# P-P WAVE AND P-S CONVERTED WAVE SEPARATION AND DE-NOISING COMBINING WITH SVD AND F-K FILTERING

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ABSTRACT

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Wave model separation and de-noising of P-P waves and P-S converted waves is a vital task in seismic data processing. In a multi-mode seismic wave field, P-P waves have the dissimilar characteristics in comparison with other seismic signals: the energy is relatively strong, and the propagation law and coherence are more obvious to be observed (with a hyperbolic law). As for P-S converted events, they have large differences in velocity and frequency characteristics compared to other seismic wave modes. This article puts forward a P-P wave, P-S converted wave separation and de-noising workflow by combining Singular Value Decomposition (SVD) and f-k filtering. First, align P-P waves by NMO correction to make they achieve the best coherence in the transverse direction. Then, by extracting the singular values of the P-P waves through SVD, the P-P waves can be separated. Finally, by using an f-k filtering method, P-S converted waves and other noises can be separated. In this way, the complementary advantages of the SVD and the f-k filtering techniques are combined in wave field separation and de-noising, and can avoid the ineffectiveness in separation the P-P waves and P-S converted waves by only SVD or only an f-k filtering technique. Model and actual seismic data have been processed with this method, and good results have been achieved.

KEY WORDS: Singular Value Decomposition (SVD), f-k filtering, P-P waves, P-S converted waves, seismic wave field separation, de-noising.

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## INTRODUCTION

The signals gathered in a seismic wave field contain abundant geological information, but at the same time it can be highly complicated to decipher them due to the interferences from surroundings. Those interferences with noises will seriously impede the identification and tracking of target wave fields, thus affecting the extraction of precise subsurface geological information (Yilmaz, 2001). Besides, seismic wave field may be converted also, and form converted wave,which is a type of important information sources too (Li et al., 2001; Grechka and Tsvankin, 2002; Li and Yuan, 2003; Stewart et al., 2003; Lu et al., 2003; DeAngelo et al., 2004; Artola et al., 2004; Meier and Lee, 2009; Zhao et al., 2011; Yan et al., 2012). Therefore, only after precise seismic wave field separation and de-noising to gain a more pure type of wave field, we can guarantee the quality of velocity analysis and imaging, and thus ensure the complete extraction of geological information.

In seismic data processing, seismic wave field separation and de-noising is an old problem urgent to be solved. Many methods and techniques have been developed, among which f-k filtering is a relatively mature technique. Pioneers such as Embree et al. (1963), Fai and Grau (1963) proposed the 2D Fourier transform filtering (also known as the f-k filtering or apparent velocity filtering) for seismic data. Especially after the proposal of Cooley-Tukey FFT algorithm (Cooley and Tukey, 1965) and the improvements from Wiggins (1966), Treitel et al. (1967), Sengbush and Foster (1968), Zhou and Greenhalgh (1994), Duncan and Beresford (1994), Askari and Siahkoohi (2008), Adizua et al. (2015), a complete f-k filtering theory has been formed which has been widely used in seismic data processing. However, for a relatively complex seismic wave field, when several different signals with overlapping frequency or wave number are present, there are only minor differences between their apparent velocities. As a result, wave field separation and de-noising effects will be ineffective, and are prone to false frequency phenomenon, and the identification and separation of P-P waves and P-S converted waves may fail.

SVD filtering is another commonly used seismic wave field separation and de-noising technique. Hemon and Mace (1978) came up with the method of KL (Karhunen-Loeve) filter, and realized the seismic wave field separation and de-noising. Jones and Levy (1987) further demonstrated that the use of KL transformation can effectively improve the signal to noise ratio of seismic records. Because SVD can easily obtain signal eigenvalues by singular values, Freire and Ulrych (1988) tried the wave field separation and de-noising of VSP by SVD, and elaborated the method of KL transformation by SVD. Then, Jackson et al. (1991) summarized the principle of seismic data de-noising via SVD. Franco and Musacchio (2001) proposed a method of polarization filter with singular value decomposition. Vrabie et al. (2006) presented a multi-linear decomposition named higher-order singular value decomposition and unimodal independent component analysis (HOSVD/unimodal ICA) to split the recorded three-component data into two orthogonal subspaces. Bekara and Baan (2007) used SVD to improve the signal to noise ratio of seismic data, and the results showed after comparing with f-x domain convolution and median filter that SVD can effectively remove background noise, yet the ability to extract weak signals was not ideal. Porsani et al. (2009) processed seismic records with NMO, then filtered the ground roll using SVD. Shen and Li (2008, 2009, 2010, 2012a) proposed seismic wave field de-noising in the frequency domain and seismic wave field separation in linear domain via SVD. Gao et al (2013) tried separating upgoing waves and downgoing waves from zero-offset VSP data based on two-step SVD transformation. Extensive research and practice has proved that using the traditional SVD filtering method alone is very difficult to separate P-P waves and P-S converted waves (Shen and Li, 2012b). Therefore, based on the differences in apparent velocity and time-distance between P-P waves and P-S converted waves, Shen and Li (2012b) processed NMO for P-P wave and P-S converted wave respectively, and then extracted the singular value of target signals by twice SVD to reconstruct signals to achieve P-P wave and P-S seismic wave separation.

