

ITERATIVE ADAPTIVE APPROACH FOR SEISMIC DATA RESTORATION

ZHIGANG DAI, ZHIHUI LIU and JINYAN WANG

School of Mathematics and Physics, China University of Geosciences, Wuhan 430074, P.R. China. zhliu@cug.edu.cn

(Received September 3, 2018; accepted May 2, 2019)

ABSTRACT

Dai, Z.G., Liu, Z.H. and Wang, J.Y., 2019. Iterative adaptive approach for seismic data restoration. *Journal of Seismic Exploration*, 28: 333-345.

Reconstruction of missing traces of seismic data from finite samples is a problem in seismic data processing. In this paper, an iterative adaptive approach is proposed to restore seismic data with randomly missing traces, specifically, which is suitable to recover a large number of missing traces. The proposed method is based upon the weighted least square theory. Unlike previous low-rank methods that use the low-rank property of the Hankel matrix on each frequency slice, we exploit the harmonic structure of frequency slice, and develop an iterative adaptive manner for seismic temporal frequency slices to obtain an accurate spectral estimation. The missing data is filled using a linear minimum mean-squared error estimator. Numerical experiments show that our method provides much better performance for reconstruction compared to that of the classical low-rank methods such as iterative soft thresholding, low-rank matrix fitting and orthogonal rank-one matrix pursuit.

KEY WORDS: seismic data restoration, iterative adaptive approach, weighted least squares, spectral estimation.

INTRODUCTION

In seismic exploration, missing traces have been a common problem because of the limitation of physical and economic constraints. Seismic data with missing traces cause substantial problems in a variety of seismic applications, including the inversion, migration, multiple suppression and so on. Therefore, recovering these missing traces is of great importance for seismic research.

Various methods based on different techniques have been developed to reconstruct missing seismic data. Prediction filter methods have been implemented by exploiting the linear predictability of the signal in different domains and can efficiently address spatially aliased regularly sampled data (Spitz, 1991; Claerbout and Nichols, 1991; Li et al., 2018). However, they are inapplicable to irregularly sampled data. Techniques based on wave-field continuation operator use the physical characteristics of seismic wave fields in subsurface media for restoration, such as the shot gather continuation (Bagaini and Spagnolini, 1993), offset continuation (Bolondi et al., 1982) and dip-move-out (DMO) processing methods (Deregowski, 1986; Canning and Gardner, 1996). Unfortunately, they require considerable computing time and the interpolation performance relies heavily on the information of the underground medium. The transform-based methods mainly include Fourier (Duijndam and Schonewille, 1999; Liu and Sacchi, 2004; Zwartjes and Sacchi, 2006), Radon (Thorson, 1985; Sacchi and Ulrych, 1995; Kabir and Verschuur, 1995) and curvelet (Hennenfent et al., 2010; Zhang et al., 2015), which find the coefficients in the transform domain via least squares inversion and obtain reconstructed data via an inverse transformation. However, they are restricted to spatial aliasing and only suitable for band limited data.

Recently, rank-reduction methods have achieved significant success in seismic data restoration. In this kind of methods, seismic data are assumed to be low-rank structure by applying some transformation such as Hankel (Trickett et al., 2010) and texture-patch transformation (Ma, 2013). The Cadzow filtering methods organize spatial data at each frequency slice into a block Hankel matrix, and then apply rank-reduction algorithms via the singular value decomposition (SVD) to reconstruct seismic data (Oropeza and Sacchi, 2011). However, they are not computationally feasible for large-scale data due to operating repeatedly SVD. Several techniques based on matrix completion have been adopted to solve the computational problem of SVD. Cai et al. (2008) develop one efficient singular value thresholding algorithm (IST) and they perform soft thresholding operations on the singular values of a certain matrix by a iterative manner. Wen et al. (2012) propose a low-rank matrix factorization algorithm (LMaFit) which exploits nonlinear successive overrelaxation to interpolate the missing data without performing the SVD. Wang et al. (2014) only computes the left and right top singular vectors by the power method and avoid the SVD, and they present orthogonal rank-one matrix pursuit algorithm (OR1MP) for restoring missing seismic data.

In this paper, an iterative adaptive approach (IAA) is introduced to reconstruct two-dimensional (2D) seismic data with randomly missing traces. The proposed IAA method based on the weighted least squares theory first obtains accurate spectral estimation from each frequency slice of given data using an iterative adaptive manner. A recovery step of missing data is performed via a linear minimum mean-squared error (MMSE) estimator. Moreover, the IAA method is applicable for restoration of a large number of missing traces. Numerical experiments on synthetic and real seismic data

show that the proposed method provides much better performance than the existing low-rank methods such as IST, LMaFit and OR1MP.

This paper is organized as follows. We first provide a description of 2D seismic data with linear events. Then, we present the IAA method to obtain spectral estimation from the available data. Subsequently, we study the missing data recovery using a linear MMSE estimator. Next, numerical experiments are provided to validate the reconstruction performance of our approach for seismic data restoration. Finally, conclusions and future work are presented.

