MULTIPLES ATTENUATION USING SHAPING REGULARIZATION WITH SEISLET DOMAIN SPARSITY CONSTRAINT

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ABSTRACT

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In this paper, we propose a novel multiples attenuation approach based on seislet domain sparsity constraint (SSC). The basic principle of the proposed method is separating primaries and multiples according to the their difference in local slopes. We use the multiples model predicted by the surfaced related multiples elimination (SRME) approach to calculate the matching filter (MF) in order to obtain the initial multiples and initial primaries. The initial multiples and primaries are then used to calculate the local slope of both multiples and primaries used in the proposed iterative inversion framework. The local slope of estimated primaries and multiples can be updated during the iterations in order to get more precise result. A field data example demonstrate a successful performance of the proposed approach. Except for the removed surface-related multiples, the internal multiples can also be attenuated.

KEY WORDS: multiples attenuation, seislet transform, sparse inversion, matching filtering.

INTRODUCTION

Multiples are multiplicative events seen in seismic profiles, which undergo more than one reflections. Instead of being incoherent along the spatial direction like random noise (Yang et al., 2014; Chen and Ma, 2014), the multiples are coherent and behave nearly exactly same as the primary reflections, which makes their removal very difficult using simple signal processing methods.

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A wave-equation-based multiple attenuation method usually consists of two steps: multiple prediction and adaptive subtraction (Verschuur et al., 1992; Huo and Wang, 2009). The difficulty of this type of demultiple approach lays in both parts: how to get a precise prediction for all types of multiples and how to design a good matching filter (MF) used for subtraction. Based on this type of approach, there have existed many approaches for improving the attenuation of multiples, either enhancing the prediction or enhancing adaptive subtraction (Foster and Mosher, 1992; Amundsen et al., 2001; Huo and Wang, 2009; Fomel, 2009; Donno, 2011).

The inverse scattering series (ISS) based demultiple approaches predicts the amplitude and phase of free surface multiples at all offsets, does not require a Radon transform or adaptive subtraction and can eliminate the multiple in the presence of interfering events (Carvalho, 1992; Weglein et al., 2003; Weglein, 2013). Recently, because of popularity in deblending (Chen et al., 2014a,b; Chen, 2014), there exists new approaches combining deblending and demultiple (Berkhout and Blacquière, 2014). In this paper, we propose a novel multiple attenuation approach using an iterative shaping regularization framework
48 based on seislet domain sparsity constraint (SSC). A field data example show successful performance of the proposed approach, compared with conventional MF based approach.

THEORY

Demultiple using shaping regularization

Suppose the recorded data can be denoted as the summation of primaries and multiples:

$$\mathbf{d} = \mathbf{p} + \mathbf{m} \ , \tag{1}$$

where **d** is the observed data, **p** and **m** denote primaries and multiples, respectively.

Eq. (1) can also be formulated with a more classic form:

$$\mathbf{d} = \mathbf{F}\mathbf{x} \ , \tag{2}$$

where $\mathbf{F} = [\mathbf{I}\ \mathbf{I}]$, $\mathbf{I}$ being an identity operator, and $\mathbf{x} = [\mathbf{p};\ \mathbf{m}]^H$.

In the sense of least-squares misfit, we need to solve the following minimization problem:

$$\min_{\mathbf{x}} \| \mathbf{d} - \mathbf{F}\mathbf{x} \|_2^2 \ . \tag{3}$$

In order to solve the problem as shown in eq. (3), a regularization term should be added such that

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \lambda\mathbf{R}(\mathbf{x}) \quad . \tag{4}$$

Here, λ is a controlling parameter, and **R** is the regularization operator.

An appropriate regularization is to ensure the least summation of the L_1 norm of sparse transform domain coefficients:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \lambda\|\mathbf{A}\mathbf{x}\|_1 \quad , \tag{5}$$

where $\mathbf{A} = [\mathbf{A}_p\ \mathbf{O};\mathbf{O}\ \mathbf{A}_m]$, $\mathbf{A}_p$ and $\mathbf{A}_m$ are the sparsity promoting transforms that correspond to primaries and multiples, and **O** denotes zero matrix.

