$L_{1/2}$ NORM REGULARIZATION FOR 3D SEISMIC DATA INTERPOLATION

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ABSTRACT

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Sparse reconstruction of seismic data aims to reconstruct the missing traces from noise-contaminated or incomplete seismic datasets with a sparsity regularization. The L_0 and L_1 regularizations are the two most widely used methods to constrain the transform-domain coefficients. However, because of the NP-hard difficulty of L_0 regularization and non-sparsest solution of L_1 regularization, the traditional approach cannot get the optimal solutions to the seismic interpolation problems. We propose a novel $L_{1/2}$ regularization model to solve the seismic interpolation problem and borrow the efficient iterative half-thresholding solver from the signal-processing field to solve the proposed $L_{1/2}$ regularization model. Both 3D irregularly sampled synthetic data and field seismic data with 50% randomly missing traces show accurate reconstructions using the proposed approach. Comparisons with the traditional L_0 and L_1 regularizations also confirm the effectiveness of the proposed approach. Because of the simple and efficient implementation of the iterative half-thresholding algorithm, the proposed approach can be conveniently used in the industry.

KEY WORDS: sparse reconstruction, irregularly sampled seismic data, $L_{1/2}$ norm regularization.

INTRODUCTION

Sparse reconstruction has attracted a lot of attentions in recent years, which aims to reconstruct the sparse solution from noise-contaminated or incomplete datasets. The sparse reconstruction problem, simply speaking, refers to recovering $\mathbf{m}$ from the observation d according to a forward modeling equation (Gan et al., 2015; Chen et al., 2015):

$$\mathbf{d} = \mathbf{Am} + \mathbf{n} \ , \tag{1}$$

where $\mathbf{A}$ is the forward operator, and $\mathbf{n}$ is the ambient noise vector. In the exploration geophysics field, the $\mathbf{m}$ usually refers to a transformed domain coefficients vector and $\mathbf{d}$ refers to the observed data with regularly or irregularly missing traces. The L_0 norm regularization can be used to solve $\mathbf{m}$ by using a balanced objective function between the L_2 norm of data misfit and the L_0 norm of the model vector (Wang et al., 2015). However, L_0 is usually NP-hard, with the complexity of the model proportional to the number of variables, and solving the model is generally intractable when N is large. Alternatively, L_0 regularization can be relaxed to L_1 regularization (Daubechies et al., 2004; Chen et al., 2014), which can be transformed into an equivalent convex quadratic optimization problem, and thus can be solved efficiently. Under some mild conditions, the sparsity-based solution can be equivalent to that from L_0 regularization (Donoho, 2005). Because of these reasons, the L_1 regularization is becoming more and more popular in the field of sparse optimization. However, it often introduces extra bias in estimation under some situations, and cannot recover a signal with the least measurements when applied to compressive sensing (Meinshausen and Yu, 2009). Thus, a further modification of the sparse recovery model is still required. One of the efforts tried in the literature is the general L_p regularization (Chartrand, 2007). In this paper, inspired from the signal-processing field, we propose a novel $L_{1/2}$ norm regularization tailored for reconstructing irregularly sampled seismic data. The fast solver of $L_{1/2}$ norm regularization is applied to 3D seismic data reconstruction and is demonstrated to be superior than the alternative iterative soft thresholding and hard thresholding methods. Both 3D synthetic and field seismic datasets with irregularly sampled traces show accurate reconstruction using the proposed approach when a high percentage of traces are missing, which is obviously better than L_0 and L_1 regularizations.

THEORY

Formulation of seismic interpolation

The basic target of seismic interpolation is to solve the following equation:

$$\mathbf{d} = \mathbf{M}\mathbf{d_r} \ , \tag{2}$$

where $\mathbf{d}$ is the observed data which is regularly or irregularly sampled, $\mathbf{d_r}$ is the unknown data needed to be reconstructed and $\mathbf{M}$ is the sampling matrix. Because of the seriously badly-posed problem of eq. (2) due to many zero entries in $\mathbf{M}$, we need to add some regularizations when solving eq. (2). An appropriate way is to enforce sparsity in a transform domain (Chen et al., 2014, 2015):

$$\min_{\mathbf{d_r}} \| \mathbf{d} - \mathbf{M}\mathbf{d_r} \|_2^2 + \mathbf{R}(\mathbf{S}\mathbf{d_r}) \ , \tag{3}$$