In order to take full advantage of f-k filtering technique and avoid its difficulty to separate of P-P waves and P-S converted waves, and at the same time exploit the advantages of SVD, this paper presents a combination method of SVD with f-k transformation to achieve P-P and P-S seismic wave field separation and de-noising. In other words, it aims to extract the P-P waves by SVD in the linear domain, then use of f-k filtering to separate P-S converted waves and other noise.

## METHODOLOGY

### SVD filtering

Singular Value Decomposition (SVD) is a filtering method that uses eigenvalues or singular values as the orthogonal basis in the signal space orthogonal decomposition to enhance the coherent energy, and suppress interfering signals (Freire and Ulrych, 1988; Jackson et al., 1991). Previous studies (Freire and Ulrych, 1988; Jackson et al., 1991; Vrabie et al., 2006; Shen and Li, 2008, 2009, 2010, 2012a, 2012b) had established SVD seismic wave field separation and de-noising theory.

If we set 2D seismic records section as **X**, total traces number as m and sampling points number as n, the SVD of the $m \times n$-order matrix **X** can be transformed into the multiplication of $m \times m$-order orthogonal matrix **U**, $m \times n$-order diagonal matrix $\begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}$ and $n \times n$-order orthogonal matrix **V**,

$$\mathbf{X} = \mathbf{U}\begin{bmatrix} \boldsymbol{\Sigma} & 0 \\ 0 & 0 \end{bmatrix}\mathbf{V}^T \quad , \tag{1}$$

where $\mathbf{U}$ is constituted by the eigenvalue vectors of $\mathbf{XX}^T$, $\mathbf{V}$ is constituted by the eigenvalue vectors of $\mathbf{X}^T\mathbf{X}$, $\boldsymbol{\Sigma} = \text{diag}(\sigma_1,\sigma_2,\ldots,\sigma_i,\ldots,\sigma_r)$ ($r \leq \min\{m, n\}$, and $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_i \geq \ldots \geq \sigma_r > 0$).

The singular values $\sigma_i$ are put in descending order on the main diagonal of the matrix. The number of non-zero singular values equal to the rank r of the matrix, and the rest positions of the elements are zero.

According to the reconstructed signal principle after SVD, the subscripts i of the $\sigma_i$ can be divided into four sections which are $1 \leq i \leq p-1$, $p \leq i \leq q$, $q+1 \leq i \leq r$ and $1 \leq i \leq p-1$, $q+1 \leq i \leq r$. Correspondingly, SVD filter can be divided into four types of filters that are SVD Low-pass filtering [eq. (2)], SVD Band-pass filtering [eq. (3)], SVD High-pass filtering [eq. (4)] and SVD Band-rejection filtering [eq. (5)].

$$\mathbf{X_{LP}} = \sum_{i=1}^{p-1} \sigma_i u_i v_i^T = \mathbf{U_L U_L^T X} = \mathbf{X V_L V_L^T} \quad , \tag{2}$$

$$\mathbf{X_{BP}} = \sum_{i=p}^{q} \sigma_i u_i v_i^T = \mathbf{U_B U_B^T X} = \mathbf{X V_B V_B^T} \quad , \tag{3}$$

$$\mathbf{X_{HP}} = \sum_{i=q+1}^{r} \sigma_i u_i v_i^T = \mathbf{U_H U_H^T X} = \mathbf{X V_H V_H^T} \quad , \tag{4}$$

$$\mathbf{X_{BR}} = \mathbf{X} - \mathbf{X_{BP}} \quad , \tag{5}$$

where $p \in Z$, $q \in Z$, and $1 < p < q < r$.

## Seismic separation and de-noising in linear domain via SVD

On the basis of the traditional SVD filtering technique, Shen and Li (2009, 2012a, 2012b) proposed seismic wave field separation and de-noising in linear domain via SVD. If the target signal shows poor coherence in transverse direction, it can be converted to the domain with coherence preferably in transverse direction and then extracted by SVD filter. Take the reflected waves with hyperbolic law as an example and it workflow are shown in Fig. 1. Firstly, make the original seismic records $\mathbf{X}_1$ transform to another processing domain $\mathbf{L}_1$ with better coherence in the transverse direction by NMO. Then take SVD for the signal in this domain, and extract the singular values $\mathbf{\Sigma}_2$ of the target signal to reconstruct this signal. Next carry out an inverse NMO transformation and get $\mathbf{L}_2$. Finally go back to the original data field $\mathbf{X}_2$. This is the method of seismic wave field separation and de-noising processing combined NMO with SVD.