We follow the shaping regularization framework (Fomel, 2007), which was proposed to solve an under-determined equation with easy control on the model property, to solve the minimization problem as shown in eq. (5).

$$\mathbf{x}_{n+1} = \mathbf{S}[\mathbf{x}_n + \mathbf{B}(\mathbf{d} - \mathbf{F}\mathbf{x}_n)] \quad , \tag{6}$$

where **S** is the shaping operator, which iterative shapes the model into its more admissible model space, and **B** is a backward operator, which give an approximate inverse mapping from data to model space. In this paper, we propose to use a transformed domain soft-thresholding operator as the shaping operator:

$$\mathbf{S} = \mathbf{A}\mathbf{T}_\alpha\mathbf{A}^{-1} \quad , \tag{7}$$

and **B** as a scaled identity operator: $\mathbf{B} = \mathbf{I}$.

$\mathbf{T}_\alpha$ can be either a soft-thresholding operator:

$$\mathbf{T}_\alpha^S(x) = \begin{cases} (|x| - \alpha) * \text{sign}(x) & \text{for } |x| \geq \alpha \\ 0 & \text{for } |x| < \alpha \end{cases} \quad , \tag{8}$$

or a hard thresholding operator:

$$\mathbf{T}_\alpha^h(x) = \begin{cases} x & \text{for } |x| \geq \alpha \\ 0 & \text{for } |x| < \alpha \end{cases} \quad . \tag{9}$$

$\mathbf{x} = [\mathbf{p}; \mathbf{m}]$ is chosen as the MF estimated primaries and multiples. In this paper, we chose $\mathbf{A}$ as the seislet transform. $\mathbf{T}_\alpha$ is chosen as a soft-thresholding operator. According to our numerical tests, the soft and hard thresholding operators do not differ too much in terms of the demultiple performance. In the next section, a short review of the seislet transform will be given. The local slope required by the seislet transform can be updated during the iterations.

Review of seislet transform

The seislet is defined with the help of the wavelet-lifting scheme (Sweldens, 1995) combined with local plane wave destruction (PWD) (Fomel and Liu, 2010; Fomel, 2002). The wavelet-lifting utilizes predictability of even components from odd components and finds a difference $\mathbf{r}$ between them. The forward and inverse seislet transforms can be expressed as:

$$\mathbf{r} = \mathbf{o} - \mathbf{P}[\mathbf{e}] \quad , \tag{10}$$

$$\mathbf{c} = \mathbf{e} + \mathbf{U}[\mathbf{r}] \quad , \tag{11}$$

$$\mathbf{e} = \mathbf{c} - \mathbf{U}[\mathbf{r}] \quad , \tag{12}$$

$$\mathbf{o} = \mathbf{r} + \mathbf{P}[\mathbf{e}] \quad , \tag{13}$$

where $\mathbf{P}$ is the prediction operator, $\mathbf{U}$ is the updating operator. $\mathbf{r}$ denotes the difference between true odd trace and predicted odd trace (from even trace), $\mathbf{c}$ denotes a coarse approximation of the data.

The above prediction and update operators can be defined as follows:

$$\mathbf{P}[\mathbf{e}]_k = (\mathbf{P}_k^{(+)}[\mathbf{e}_{k-1}] + \mathbf{P}_k^{(-)}[\mathbf{e}_k])/2 \quad , \tag{14}$$

$$\mathbf{U}[\mathbf{r}]_k = (\mathbf{P}_k^{(+)}[\mathbf{r}_{k-1}] + \mathbf{P}_k^{(-)}[\mathbf{r}_k])/4 \quad , \tag{15}$$

where $\mathbf{P}_k^{(+)}$ and $\mathbf{P}_k^{(-)}$ are operators that predict a trace from its left and right neighbors, correspondingly, by shifting seismic events according to their local slopes.