where $\mathbf{R}$ denotes a regularization function enforcing a sparsity constraint and $\mathbf{S}$ denotes the forward sparse transform. Let $\mathbf{m} = \mathbf{S}\mathbf{d_r}$, then eq. (3) can be transformed into:

$$\min_{\mathbf{m}} \| \mathbf{d} - \mathbf{A}\mathbf{m} \|_2^2 + \mathbf{R}(\mathbf{m}) \ , \tag{4}$$

where $\mathbf{A} = \mathbf{M}\mathbf{S}^{-1}$, $\mathbf{S}^{-1}$ is the inverse of the sparse transform $\mathbf{S}$. Selection of the regularization function is of great importance when solving problem (4). There existed a lot of methods in the literature according to different regularization functions. Two most widely used approaches are based on L_0 and L_1 regularizations.

Several regularized optimization models

The L_0 regularization problem aims to find the solution to the following model:

$$\min_{\mathbf{m}} \| \mathbf{d} - \mathbf{A}\mathbf{m} \|_2^2 + \lambda \| \mathbf{m} \|_0 \ , \tag{5}$$

where $\| \cdot \|_0$ denotes the L_0-norm, which corresponds to the number of non-zero entries for a given vector and $\| \cdot \|_2$ denotes the L_2-norm. λ is a regularization parameter. Eq. (5) can be understood as a penalized least-squares data misfit with penalty $\| \mathbf{m} \|_0$, and λ balances the two objective terms. To deal with such NP-hard difficulty, eq. (5) is relaxed to the following L_1 regularization:

$$\min_{\mathbf{m}} \| \mathbf{d} - \mathbf{A}\mathbf{m} \|_2^2 + \lambda \| \mathbf{m} \|_1 \ , \tag{6}$$

where $\| \cdot \|_1$ denotes the L_1-norm. Even though L_1 is becoming more and more popular,there are still some problems. For example, it often introduces extra bias in estimation under some situations, and cannot recover a signal with the

least measurements when applied to compressive sensing (Meinshausen and Yu, 2009). Thus, a further modification of the sparse recovery model is still required. One of the efforts tried in the literature is the general L_p regularization (Chartrand, 2007; Chartrand and Staneva, 2008):

$$\min_{m} \| \mathbf{d} - \mathbf{Am} \|_2^2 + \lambda \| \mathbf{m} \|_p^p , \tag{7}$$

where $\| \cdot \|_p^p \, (0 < p < 1)$ is the L_p quasi-norm.

$L_{1/2}$ regularization and its fast solver

Krishnan and Fergus (2009) demonstrated the very high efficiency of $L_{1/2}$ and $L_{2/3}$ regularization when applied to image deconvolution. Xu et al. (2012) showed the representativeness of $L_{1/2}$ regularization among all L_p regularizations. Their results basically revealed that the L_p regularizations can assuredly generate more sparse solutions than L_1 regularization, and 1/2 plays a representative role because whenever $1/2 < p < 1$, the smaller the p is, the sparser the solutions yielded by L_p regularizations can be and the solutions stay relatively constant when $0 < p < 1/2$. Inspired by these discoveries, we proposed a novel $L_{1/2}$ regularization for seismic interpolation here:

$$\min_{m} \| \mathbf{d} - \mathbf{Am} \|_2^2 + \lambda \| \mathbf{m} \|_{½}^{½} . \tag{8}$$

In order to solve eq. (8) efficiently, similar to the iterative shrinkage-thresholding method for L_1 regularization. We use an efficient iterative half-thresholding method for $L_{1/2}$ regularization, proposed by Xu et al. (2012):

$$\mathbf{m}_{n+1} = \mathbf{T}[\mathbf{m}_n + \mathbf{A}^H(\mathbf{d} - \mathbf{Am}_n)] , \tag{9}$$

where $[\cdot]^H$ denotes adjoint and $\mathbf{T}$ denotes the half-thresholding operator:

$$\mathbf{T}[v(\mathbf{m})] = \tag{10}$$

$$\begin{cases} (2/3)v(\mathbf{m})\{1 + \cos[(2/3)\pi - (2/3)\arccos((\tau/8)(|v(\mathbf{m})|/3)^{-3/2})]\} \\ \qquad\qquad \text{for } |v(\mathbf{m})| > \gamma \\ \\ 0 \qquad\qquad \text{for } |v(\mathbf{m})| \le \gamma \end{cases} \tag{11}$$

$v(\mathbf{x})$ denotes the amplitude of each position-coordinate vector $\mathbf{x}$ and γ is a pre-defined threshold value.