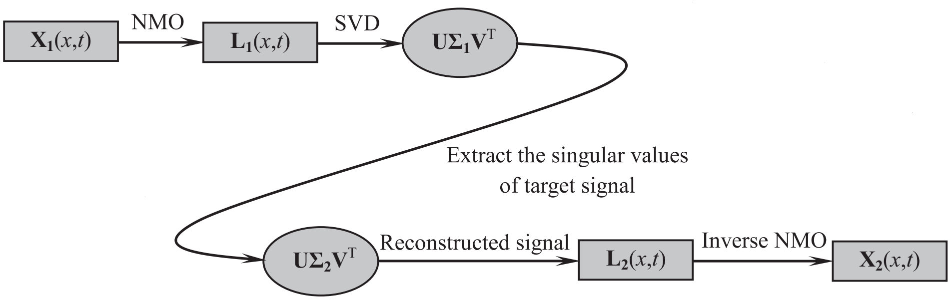


Fig. 1. Processing flowchart of seismic wave field separation and de-noising combining NMO correction with SVD filtering.

## P-P wave, P-S converted wave separation and de-noising combining SVD and f-k transform

For the seismic records in which P-P waves and P-S converted waves developed, P-P waves have hyperbolic time-distance characteristics in the t-x domain. While P-S converted waves have none because the propagation velocity between P-wave and S-wave are difference. The coherence of the P-P waves in the transverse direction will become strong after NMO correction processing. However, P-S waves, direct waves, surface waves, refracted waves, random noises and other interference noises will show weak coherence with varying degrees or even no coherence, because they are difficult to be aligned on horizontal. At this time, extracting the first few singular values in the SVD domain to reconstruct the signal will completely extract the P-P waves. Next, choosing an f-k filtering method for the residual wave field can effectively separate P-S converted waves and other noises, because there are significant differences on apparent velocity and frequency between them. The processing flowchart of the method is shown in Fig. 2.

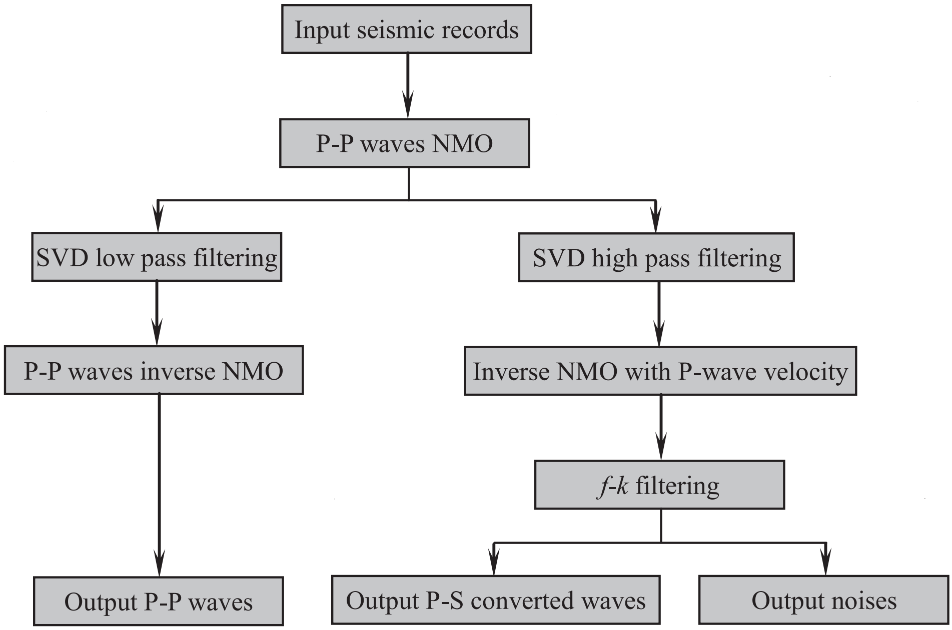


Fig. 2. Processing flowchart of P-P wave and P-S converted wave separation and de-noising combining SVD and f-k filtering.

## APPLICATION TO MODEL DATA

A shot of model data was processed, and the original seismic records are shown in Fig. 3b. The total number of channels is 100, number of sampling points is 2048 and the sampling rate is 0.2 ms. Various types of wave fields (P-P waves, P-SV converted waves, direct waves, surface waves, random noises, etc.) are visible in the data. Fig. 3a is the velocity analysis result with P-P wave time-distance equation. Fig. 3c is the seismic records after NMO correction with P-P wave velocity, in which the P-P wave arrivals have been aligned, but other signals show residual move-out to different degrees.