EXAMPLE

We use a marine CMP gather to demonstrate the performance of the proposed approach. Fig. 1a shows the raw CMP gather. Fig. 1d shows the SRME predicted multiple model. Using the predicted multiple model, we can obtain the initial estimated primaries and multiples as shown in Figs. 1c and 1f, respectively. With the initial primary and multiple model, we can estimated the local slope of both primaries and multiples, as the input of proposed iterative inversion framework. The estimated primaries and multiples after 10 iterations are shown in Figs. 1b and 1e, respectively. Compared with MF estimated results, we can observe that the estimated primaries are much cleaner, with most of the multiples removed (Fig. 1e). Fig. 2 shows the initial and final local slope estimations of the primaries and multiples. It is clear that the final slope of primaries has smaller value than that of the initial slope of primaries. The final slope of multiples has obviously higher value than that of the initial slope of multiples. The slope comparison shown in Fig. 2 suggests a higher level of the

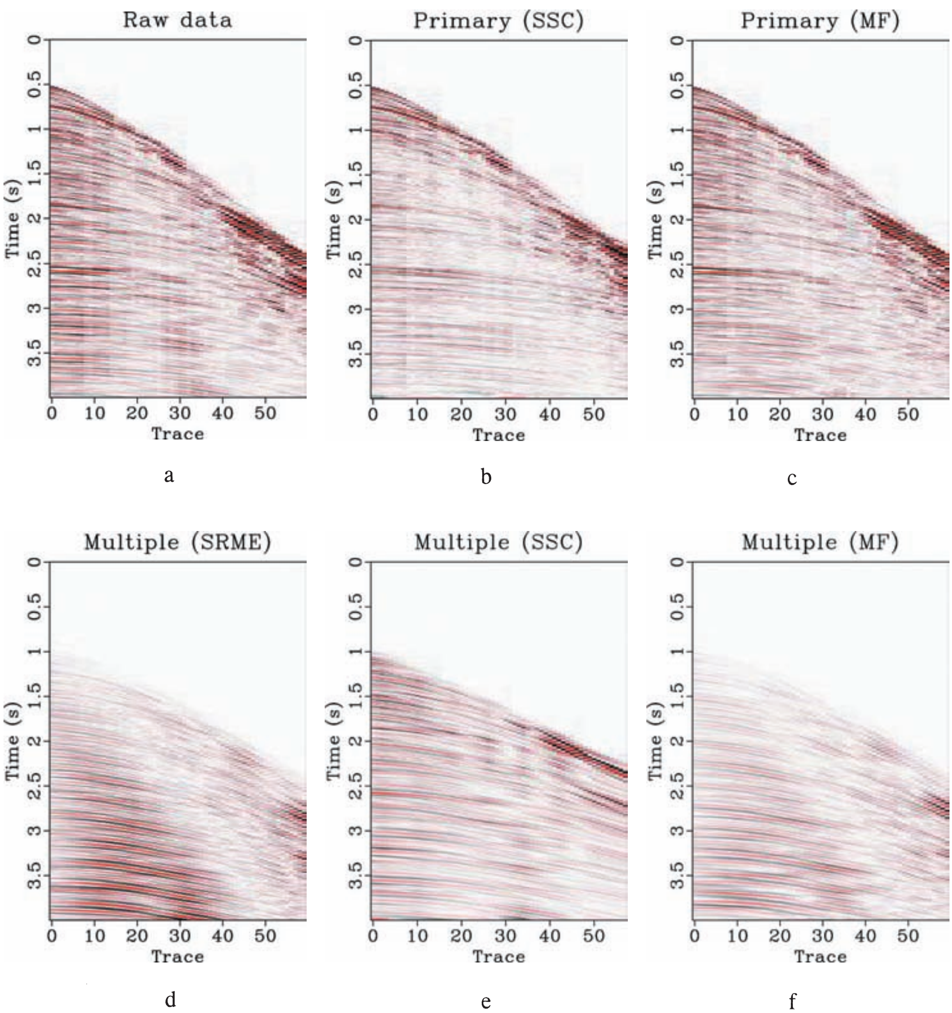

Fig. 1. (a) Raw CMP gather. (b) Estimated primaries using SSC. (c) Estimated primaries using MF. (d) Estimated multiples using SRME. (e) Estimated multiples using SSC. (f) Estimated multiples using MF.