EXAMPLE

We use one synthetic example and one field data example to demonstrate the performance of the $L_{1/2}$ norm regularizations for reconstructing missing seismic data. Both synthetic and field data examples are based on 3D datasets. We compare the $L_{1/2}$ norm regularization with both L_0 and L_1 regularizations. We use the well-known iterative hard thresholding operator to enforce the L_0 regularization and use iterative soft thresholding operator to enforce the L_1 norm regularization. For choosing the threshold value, we use the percentile thresholding strategy, assuming that a constant percentage of the coefficients in the transformed domain are significant to represent the whole seismic records. For the fair comparison, we choose the best results for each different approaches. In order to numerically compare the reconstruction performance, we define the signal-to-noise ratio (SNR) (Zhong et al., 2015) as:

$$\text{SNR}_n = 10 \log_{10}\{ \| \mathbf{d}_r \|_2^2 \, / \, \| \mathbf{d}_r - \mathbf{d}_n \|_2^2 \} \quad , \tag{12}$$

where $\mathbf{d}_n = \mathbf{S}^{-1}\mathbf{m}_n$ is the reconstructed data after the n-th iteration.

The synthetic example is a post-stack 3D dataset, as shown in Fig. 1. Fig.1a shows the original 3D synthetic data example, and Fig. 1b shows the irregularly sampled data with 50% randomly missing traces. The synthetic data contains three plane-wave components. We apply the $L_{1/2}$ regularization with its fast solver onto the decimated 3D dataset and obtain the recovered data as shown in Fig. 2c. As a reference, we also obtain the result by L_1 and L_0 regul-
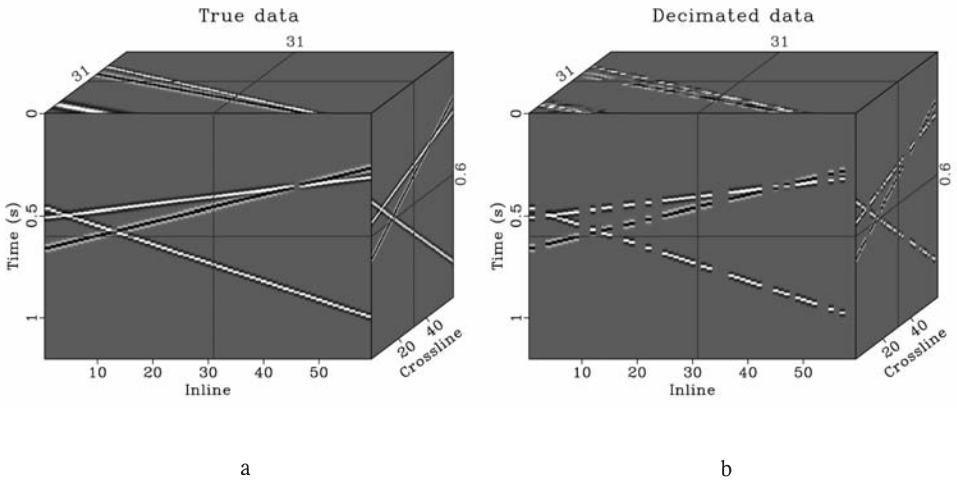

Fig. 1. Synthetic data example. (a) Original 3D synthetic data example. (b) Data with 50% randomly missing traces.

