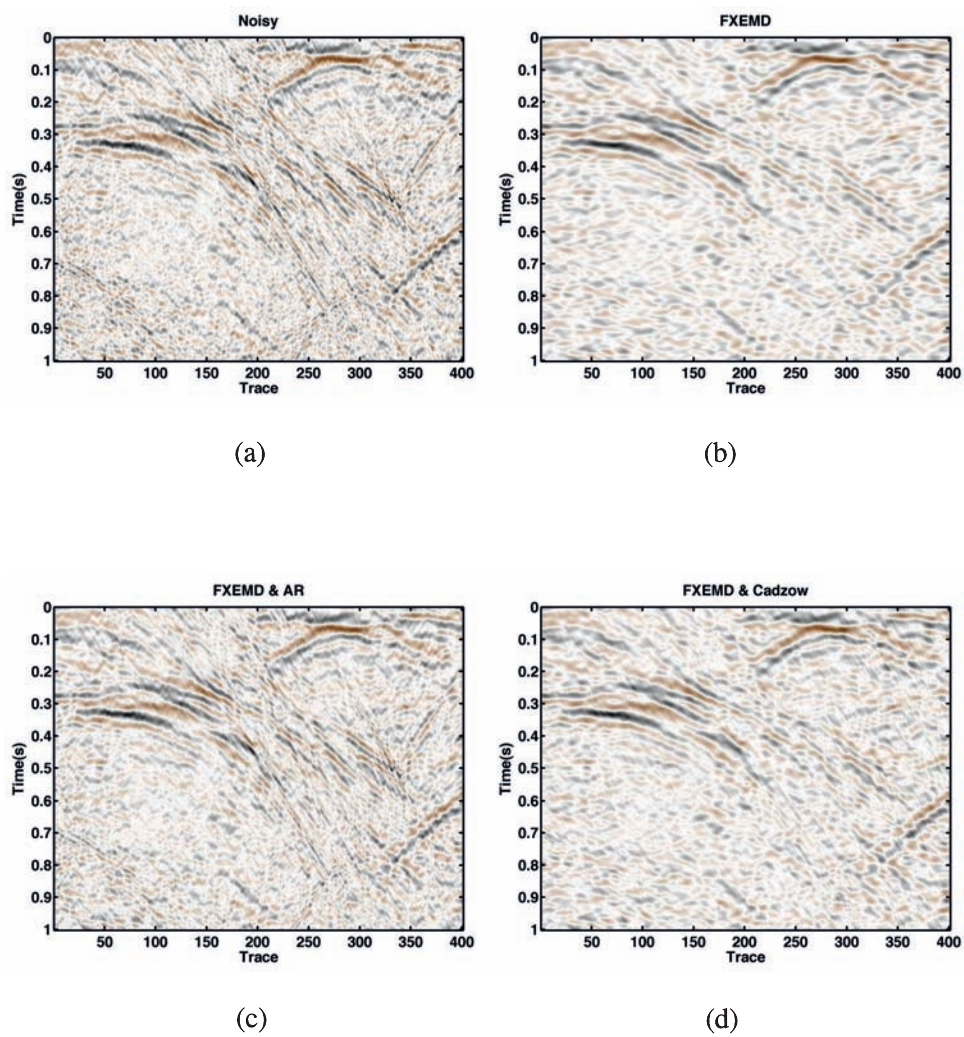


Fig. 9. (a) Noisy section. (b) Denoised section using f-x EMD. (c) Denoised section using combined f-x EMD and AR. (c) Denoised section using combined f-x EMD and Cadzow filtering.

CONCLUSIONS

We have provided an overall introduction of the applications of EMD in random noise attenuation of seismic data. We have introduced two t-x domain approaches and one f-x domain method based on EMD, analyzed and compared their differences. We have introduced a hybrid denoising approach using f-x domain EMD in order to deal with the complex seismic profile. Numerical results show that the CPU cost for the common hybrid denoising approaches are similar but the combined f-x EMD and AR can get better result. We have also applied the f-x EMD based denoising approach to the iterative blending noise attenuation process. Field data example also show that the hybrid f-x domain EMD can help preserve much more useful energy.

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