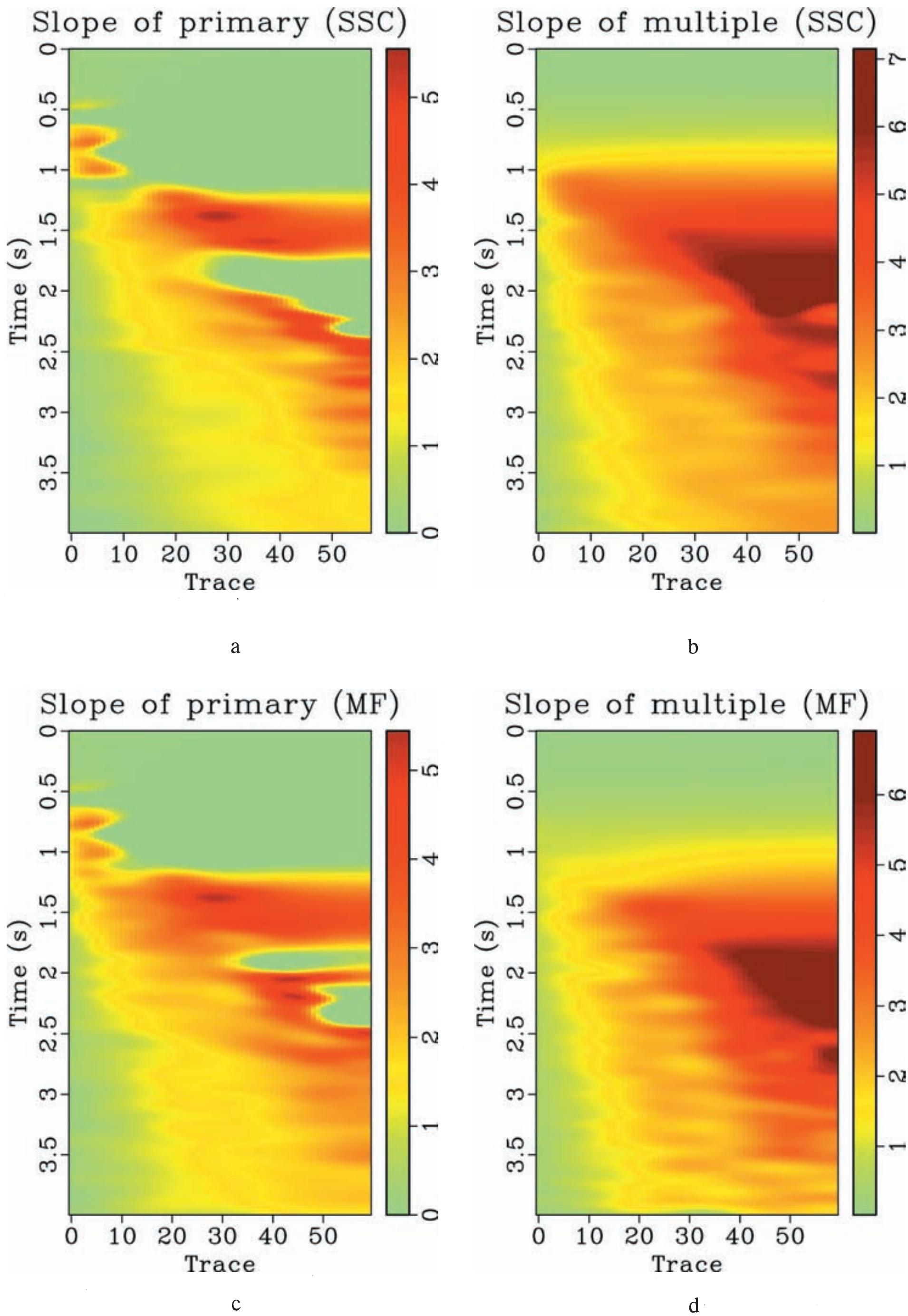

Fig. 2. (a) Final local slope of primaries. (b) Final local slope of multiples. (c) Initial local slope of primaries. (d) Initial local slope of multiples.

multiples removal (higher local slope) and a cleaner demultipled section 108
(smaller local slope). The corresponding velocity spectrum for each section shown in Fig. 1 are shown in Fig. 3. The comparison of velocity spectrum confirms the effectiveness of the proposed approach. The velocity spectrum of the estimated primaries by SSC does not contain too much low-velocity components, however, for the MF estimated primaries, there are still many low-velocity components. The velocity spectrum of MF estimated multiples are very much similar to that of SRME estimated multiples, which indicates that MF is only capable of removing surface-related multiples. However, as can be seen from both data sections and velocity spectrum, the proposed approach can remove both surface-related and internal multiples.