arizations with the traditional iterative soft and hard thresholding solvers and show them in Figs. 2a and 2b. Fig. 3 shows the error sections for three different approaches, after amplifying the error sections 20 times. It is obvious that the $L_{1/2}$ regularization causes the least reconstruction error. It is apparent that the $L_{1/2}$ norm regularization can obtain the least reconstruction error, and the L_1 norm regularized result seems to obtain the largest reconstruction error. The SNRs of Figs. 2a, 2b, and 2c are 22.45 dB, 25.03 dB, and 28.21 dB, respectively. The $L_{1/2}$ regularization obtains the highest SNR for the synthetic example.
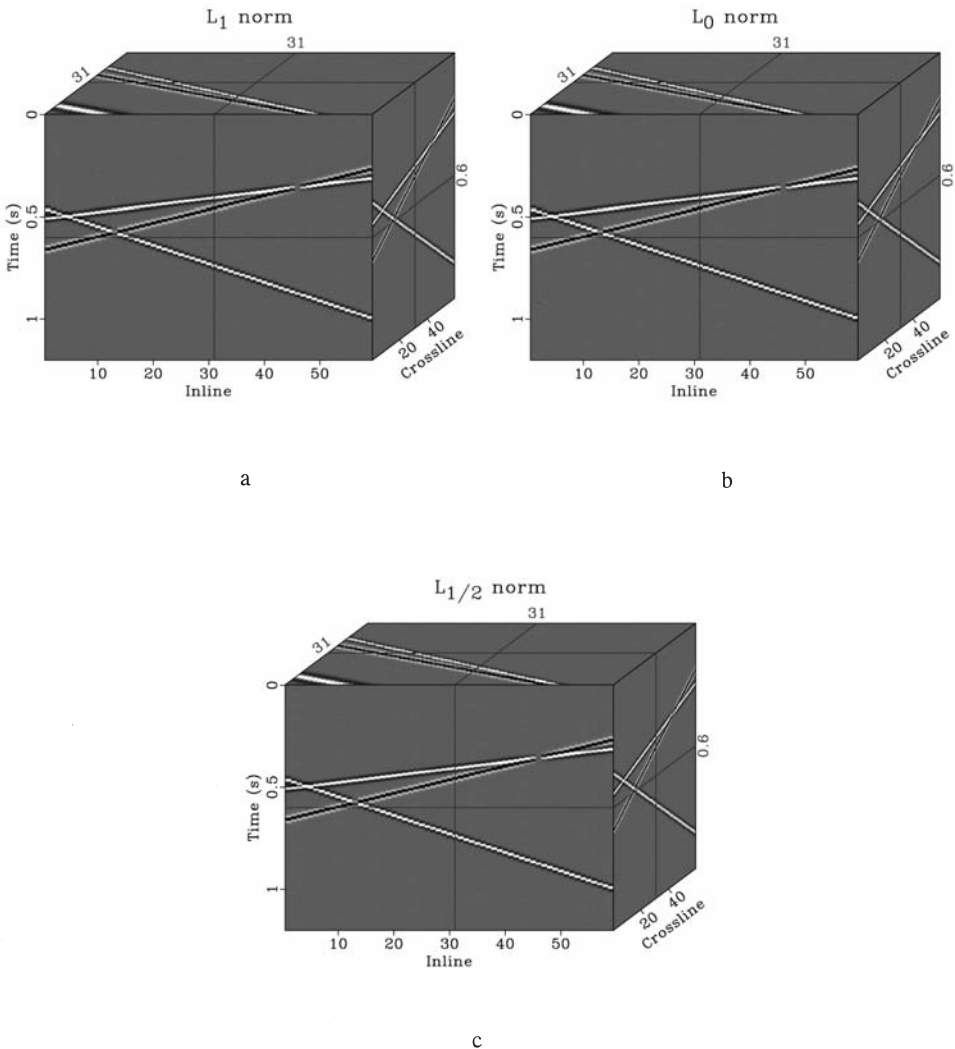
Fig. 2. Reconstructed sections for the synthetic example using (a) L_1-norm regularization; (b) L_0-norm regularization, and (c) $L_{1/2}$-norm regularization.
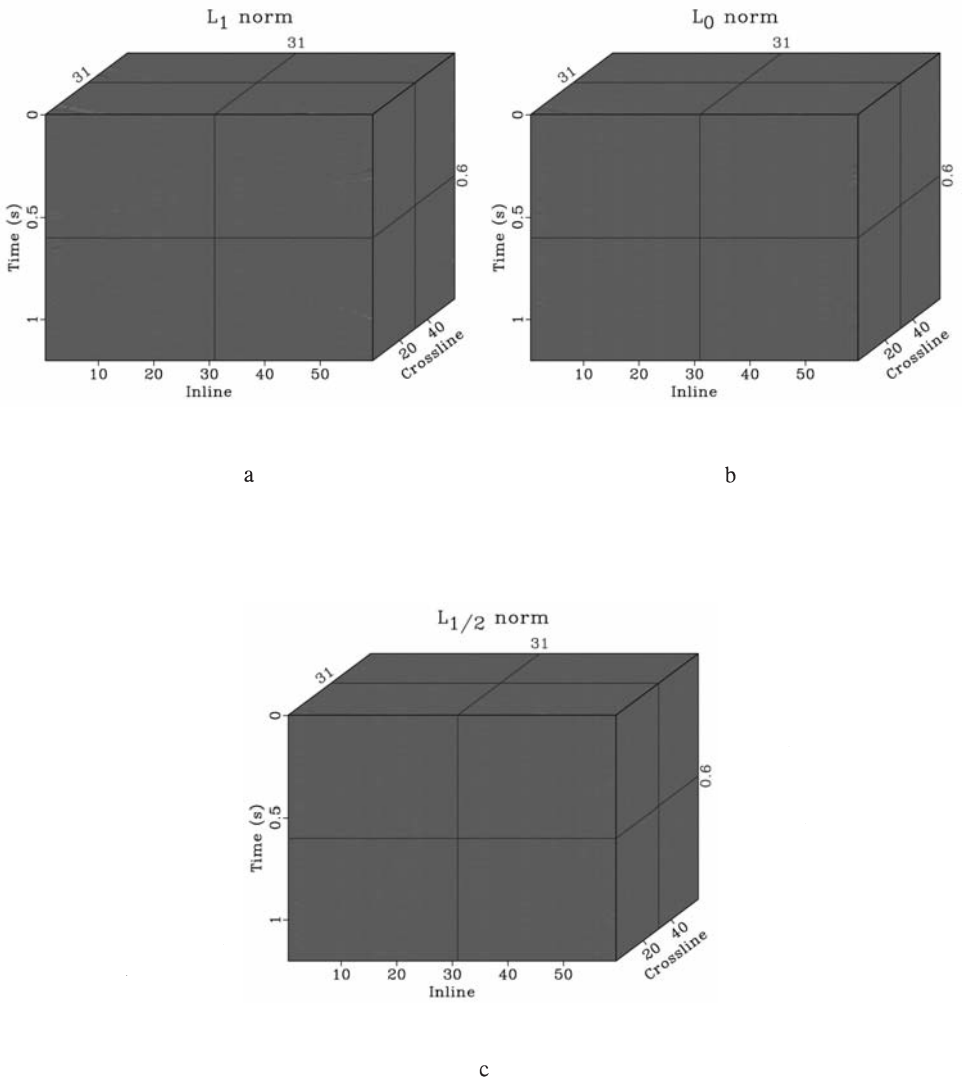
Fig. 3. Error sections for the synthetic example (amplified 20 times) using (a) L_1-norm regularization; (b) L_0-norm regularization, and (c) $L_{1/2}$-norm regularization.