Fig. 4a shows the SVD singular value spectrum from the original seismic records in Fig. 3b. The energy of the singular value spectrum is dispersed because the seismic signal coherence in the horizontal direction is weak. Extraction of the first 20 $\sigma$ values to reconstruct the signal (Fig. 6a) is ineffective to carry out wave field separation. Fig. 4b shows the singular values characteristic curve after NMO correction (Fig. 3c), where it is visible that the singular value energy is relatively concentrated due to strong P-P wave coherence along the horizontal direction. Fig. 7a represents the seismic records

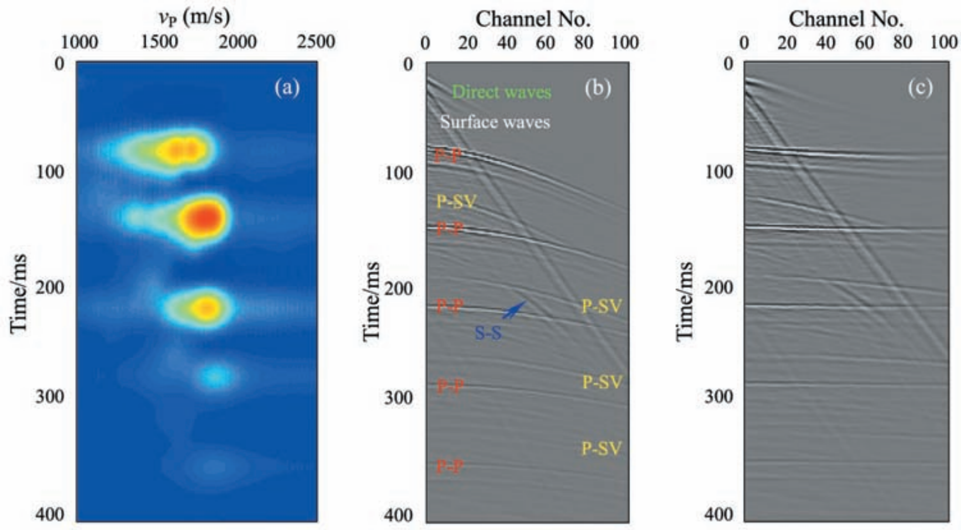


Fig. 3. Original seismic records and NMO correction processing with P-P wave velocity. (a) Velocity spectrum with P-P wave time-distance equation. (b) Original seismic record. (c) The seismic record after NMO correction with P-P wave velocity.

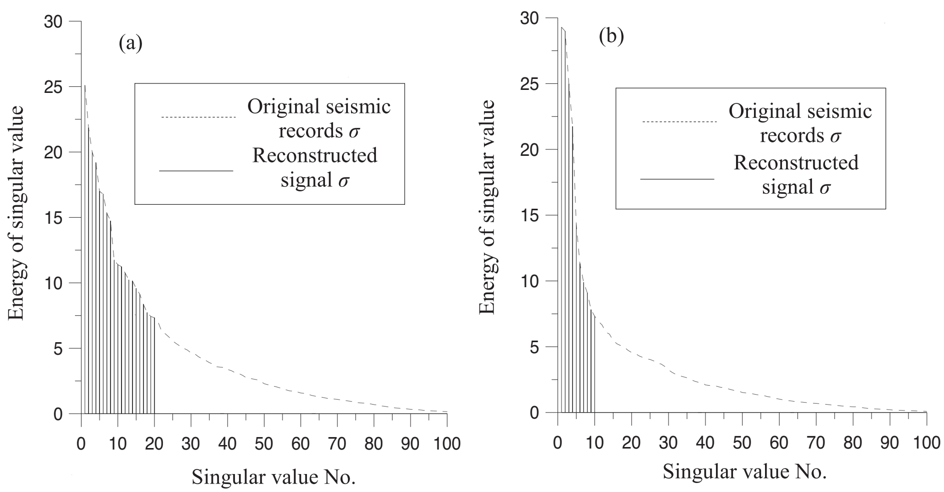


Fig. 4. Singular value features of seismic records. (a) Singular value spectrum of Fig. 3b (before NMO correction). (b) Singular value spectrum of Fig. 3c (after NMO correction).

by inverse NMO correction with extracting the first 10 $\sigma$ values to reconstruct the signal after NMO correction. The wave field separation effect is good because the direct waves, surface waves, P-S converted waves and other noises are removed, and only the P-P waves remain. Fig. 5 shows the f-k spectrum of the original seismic records (Fig. 3b) before and after f-k filtering from which it is clear that it is difficult to conduct efficient wave separation by the conventional f-k filter technique. This is because the apparent velocities of the P-P waves and P-SV converted waves are similar, and there is an overlap region on frequency and wave number. Fig. 6b demonstrates the f-k filtering result in which the P-P wave and P-SV wave field separation does not achieve the desired results.