THEORY ANALYSIS

2D seismic data model

We consider that 2D seismic data with ray parameter which in a small window can be represented as follows (Chen and Sacchi, 2015):

$$y_n(\omega) = \sum_{k=1}^K s_k(\omega) e^{-j2\pi\omega p_k(n-1)\Delta x}, \quad n = 1, 2, \dots, N, \quad (1)$$

where $j = \sqrt{-1}$, N , p_k and ω denote trace numbers, ray parameter and temporal frequency, respectively. Here, Δx is the spatial interval between two adjacent traces, and $s_k(\omega)$ denotes complex amplitude corresponding to the k -th plane wave. Because the following analysis is valid for all frequencies, we omit the symbol ω and let $\tau_k = \omega p_k \Delta x$, and then rewrite the model (1) as follows:

$$y_n = \sum_{k=1}^K s_k e^{-j2\pi\tau_k(n-1)}, \quad n = 1, 2, \dots, N \quad (2)$$

The data model in (2) can be equivalently written as follows:

$$\mathbf{y} = \sum_{k=1}^K s_k \mathbf{a}(\tau_k) = \mathbf{A} \mathbf{s}, \quad (3)$$

where, $\mathbf{a}(\tau_k) = [1, e^{-j2\pi\tau_k}, \dots, e^{-j2\pi(N-1)\tau_k}]^T$ $\mathbf{A} = [\mathbf{a}(\tau_1), \mathbf{a}(\tau_2), \dots, \mathbf{a}(\tau_K)]$,

$\mathbf{y} = [y_1, y_2, \dots, y_N]^T$ and $\mathbf{s} = [s_1, s_2, \dots, s_K]^T$, and $(\cdot)^T$ denotes the transpose.

The number of linear events K is usually unknown in practice. Therefore, the region of interest is discretized into some grid points, and the grid point is considered as a potential τ_k .

We consider a reconstruction problem of missing traces and the observed samples, denoted by \mathbf{y}_Ω , is corrupted seismic data with missing traces. Ω denotes subscript for available seismic traces and $\Omega \subset \{1, 2, \dots, N\}$ (of size $L < N$). Given partial samples \mathbf{y}_Ω , our object is to recover the complete data \mathbf{y} .

We define the sampling matrices \mathbf{U}_Ω and $\mathbf{U}_{\bar{\Omega}}$, which are composed of 0 and 1, and obtain the following formula:

$$\mathbf{y}_\Omega = \mathbf{U}_\Omega \mathbf{y}, \quad \mathbf{y}_{\bar{\Omega}} = \mathbf{U}_{\bar{\Omega}} \mathbf{y} \quad , \quad (4)$$

where $\bar{\Omega} \triangleq \{1, 2, \dots, N\} \setminus \Omega$ is the complementary set of Ω .

Because the sampling matrices satisfy $\mathbf{U}_\Omega^T \mathbf{U}_\Omega = \mathbf{I}_L$ and $\mathbf{U}_{\bar{\Omega}}^T \mathbf{U}_{\bar{\Omega}} = \mathbf{I}_{N-L}$, the missing data model (4) can be further reformulated as follows:

$$\mathbf{y}_\Omega = \mathbf{U}_\Omega \mathbf{y} = \mathbf{U}_\Omega \mathbf{A} \mathbf{s} \triangleq \mathbf{A}_\Omega \mathbf{s} \quad , \quad (5)$$

where $\mathbf{A}_\Omega = [\mathbf{a}_\Omega(\tau_1), \mathbf{a}_\Omega(\tau_2), \dots, \mathbf{a}_\Omega(\tau_K)]$ and $\mathbf{a}_\Omega(\tau_k)$ is a subvector of $\mathbf{a}(\tau_k)$ indexed by Ω .

IAA for spectral estimation

IAA is a non-parametric iterative adaptive approach based on weighted least squares, which has been recently attracting attention (Zhang et al., 2016; Feng et al., 2018). IAA first obtains an accurate spectral estimation from the given data by an iterative adaptive manner, and then reconstructs the missing data using a linear MMSE estimator.

Let

$$p_k = |s_k|^2, \quad k = 1, 2, \dots, K \quad (6)$$

denote the signal power estimate at the grid point τ_k . The interference covariance matrix at τ_k in the given data has the following expression (Stoica et al., 2009; Xue et al., 2011):

$$\mathbf{Q}_\Omega(\tau_k) = \mathbf{R}_\Omega - p_k \mathbf{a}_\Omega(\tau_k) \mathbf{a}_\Omega^H(\tau_k) \quad . \quad (7)$$

The covariance matrix of \mathbf{y}_Ω , denoted by \mathbf{R}_Ω , is expressed as:

$$\mathbf{R}_\Omega = E\{\mathbf{y}_\Omega \mathbf{y}_\Omega^H\} = \sum_{k=1}^K p_k \mathbf{a}_\Omega(\tau_k) \mathbf{a}_\Omega^H(\tau_k) = \mathbf{A}_\Omega \mathbf{P} \mathbf{A}_\Omega^H \quad , \quad (8)$$

where $\mathbf{P} = \text{diag}(\mathbf{p})$ with $\mathbf{p} = [p_1, p_2, \dots, p_K]^T$, and $\text{diag}(\cdot)$ denotes the diagonal matrix, and $(\cdot)^H$ denotes conjugate transpose.

IAA estimates the spectral amplitude S_k at the grid point τ_k which is based on a weighted least squares criterion can be formulated as (Yardibi et al., 2010):

$$\min_{s_k} \left\| \mathbf{y}_\Omega - s_k \mathbf{a}_\Omega(\tau_k) \right\|_{\mathbf{Q}_\Omega^{-1}}^2, \quad (9)$$

where $\| \mathbf{x} \|_{\mathbf{W}}^2 \triangleq \mathbf{x}^H \mathbf{W} \mathbf{x}$.