The field data example is a pre-stack 3D dataset coming from a 2D survey. The original field data is shown in Fig. 4a and the decimated data with 50% traces randomly removed is shown in Fig. 4b. After using three different regularization approaches, we can obtain the three reconstructed results as

shown in Fig. 5. Fig. 6 shows the reconstruction error sections using three different approaches. As we can see, the recovered data using $L_{1/2}$ regularization is better than L_1 and L_0 regularizations, because of the more continuous recovered events and less estimation error. The SNRs of Figs. 5a, 5b, and 5c are 10.77 dB, 11.53 dB, and 12.76 dB, respectively. The $L_{1/2}$ regularization still obtains the highest SNR.

　　　In the examples, we choose the 3D FFT transform as the sparsity promoting transform. Please note that other sparsity-promoting transforms will cause similar performance. We do not use local windows to process the data. Although the plane assumption of the 3D FFT transform can be better satisfied in local windows, processing in local windows lacks the global constraint since FFT in local windows is more like a local transform. Since the difference between the three regularization methods when using the fast iterative solvers is just the thresholding function, the run time of the three methods are almost the same.
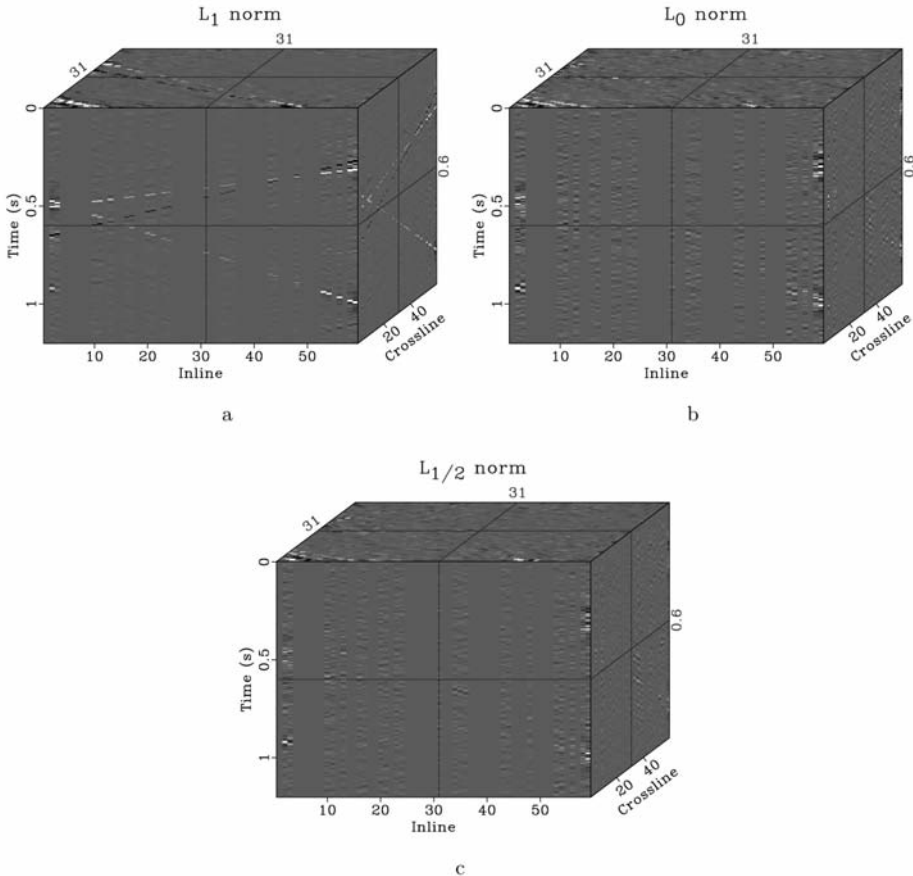

Fig. 4. Error sections for the synthetic example (amplied 20 times) using
(a) L1 norm regularization, (b) L0 norm regularization, and (c) L1=2 norm regularization.
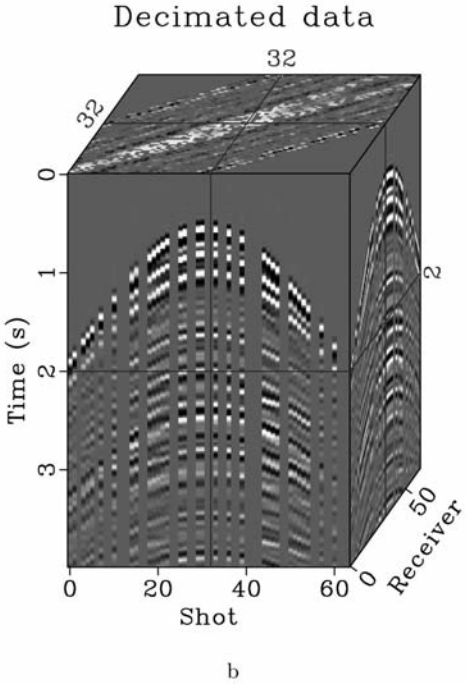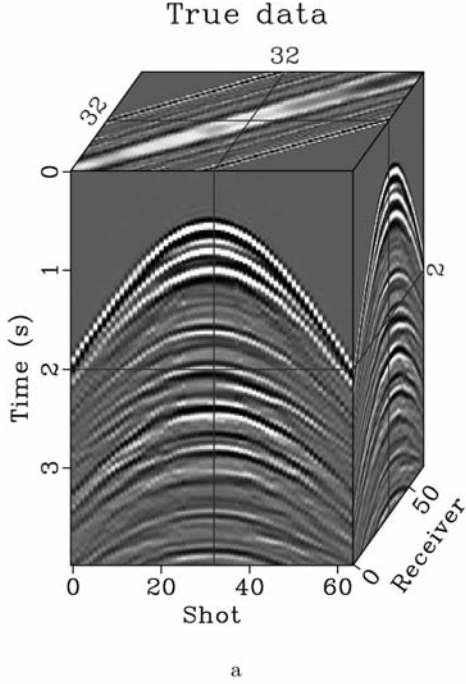
True data