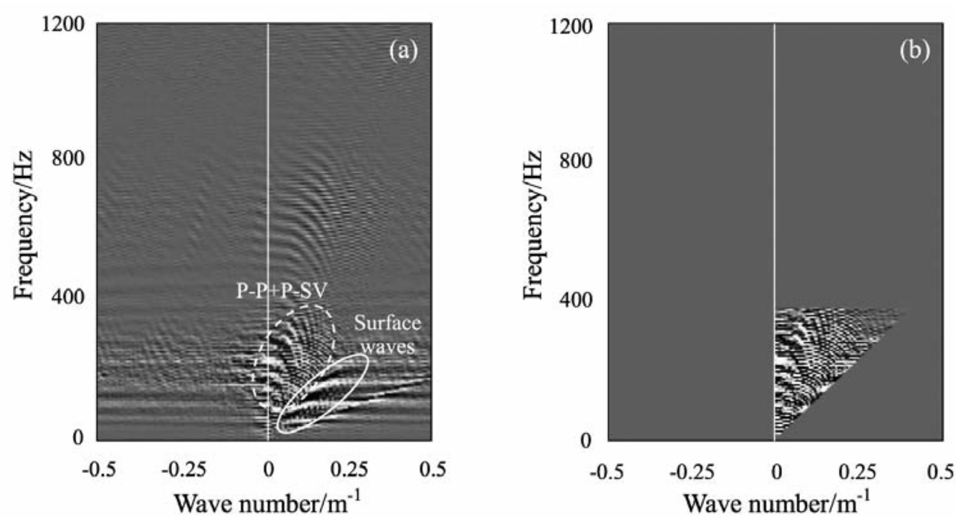


Fig. 5. f-k spectrums of the original seismic records before and after filtering. (a) The f-k spectrum before filtering. (b) The f-k spectrum after filtering.

Fig. 7b represents the seismic signals after inverse NMO correction after removing P-P waves in which the P-P wave removed clean and the relatively weak P-SV converted wave energy is also highlighting. And Fig. 8a shows the f-k spectrum after the removal of the P-P waves of Fig.7b. Compared with the f-k spectrum of the original seismic record in Fig. 5a, it can be seen that the energy of the P-P waves, which overlapped with the P-SV converted waves, has been stripped. Fig. 8b represents the corresponding f-k spectrum of extracting P-SV converted waves. Fig. 9a demonstrates the extracted P-SV converted waves, and the extraction of the wave field is complete. Fig. 9b shows the removed noises that include direct waves, surface waves, S-S waves, and so on.

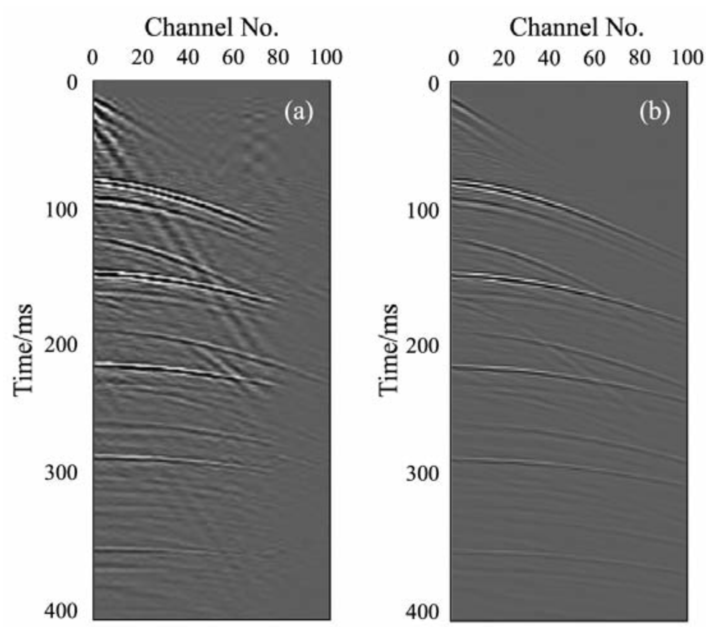


Fig. 6. Filtering results by traditional filtering technique. (a) P-P waves extracted via SVD filtering without NMO correction. (b) f-k filtering results.

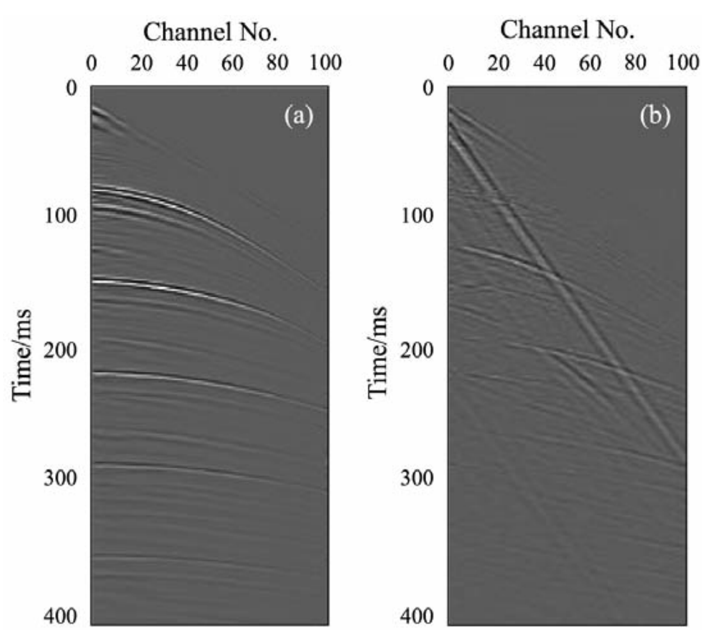


Fig. 7. Filtering results via SVD filtering after NMO correction. (a) The extracted P-P waves via SVD filtering after NMO correction with P-P wave velocity. (b) The seismic wave field after extracted P-P waves.