The solution to the optimization problem (9) at τ_k is as follows (Stoica et al., 2009; Karlsson et al., 2014; Glentis et al., 2011):

$$\hat{s}_k = \frac{\mathbf{a}_\Omega^H(\tau_k) \mathbf{Q}_\Omega^{-1}(\tau_k) \mathbf{y}_\Omega}{\mathbf{a}_\Omega^H(\tau_k) \mathbf{Q}_\Omega^{-1}(\tau_k) \mathbf{a}_\Omega(\tau_k)}, \quad k=1, 2, \dots, K \quad (10)$$

It follows from that eq. (7) and the matrix inversion lemma, the amplitude spectral in eq. (10) becomes the following:

$$\hat{s}_k = \frac{\mathbf{a}_\Omega^H(\tau_k) \mathbf{R}_\Omega^{-1} \mathbf{y}_\Omega}{\mathbf{a}_\Omega^H(\tau_k) \mathbf{R}_\Omega^{-1} \mathbf{a}_\Omega(\tau_k)}, \quad k=1, 2, \dots, K \quad (11)$$

Because \mathbf{R}_Ω depends on S_k which is unknown, eq. (11) is implemented using an iterative manner. The initialization is completed by setting $\mathbf{R}_\Omega = \mathbf{I}_L$ and

$$s_k^{(0)} = \frac{\mathbf{a}_\Omega^H(\tau_k) \mathbf{y}_\Omega}{\| \mathbf{a}_\Omega(\tau_k) \|^2}, \quad k=1, 2, \dots, K \quad (12)$$

where $\| \cdot \|$ denotes the Euclidean norm.

Seismic missing traces recovery

In this section, we use the estimated amplitude spectral \hat{s}_k to restore the missing samples $\mathbf{y}_{\bar{\Omega}}$. The linear relationship between the missing data and the available data can be expressed as follows (Stoica et al., 2009; Karlsson et al., 2014):

$$\hat{\mathbf{y}}_{\bar{\Omega}} = \mathbf{T}\mathbf{y}_{\Omega} \quad , \quad (13)$$

where \mathbf{T} is a linear operator. The mean-squared error (MSE) of $\hat{\mathbf{y}}_{\bar{\Omega}}$ is as follows (Stoica et al., 2009):

$$\begin{aligned} \text{MSE}(\hat{\mathbf{y}}_{\bar{\Omega}}) &= E\{(\mathbf{T}\mathbf{y}_{\Omega} - \mathbf{y}_{\bar{\Omega}})^H(\mathbf{T}\mathbf{y}_{\Omega} - \mathbf{y}_{\bar{\Omega}})\} \\ &= \text{Tr}\{\mathbf{T}\mathbf{R}_{\Omega}\mathbf{T}^H - \mathbf{T}\mathbf{R}_{\bar{\Omega}\Omega} - \mathbf{R}_{\bar{\Omega}\Omega}\mathbf{T}^H + \mathbf{R}_{\bar{\Omega}}\} \\ &= \text{Tr}\left\{(\mathbf{T} - \mathbf{R}_{\bar{\Omega}\Omega}\mathbf{R}_{\Omega}^{-1})\mathbf{R}_{\Omega}(\mathbf{T} - \mathbf{R}_{\bar{\Omega}\Omega}\mathbf{R}_{\Omega}^{-1})^H\right\} + \text{Tr}\{\mathbf{R}_{\bar{\Omega}} - \mathbf{R}_{\bar{\Omega}\Omega}\mathbf{R}_{\Omega}^{-1}\mathbf{R}_{\bar{\Omega}\Omega}^H\} \quad (14) \\ &\geq \text{Tr}\{\mathbf{R}_{\bar{\Omega}} - \mathbf{R}_{\bar{\Omega}\Omega}\mathbf{R}_{\Omega}^{-1}\mathbf{R}_{\bar{\Omega}\Omega}^H\} \end{aligned}$$

where $\text{Tr}\{\cdot\}$ is the trace of a matrix, $\mathbf{R}_{\Omega} = E\{\mathbf{y}_{\Omega}\mathbf{y}_{\Omega}^H\}$ and

$$\mathbf{R}_{\bar{\Omega}\Omega} = E\{\mathbf{y}_{\bar{\Omega}}\mathbf{y}_{\Omega}^H\} = \sum_{k=1}^K p_k \mathbf{a}_{\bar{\Omega}}(\tau_k) \mathbf{a}_{\Omega}(\tau_k)^H = \mathbf{A}_{\bar{\Omega}} \mathbf{P} \mathbf{A}_{\Omega}^H \quad . \quad (15)$$

The lower bound in eq. (14) can be achieved when

$$\hat{\mathbf{T}} = \mathbf{R}_{\bar{\Omega}\Omega} \mathbf{R}_{\Omega}^{-1} \quad , \quad (16)$$

which can ensure the MMSE estimate of $\mathbf{y}_{\bar{\Omega}}$ (Sayed, 2003).

Using eqs. (13), (15) and (16), the estimation of the missing data can be computed as follows:

$$\hat{\mathbf{y}}_{\bar{\Omega}} = \sum_{k=1}^K p_k \left[\mathbf{a}_{\Omega}^H(\tau_k) \mathbf{R}_{\Omega}^{-1} \mathbf{y}_{\Omega} \right] \mathbf{a}_{\bar{\Omega}}(\tau_k) = \mathbf{A}_{\bar{\Omega}} \mathbf{P} \mathbf{A}_{\Omega}^H \mathbf{R}_{\Omega}^{-1} \mathbf{y}_{\Omega} \quad . \quad (17)$$

Because of $\mathbf{U}_{\bar{\Omega}}^T \mathbf{U}_{\bar{\Omega}} + \mathbf{U}_{\Omega}^T \mathbf{U}_{\Omega} = \mathbf{I}$, the estimation of complete data $\hat{\mathbf{y}}$ can be obtained by using the following formula:

$$\begin{aligned} \hat{\mathbf{y}} &= (\mathbf{U}_{\Omega}^T \mathbf{U}_{\Omega} + \mathbf{U}_{\bar{\Omega}}^T \mathbf{U}_{\bar{\Omega}}) \hat{\mathbf{y}} \\ &= \mathbf{U}_{\Omega}^T \mathbf{y}_{\Omega} + \mathbf{U}_{\bar{\Omega}}^T \mathbf{y}_{\bar{\Omega}} \\ &= (\mathbf{U}_{\Omega}^T + \mathbf{U}_{\bar{\Omega}}^T \mathbf{A}_{\bar{\Omega}} \mathbf{P} \mathbf{A}_{\Omega}^H \mathbf{R}_{\Omega}^{-1}) \mathbf{y}_{\Omega} \quad . \quad (18) \end{aligned}$$

The IAA method for recovering missing traces is summarized as follows:

Table 1. IAA algorithm for seismic data restoration.