Decimated data

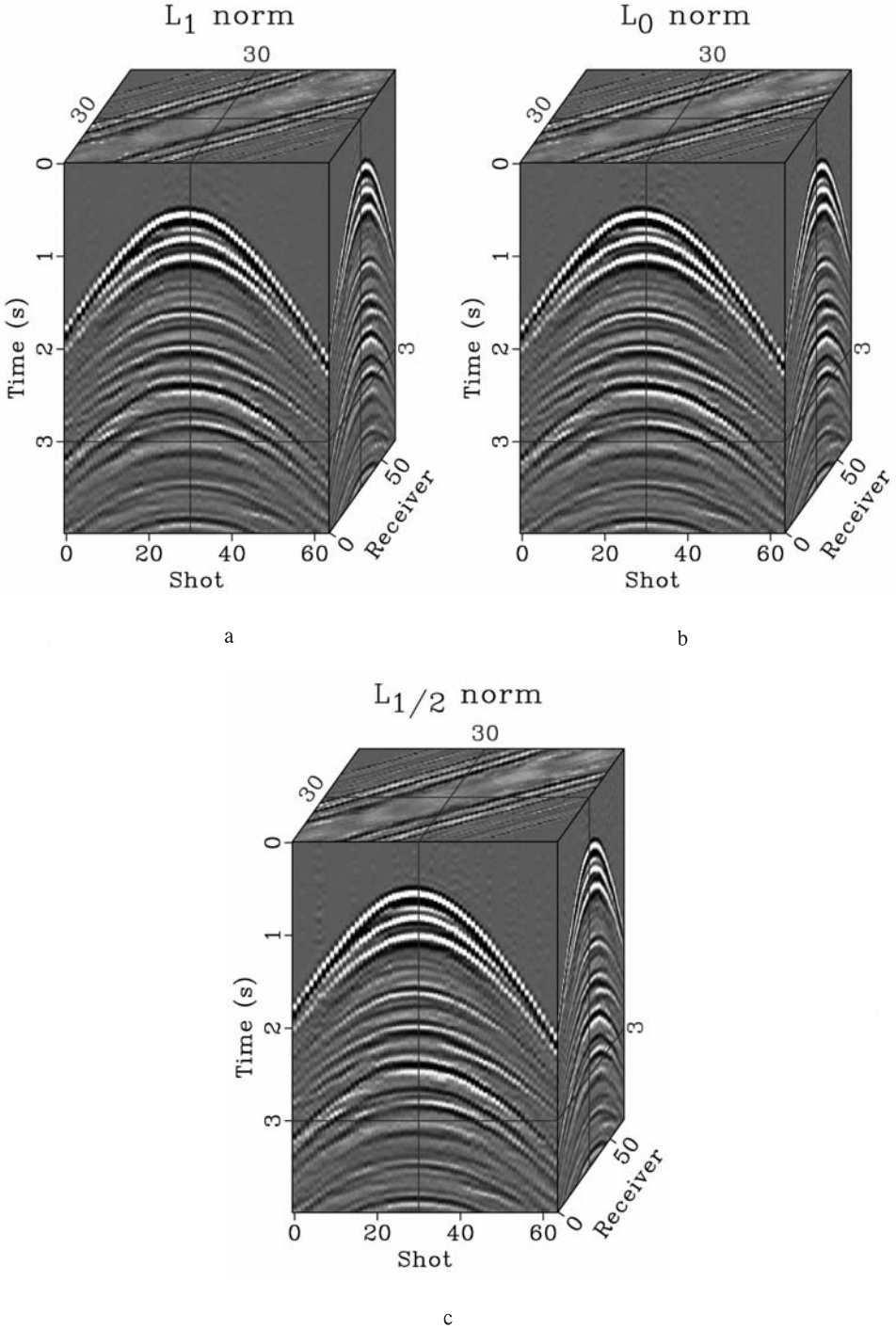Fig. 5. Field data example. (a) Original 3D field data example. (b) Data with 50% randomly missing traces.