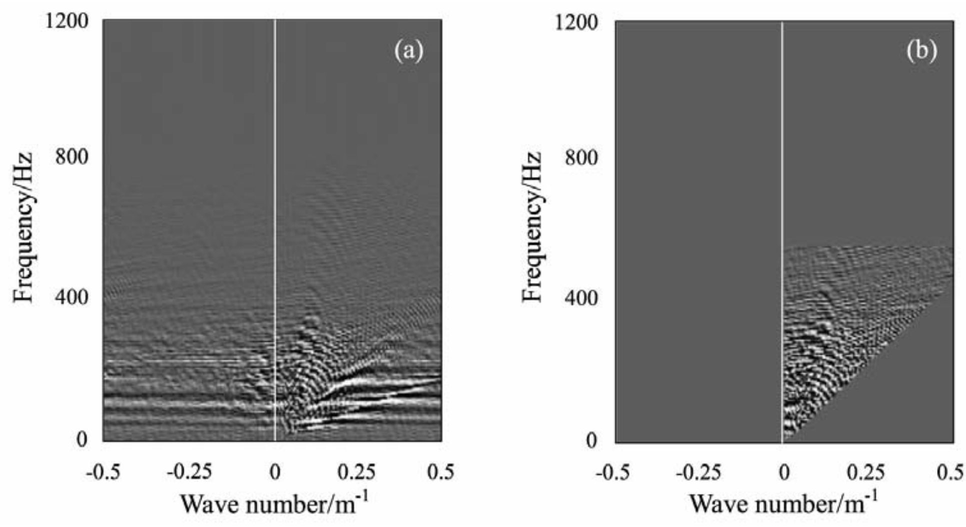


Fig. 8. The f-k spectrums before and after extracting P-SV converted waves. (a) The f-k spectrum after extracted P-P waves. (b) The f-k spectrum of extracting P-SV converted waves.

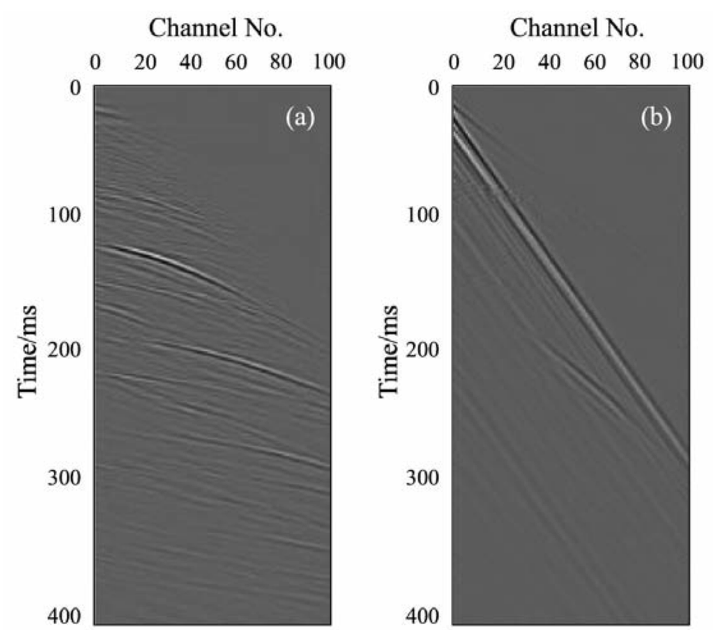


Fig. 9. f-k filtering results after extracted P-P waves in Fig. 7b. (b) The extracted P-SV converted waves by f-k filtering. (c) The seismic wave fields after extracted P-SV converted waves (Noises).

We can see from Fig. 7a and Fig. 9 that the seismic wave field separation and de-noising are more thorough for the P-P waves and the P-SV converted waves by the method, the SNR of the effective wave has increased, simultaneously, and the valid signals have scarcely lost.

## APPLICATION TO ACTUAL DATA

Next, a set of seismic data was processed that was acquired in a mountain frontal fault zone in North China. The surface layer is farming soil in the area. A 2D seismic survey line was arranged in the area in order to find the contact surface of the bedrock with the Quaternary soil, ascertain the mountain frontal fault location and geological structural characteristics near the bedrock surface. 152 shots were collected with each shot 84 channels. The full coverage number is 21, the shot spacing is 6 m, the minimum offset is 30 m, the channel spacing is 3 m, the sampling rate is 0.5 ms, and the sampling length is 1000 ms. Fig. 10a shows one original single shot seismic record (Shot No. 69). The seismic wave field is complex, the records containing direct waves, P-P waves (target layer), P-SV converted waves, surface waves, acoustic, random interference noises, etc. The P-P reflected wave event (200-300 ms) of the target layer (bedrock interface) is relatively clear, and the energy is strong. However, the