Step 1: Input the observed data $\mathbf{X}_\Omega \in \mathbb{R}^{M \times L}$ and set \mathbf{U}_Ω , $\mathbf{U}_{\bar{\Omega}}$, \mathbf{A}_Ω , $\mathbf{A}_{\bar{\Omega}}$ and ε

Step 2: Apply Fourier transform to \mathbf{X}_Ω and obtain $\mathbf{Y}_\Omega = \text{FFT}(\mathbf{X}_\Omega)$.

Step 3: **for** $t = 1 \rightarrow M$

$$\mathbf{y}_\Omega = \mathbf{Y}_\Omega(t, :).$$

$$p_k^{(0)} = |s_k^{(0)}|^2 = \left| \mathbf{a}_\Omega^H(\tau_k) \mathbf{y}_\Omega \right|^2 / \left\| \mathbf{a}_\Omega(\tau_k) \right\|^4, \quad k = 1, 2, \dots, K.$$

$$\mathbf{P}^{(1)} = \text{diag}(\mathbf{p}^{(0)}).$$

for $i = 1 \rightarrow t$

$$\mathbf{R}_\Omega^{(i)} = \mathbf{A}_\Omega \mathbf{P}^{(i)} \mathbf{A}_\Omega^H.$$

$$p_k^{(i)} = \left| \mathbf{a}_\Omega^H(\tau_k) \mathbf{R}_\Omega^{-1} \mathbf{y}_\Omega / \mathbf{a}_\Omega^H(\tau_k) \mathbf{R}_\Omega^{-1} \mathbf{a}_\Omega(\tau_k) \right|^2, \quad k = 1, 2, \dots, K.$$

if $\|\mathbf{p}^{(i)} - \mathbf{p}^{(i-1)}\| / \|\mathbf{p}^{(i-1)}\| < \varepsilon$

break.

else

$$\mathbf{P}^{(i+1)} = \text{diag}(\mathbf{p}^{(i)}).$$

end

$$\hat{\mathbf{y}} = (\mathbf{U}_\Omega^T + \mathbf{U}_{\bar{\Omega}}^T \mathbf{A}_{\bar{\Omega}} \mathbf{P} \mathbf{A}_\Omega^H \mathbf{R}_\Omega^{-1}) \mathbf{y}_\Omega.$$

$$\hat{\mathbf{Y}}(t, :) = \hat{\mathbf{y}}.$$

end

Step 4: Output the reconstructed seismic data $\hat{\mathbf{X}} = \text{IFFT}(\hat{\mathbf{Y}})$.

NUMERICAL SIMULATION

We conduct two synthetic examples and one real seismic dataset to test the reconstruction performance of the proposed approach. We compares the proposed IAA with the IST, OR1MP and LMaFit to restore the seismic data with randomly missing traces. To evaluate the restoration performance, we define the signal-to-noise ratio (SNR) as follows:

$$\text{SNR} = 10 \cdot \log_{10} \left(\frac{\|\mathbf{X}\|_F^2}{\|\mathbf{X} - \hat{\mathbf{X}}\|_F^2} \right), \quad (19)$$

where $\|\cdot\|_F$ denotes the Frobenius norm, and \mathbf{x} and $\hat{\mathbf{x}}$ denote the complete seismic data and its reconstruction, respectively.

Fig. 1(a) displays a synthetic seismic data with two linear events. The data size is 128×128 . Fig. 1(b) shows the results under missing ratio 0.5. In this case, the rank parameter of OR1MP and LMaFit is set to 2. Figs. 1(c)-(f) show the reconstruction results via IST, OR1MP, LMaFit, and the IAA methods, respectively. In this example, IST and OR1MP obtain better reconstructed results compared to those of the LMaFit method, and the proposed IAA method achieves the best reconstructed result with a reconstruction SNR value of 83.04 dB. To more clearly illustrate the results, we present the SNR values of the random 10 single-trace original seismic data which are removed in observed data using the IST, OR1MP, LMaFit and IAA, respectively.