Fig. 6. Reconstructed sections for the field data example using (a) L_1-norm regularization; (b) L_0-norm regularization, and (c) $L_{1/2}$-norm regularization.
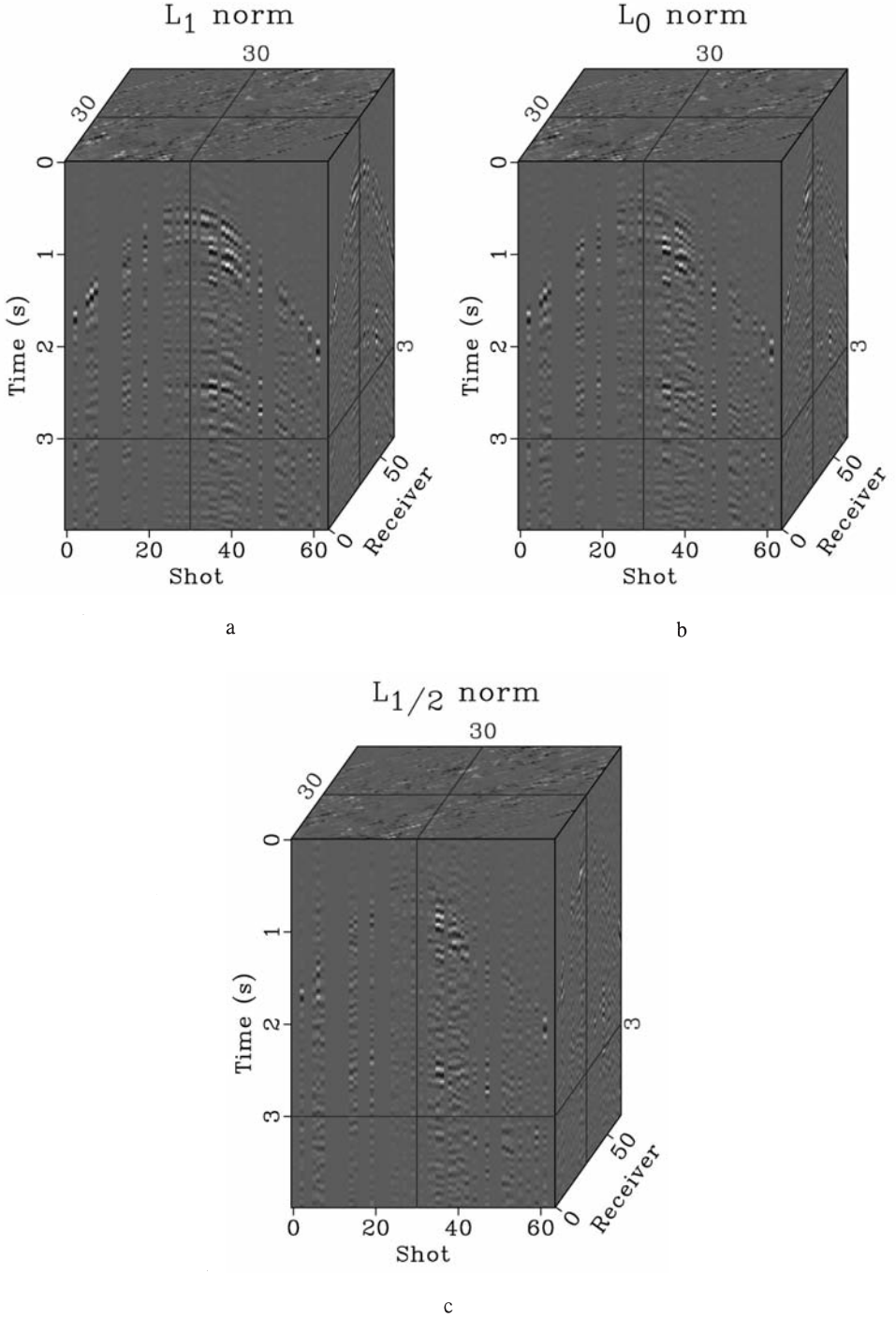
Fig. 7. Error sections for the field data example using (a) L_1-norm regularization, (b) L_0-norm regularization, and (c) $L_{1/2}$-norm regularization.

CONCLUSIONS

Conventional L_0 and L_1 regularizations cannot obtain the optimal solutions when applied to solve the seismic interpolation problem because of the NP-hard difficulty of L_0 regularization and the non-optimal sparsity constraint of L_1 regularization. We borrow the $L_{1/2}$ norm regularization and its fast thresholding solver from the signal-processing field and apply them onto the seismic interpolation problem for reconstructing highly incomplete seismic data. The performance on both 3D synthetic and field seismic data with 50% randomly missing traces show very promising results. Compared with L_0 and L_1 regularizations, the $L_{1/2}$ regularization can obtain obviously better result. The fast iterative half-thresholding solver enforcing $L_{1/2}$ regularization can be conveniently used in the industry.

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