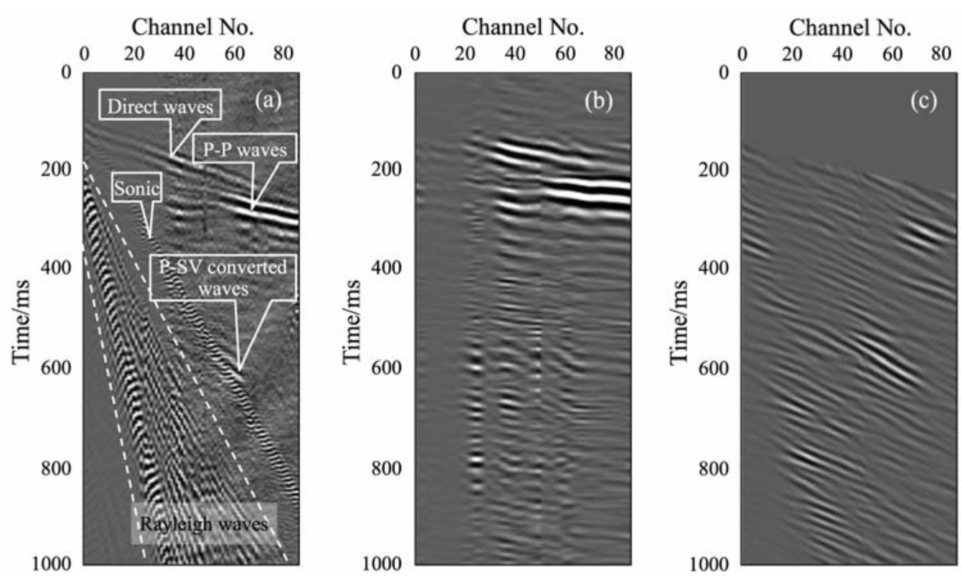


Fig. 10. Results of field seismic data processing combining with SVD and f-k filtering. (a) Original seismic data. (b) Extracted P-P waves. (c) Extracted P-SV converted waves.

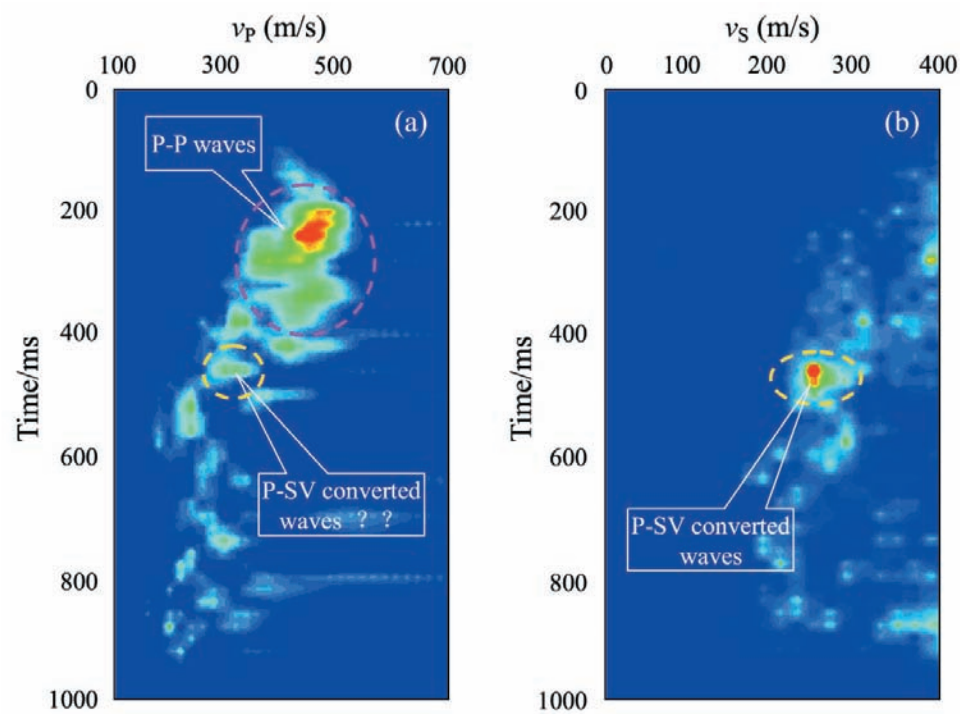


Fig. 11. P-wave and S-wave velocity analysis spectrum. (a) P-wave velocity spectrum of CMP gather NO.180. (b) S-wave velocity spectrum of CCP gather No.180.

energy of the P-SV converted waves is weaker (500-700 ms). The velocity spectrum of CMP No.180 and CCP No.180 in Fig. 11 shows that the P-wave velocity is around 470 m/s, and the SV converted wave velocity is around 250 m/s.