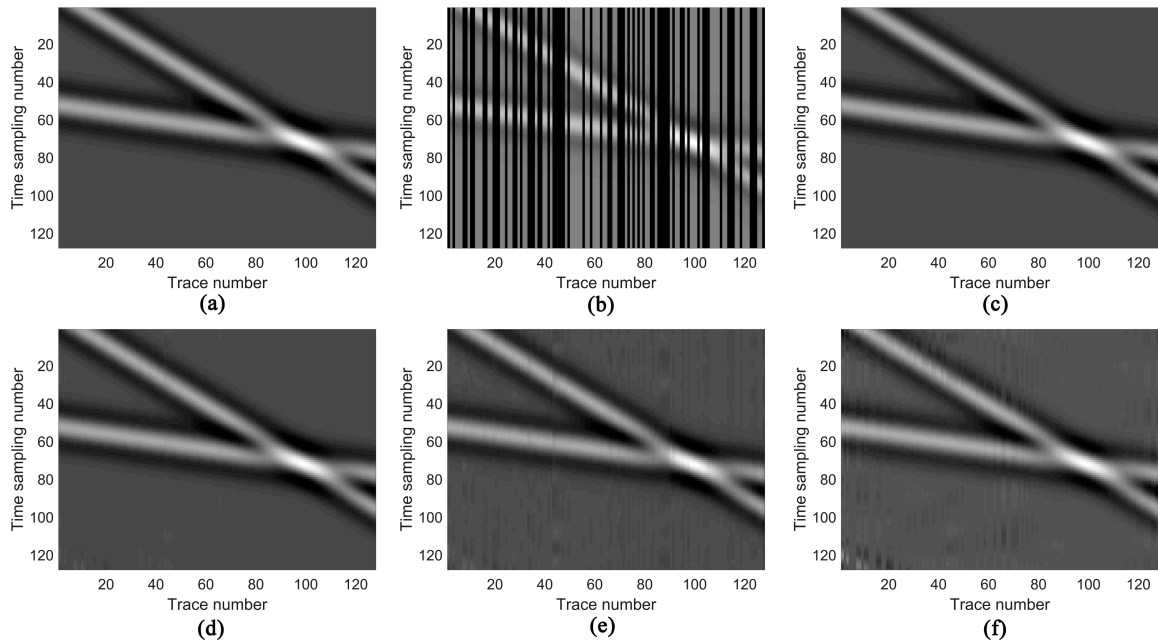


Fig. 1. Reconstructions for synthetic data with two linear events. (a) Original data. (b) missing ratio: 0.5. (c) IAA, SNR = 83.04 dB. (d) IST, SNR = 48.75 dB. (e) OR1MP, SNR = 38.14 dB. (f) LMaFit, SNR = 36.04 dB.

The corresponding results are shown in Fig. 2(a). It can be seen clearly that the proposed IAA method produces the highest reconstruction SNR value among the 10 single traces. Fig. 2(b) shows the SNR values using the four different methods as the missing ratio increases from 0.1 to 0.7. Fig. 2(b) shows that increasing samples result in higher SNR values. It is evident from Fig. 2(b) that the reconstruction SNR values obtained by the proposed IAA consistently outperform those of the other three methods. In particular, the proposed IAA method preserves the high quality reconstruction even though the missing ratio is more than 0.6.

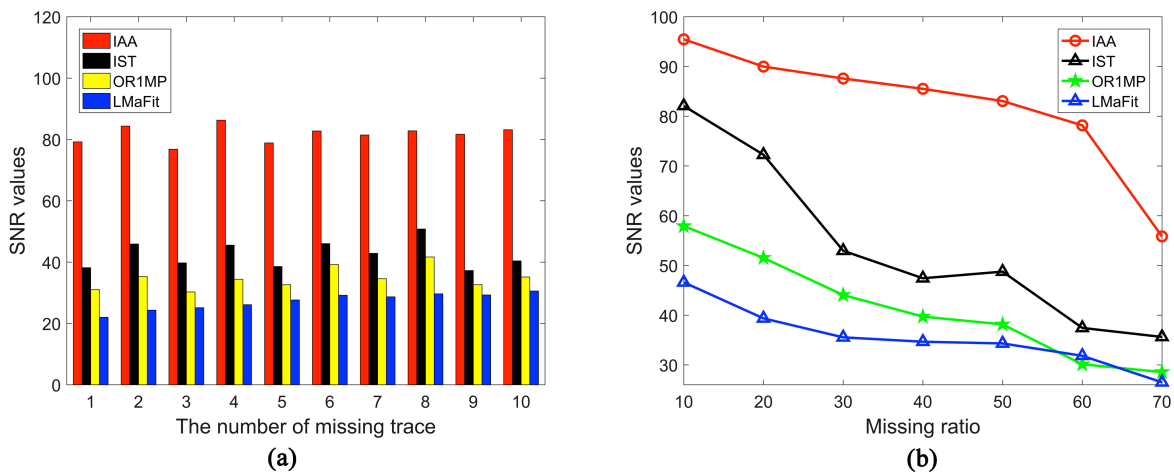


Fig. 2. (a) SNR values of 10 missing traces via four different methods for linear events. (b) Comparisons of SNR values versus missing ratio for linear events.

Next, we consider the synthetic seismic data with nonlinear events. The data size is 131×100 as shown in Fig. 3(a). The corrupted data in which 50% of the random traces are missing and is shown in Fig. 3(b). The rank parameter of the OR1MP and LMaFit methods is set to 2. The reconstructed results using IST, OR1MP, LMaFit and IAA are shown in Figs. 3(c)-(f). It is observed that our IAA method has the best reconstructed result with an SNR value of 25.82 dB. In addition, the SNR values of the random 10 single-trace