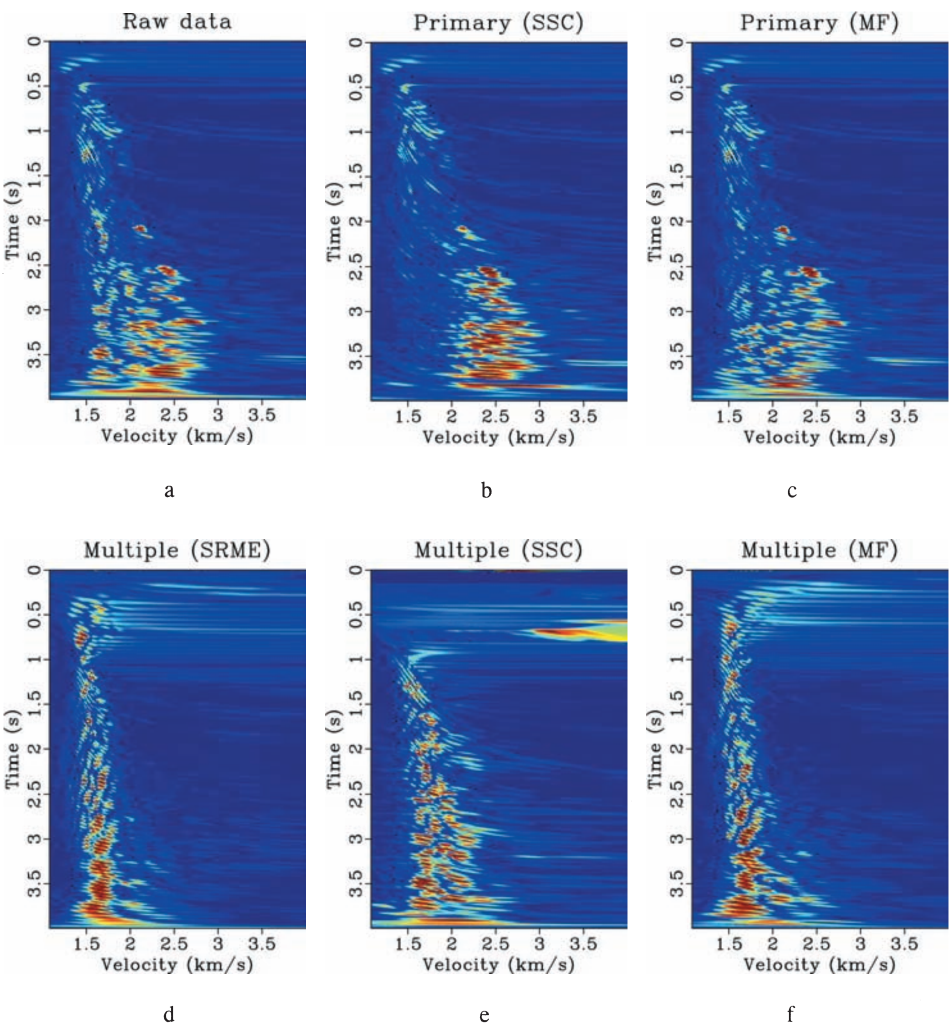

Fig. 3. Velocity spectrum corresponding to each section shown in Fig. 1.

CONCLUSIONS

We have proposed a new multiples attenuation approach using shaping regularization with a seislet domain sparsity constraint (SSC). The primaries and multiples can be iteratively estimated based on the proposed inversion framework. The inversion process requires a precise local slope estimation of both primaries and multiples, which can be updated by calculated the latest primaries and multiples model. The initial primaries and multiples models are set to be the ones estimated by a conventional matching filtering (MF) approach. Field data example shows successful result of the proposed approach, in both removing surface-related multiples and internal multiples.

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