NMO correction was carried out for the raw data (in CMP gathers) based on the obtained P-P wave velocity. Then the first 10 $\sigma$ values were extracted to reconstruct the P-P signal after inverse NMO correction and resorting to shot records. The P-P wave separation was achieved as displayed in Fig. 10b. Next f-k filtering was carried out to separate P-SV converted waves with other noises after this first step (still in NMO-corrected CMP gathers). After inverse NMO and resorting, the separated P-SV converted wave result is shown in Fig. 10c. It can be seen from the wave field separation results (Fig. 10b and Fig. 10c) that the wave field separation is satisfactory. Fig. 12 displays the post-stack migration sections of P-P wave and P-SV converted wave field. Although the energy of the post-stack migration section is weak for the P-SV converted wave compared to the post-stack migration section of the P-P wave, the resolution of the P-SV converted wave imaging result is high and the slight tectonic is clearer (small faults) which is a strong complement to the P-P wave section data interpretation. The objective of the seismic wave field separation and de-noising has been achieved, and the SNR of the seismic data has significantly improved. In addition, the existing data has been fully utilized by combining SVD filter and f-k filter to achieve P-P wave, P-SV converted wave separation and de-noising processing.

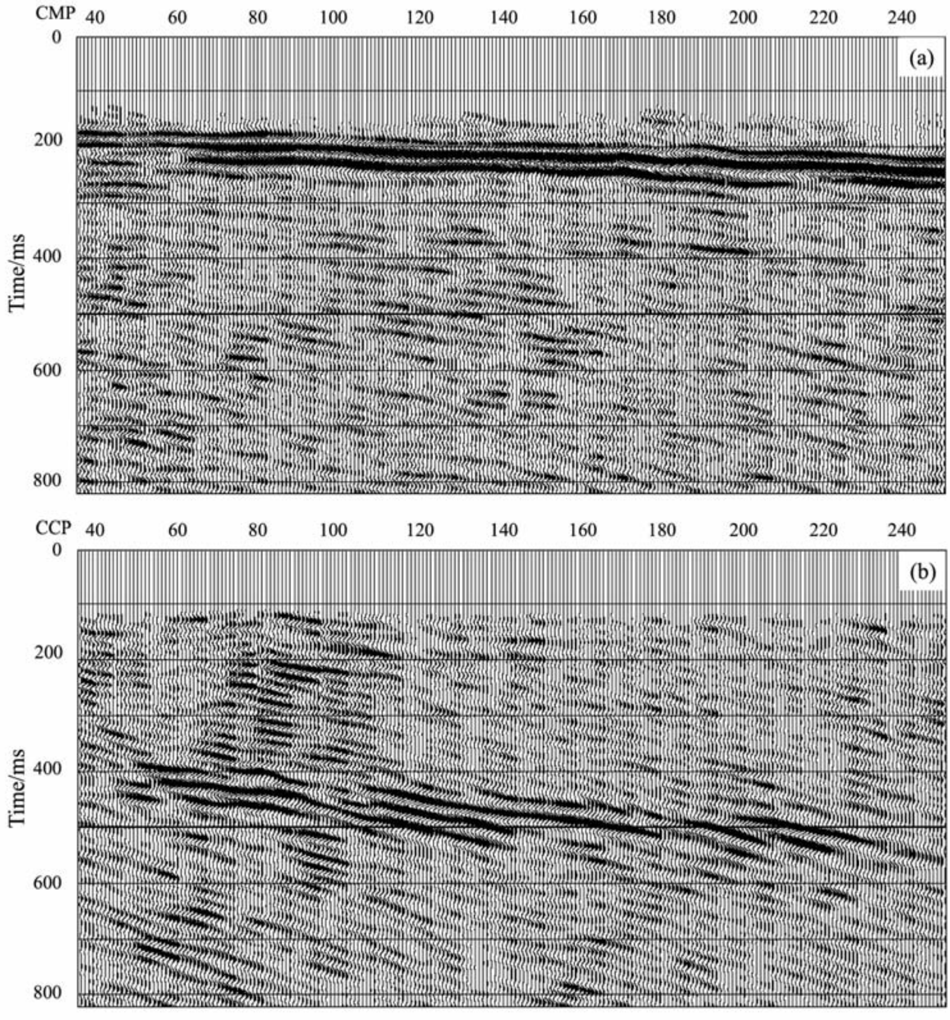


Fig. 12. Part post-stack migration sections of P-P waves and P-SV converted waves. (a) P-P wave post-stack migration section. (b) P-SV converted wave post-stack migration section.

## CONCLUSIONS

Because there are differences in propagation effects, apparent velocity, coherence and frequency for different types of seismic signals, a new workflow is introduced, that is to extract P-P waves using NMO combining with SVD, and then to separate the P-S converted waves with other noises using f-k filtering. In this way, the complementary advantages of the SVD and the f-k filtering techniques are utilized in wave field separation and de-noising, and can avoid the ineffectiveness in separation between the P-P waves and P-S converted waves by only SVD or only the f-k filtering technique. Good results have been achieved that the P-P wave and P-S wave separation are thorough, and the target signal is barely lost with model and actual data test processing. Meanwhile, the obtained data is fully utilized, not only using the P-P reflected waves with strong energy, but also using the P-S converted waves with weaker energy. Therefore, the precision of geological information extraction can be improved, and the obtained geological information is more useful.

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