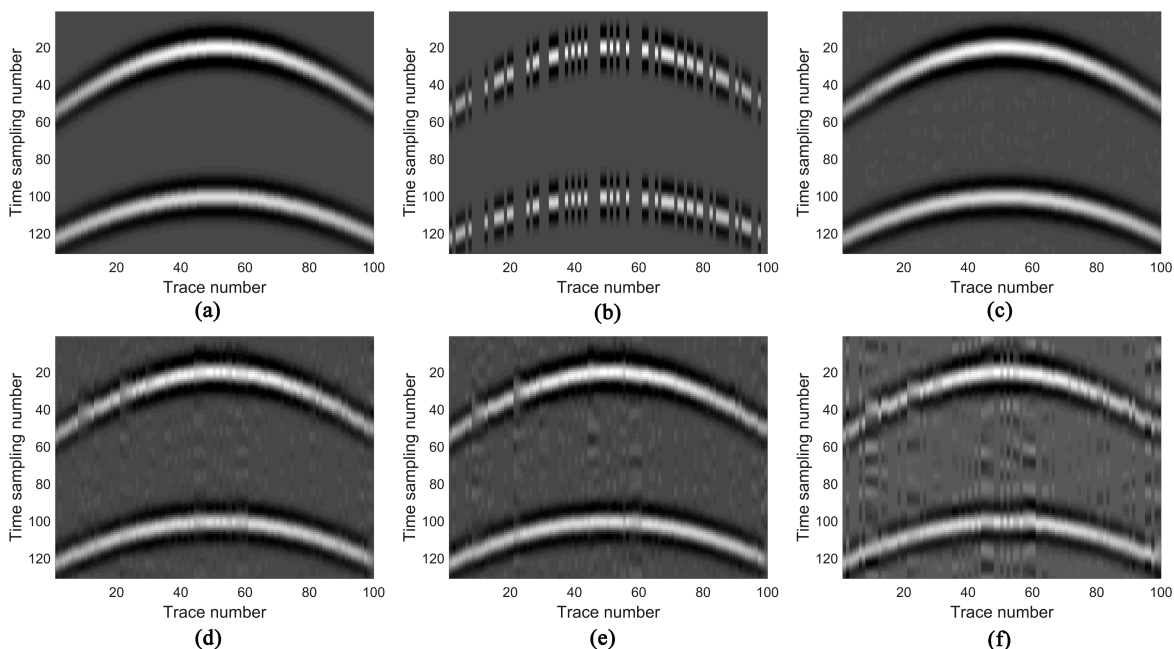


Fig. 3. Reconstructions for synthetic data with tow nonlinear events. (a) Original data. (b) Missing ratio: 0.5. (c) IAA , SNR = 25.82 dB. (d) IST, SNR = 15.79 dB. (e) OR1MP, SNR = 14.91 dB. (f) LMaFit, SNR = 9.17 dB.

original seismic data that are missing in the observed data using the four different methods are shown in Fig. 4(a). The results show that IAA performs better on the 10 selected random traces than the other methods. In addition, the reconstruction SNR changes as the missing ratio increases from 0.1 to 0.7, as shown in Fig. 4(b). Obviously, Fig. 4(b) shows that the IAA method achieves higher SNR values under different missing rates, which illustrates its good reconstruction performance.

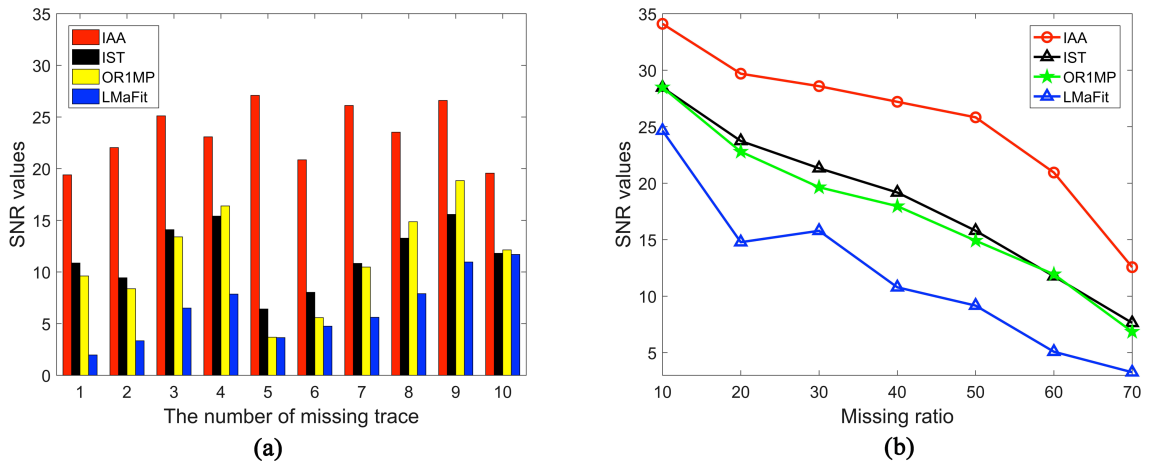


Fig. 4. (a) SNR values of 10 missing traces via four different methods for nonlinear events. (b) Comparisons of SNR values versus missing ratio for nonlinear events.

During the next experiment, we assessed numerical results on real seismic data. Fig. 5(a) shows a 2D post-stack seismic data from east Texas. The data size is 1000×60 . Fig. 5(b) shows the result under the missing ratio 0.5. For the OR1MP and LMaFit methods, the rank parameter is set to 20. Figs. 5(c)-(f) show the reconstructions using IST, OR1MP, LMaFit and IAA, respectively. The SNR value for restoring data using the IAA approach is 24.36 dB. From an SNR point of view, the proposed IAA method achieves high-quality reconstruction. Fig. 6(a) shows a comparison of SNR values of 10 missing traces using four different methods and the proposed IAA method obtains the highest SNR value. we consider the missing ratio varying from 0.1 to 0.7. It is clearly seen from Fig. 6(b) that the IAA significantly outperforms the IST, OR1MP, LMaFit methods.

CONCLUSIONS

In this paper, we introduce a new IAA-based method and show it could to be used to restore 2D seismic data with randomly missing traces. The IAA first estimates the spectral amplitudes from the seismic temporal frequency slices by an iterative adaptive manner. Then, the missing seismic data are obtained using a linear MMSE estimator. We test our method on synthetic and real post-stack seismic data. Our numerical results show that IAA is an effective method which can recovery of the missing traces from 2D seismic

data. The IAA results are the best compared to those obtained from the IST, OR1MP, and LMaFit methods, particularly for high missing ratios. Finally, our proposed IAA approach can be extended to three-dimensional seismic data and data contaminated with noise, which are being investigated and will be introduced in a future work.

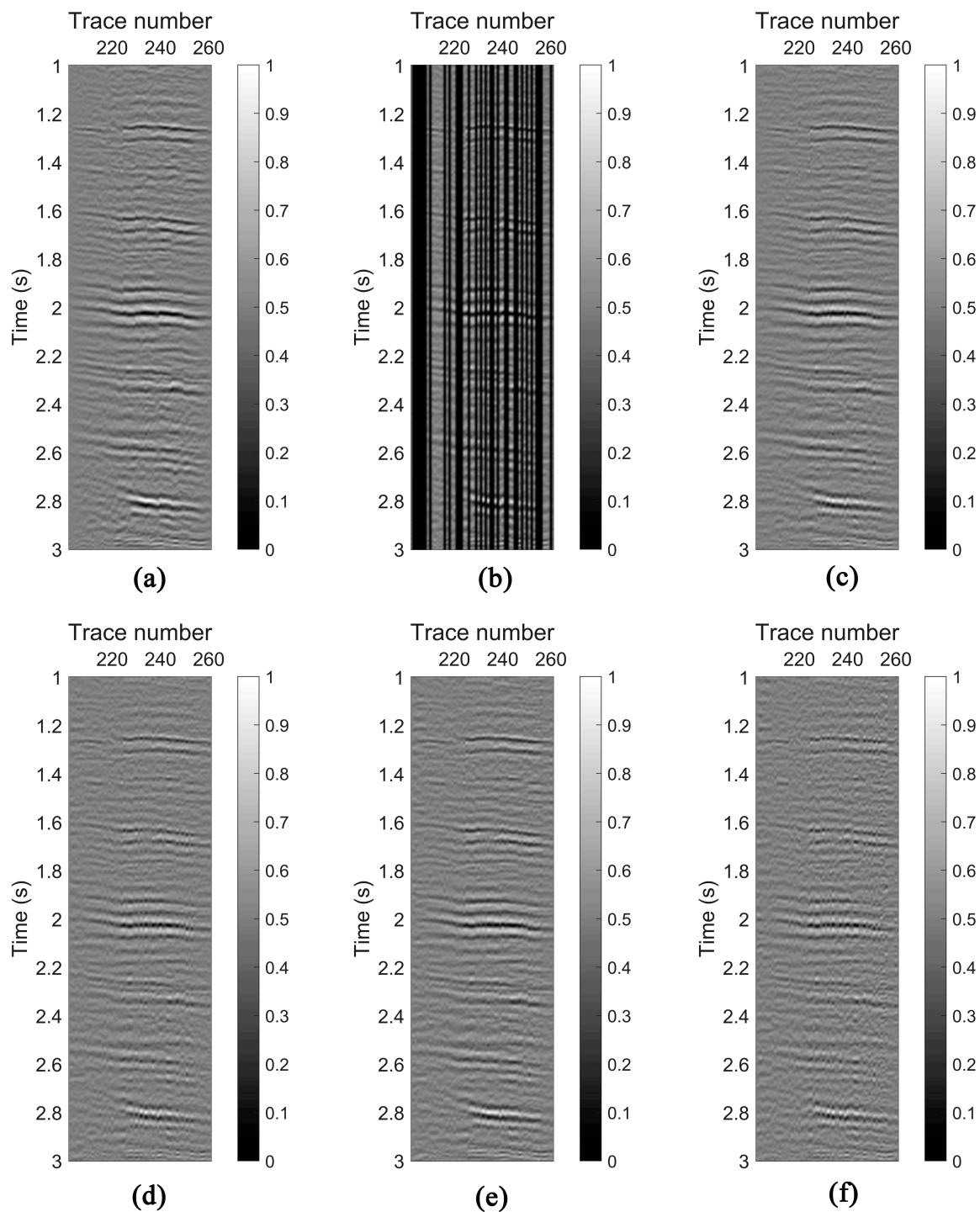


Fig. 5. Reconstructions for post-stack seismic data. (a) Original data. (b) Missing ratio:0.5 (c) IAA , SNR = 24.36 dB. (d) IST, SNR = 22.51 dB. (e) OR1MP, SNR = 22.96 dB. (f) LMaFit, SNR = 19.98 dB.

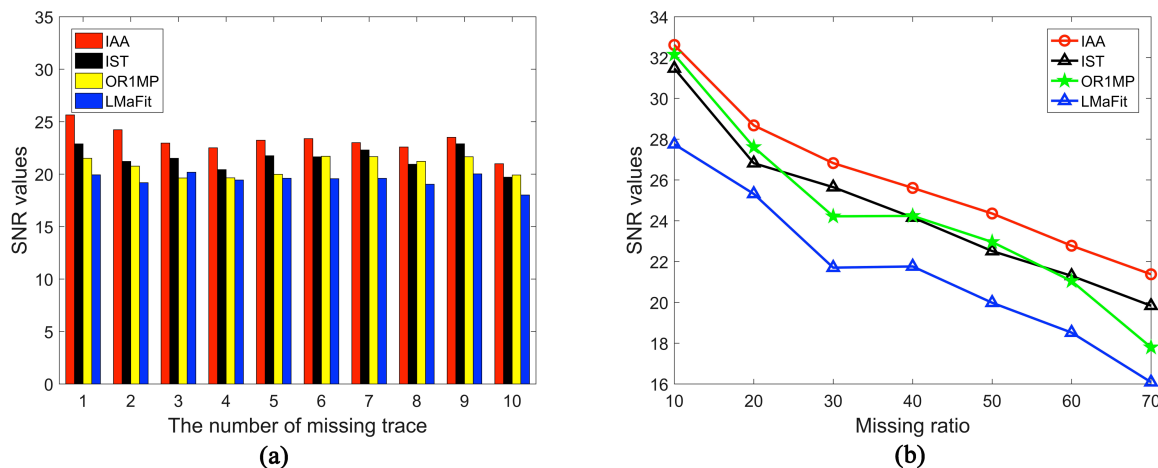


Fig. 6. (a) SNR values of 10 missing traces via four different methods for post-stack seismic data. (b) Comparisons of SNR values versus missing ratio for post-stack seismic data.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Fund of China (Grant Nos. 61601417, 61603358, 61702212).

REFERENCES

- Bagaini, C. and Spagnolini, U., 1993. Common shot velocity analysis by shot continuation operator. Expanded Abstr., 63rd Ann. Internat. SEG Mtg., Washington D.C.: 673-676.
- Bolondi, G., Loinger, E. and Rocca, F., 1982. Offset continuation of seismic sections. *Geophys. Prosp.*, 30: 813-828.
- Claerbout, J.F. and Nichols, D., 1991. Interpolation beyond aliasing by (t,x)-domain PEFs. Extended Abstr., 53rd EAEG Conf., Florence: 2-3.
- Canning, A. and Gardner, G., 1996. Regularizing 3D data sets with DMO. *Geophysics*, 61: 1846-1858.
- Cai, J.F., Candès, E.J. and Shen, Z., 2010. A singular value thresholding algorithm for matrix completion. *Siam J. Opt.*, 20: 1956-1982.
- Chen, K. and Sacchi, M.D., 2015. Robust reduced-rank filtering for erratic seismic noise attenuation. *Geophysics*, 80: 1-11.
- Deregowski, S.M., 1986. What is DMO? *First Break*, 4: 7-24.
- Duijndam, A.J.W. and Schonewille, M.A., 1999. Nonuniform fast Fourier transform. *Geophysics*, 64: 539-551.
- Feng, W.K., Guo, Y.D., He, X.Y., Liu, H.W. and Guo, J., 2018. Jointly iterative adaptive approach based space time adaptive processing using MIMO Radar. *IEEE Access*, 6: 26605-26616.
- Glentis, G.O. and Jakobsson, A., 2011. Superfast approximative implementation of the iaa spectral estimate. *IEEE Transact. Signal Process.*, 60: 472-478.
- Herrmann, F.J. and Hennenfent, G., 2008. Non-parametric seismic data recovery with curvelet frames. *Geophys. J. Internat.*, 173: 233-248.
- Hennenfent, G., Fenelon, L. and Herrmann, F.J., 2010. Nonequispaced curvelet transform for seismic data reconstruction: A sparsity-promoting approach. *Geophysics*, 75: WB203-WB210.

- Kabir, M.M.N. and Verschuur, D.J., 1995. Restoration of missing offsets by parabolic Radon transform. *Geophys. Prospect*, 43: 347-368.
- Karlsson, J., Rowe, W., Xu, L. and Glentis, G. O., 2014. Fast missing-data iaa with application to notched spectrum sar. *IEEE Trans. Aerosp. Electron. Syst.*, 50: 959-971.
- Liu, and Sacchi, M.D., 2004. Minimum weighted norm interpolation of seismic records. *Geophysics*, 69: 1560-1568.
- Li, C., Liu, G.C., Hao, Z.J., Zu, S.H., Mi, F. and Chen, X.H., 2018. Multidimensional seismic data reconstruction using frequency-domain adaptive prediction-error filter. *IEEE Transact. Geosci. Remote Sens.*, 56: 2328-2336.
- Ma, J.W., 2013. Three-dimensional irregular seismic data reconstruction via low-rank matrix completion. *Geophysics*, 78: 181-192.
- Naghizadeh, M. and Sacchi, M.D., 2010. Beyond alias hierarchical scale curvelet interpolation of regularly and irregularly sampled seismic data. *Geophysics*, 75: WB189-WB202.
- Oropeza, V. and Sacchi, M.D., 2011. Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis. *Geophysics*, 76: 25-32.
- Spitz, S., 1991. Seismic trace interpolation in the FX domain. *Geophysics*, 56: 785-794.
- Sacchi, M.D. and Ulrych, T.J., 1995. High-resolution velocity gathers and offset space reconstruction. *Geophysics*, 60: 1169-1177.
- Sayed, A., 2003. *Fundamentals of Adaptive Filtering*. Wiley, New York.
- Stoica, P., Li, J. and Ling, J., 2009 Missing data recovery via a non-parametric iterative adaptive approach. *IEEE Sign. Process. Lett.*, 16: 241-244.
- Thorson, J.R., 1985. Velocity stack and slant stochastic inversion. *Geophysics*, 50: 2727-2741.
- Trickett, S., Burroughs, L., Milton, A., Walton, L. and Dack, R., 2010. Rank-reduction-based trace interpolation. *Expanded Abstr.*, 80th Ann. Internat. SEG Mtg., Denver: 3829-3833.
- Wen, Z.W., Yin, W.T. and Zhang, Y., 2012. Solving a low-rank factorization model for matrix completion by a nonlinear successive over-relaxation algorithm. *Math. Prog. Computat.*, 4: 333-361.
- Wang, Z., Lai, M.J., Lu, Z.S., Fan, W., Davulcu, H. and Ye, J.P., 2014. Orthogonal rank-one matrix pursuit for low rank matrix completion. *SIAM J. Sci. Comput.*, 37: A488-A514.
- Xue, M., Xu, L. and Li, J. 2011. Iaa spectral estimation: fast implementation using the gohberg–semencul factorization. *IEEE Transact. Signal Process.*, 59: 3251-3261.
- Yardibi, T., Li, J., Stoica, P. and Xue, M., 2010. Source localization and sensing: A non-parametric iterative adaptive approach based on weighted least squares. *IEEE Transact. Aerosp. Electron. Syst.*, 46: 425-443.
- Zwartjes, P.M. and Sacchi, M.D. 2006. Fourier reconstruction of nonuniformly sampled, aliased seismic data. *Geophysics*, 72: V21-V32.
- Zhang, H., Chen, X. and Li, H., 2015. 3D seismic data reconstruction based on complex-valued curvelet transform in frequency domain. *J. Appl. Geophys.*, 113: 64-73.
- Zhang, Y.C., Li, W.C., Zhang, Y., Huang, Y.L. and Yang, J.Y., 2016. A fast iterative adaptive approach for scanning radar angular superresolution. *IEEE J. Select. Topics Appl. Earth Observ.*, 8: 5336-5345.