TRANSMISSION AMPLITUDE VARIATION WITH OFFSET (TAVO)

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ABSTRACT

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Amplitude Variation with Offset (AVO) has been applied successfully in reservoir geophysics for various applications including fluid detection and lithology typing. Existing AVO methods utilize seismic waves reflected off elastic interfaces. Amplitudes of transmitted waves, usually recorded in vertical seismic profiling (VSP) surveys, also show angle dependence and can be used for AVO-like analysis. In this study, we derive new approximations of the exact Zoeppritz transmission PP (T_{PP}) and PS (T_{PS}) coefficients that are convenient for conventional AVO analysis. Testing on a published reservoir model showed that the new approximations deviated from their corresponding exact coefficients by less than 7% at an incidence angle equal to 90% of the critical angle.

Furthermore, a quantitative approach is described to invert the new T_{PP} and T_{PS} approximations for the P-wave velocity contrast ($\Delta\alpha/\alpha$), density contrast ($\Delta\rho/\rho$), S-wave velocity contrast ($\Delta\beta/\beta$) and S-wave/P-wave velocity ratio (β/α) across a subsurface interface. Testing this approach using the same reservoir model resulted in error amounts of 0%, 0%, 6% and 6.5% for $\Delta\alpha/\alpha$, $\Delta\rho/\rho$, $\Delta\beta/\beta$ and β/α, respectively. These small errors show that the new approximations and inversion approach are relatively accurate.

KEY WORDS: AVO analysis, transmitted seismic wave, Zoeppritz equation, reservoir parameter estimate, VSP survey.

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INTRODUCTION

Amplitude variation with offset (AVO) involves the analysis of the behavior of seismic wave amplitude as a function of the incident angle. Conventional AVO analysis involves the analysis of the variation in amplitudes of reflected signals with incidence angle and is used in many exploration applications including fluid detection, lithology typing and fracture mapping.

The Zoeppritz equations express the exact plane wave amplitudes of reflected and transmitted waves as functions of angles of incidence and transmission, but do not give an intuitive understanding of how these amplitudes relate to the physical properties of subsurface reservoirs such as velocity and density. In order to make these important Zoeppritz equations more useful, different approximations for these equations have appeared in the literature, especially for the reflected amplitudes.

Partitioning of an incident seismic P-wave at an interface is presented in Fig. 1. Most existing AVO studies use reflected PP, PS or both. With the proliferation of vertical seismic profiling (VSP) surveys, analysis of transmitted PP, PS or both is becoming feasible. Analysis of transmitted modes avoids several undesirable effects associated with reflected arrivals because it only utilizes the downgoing (direct) wavefield. These undesirable effects include:

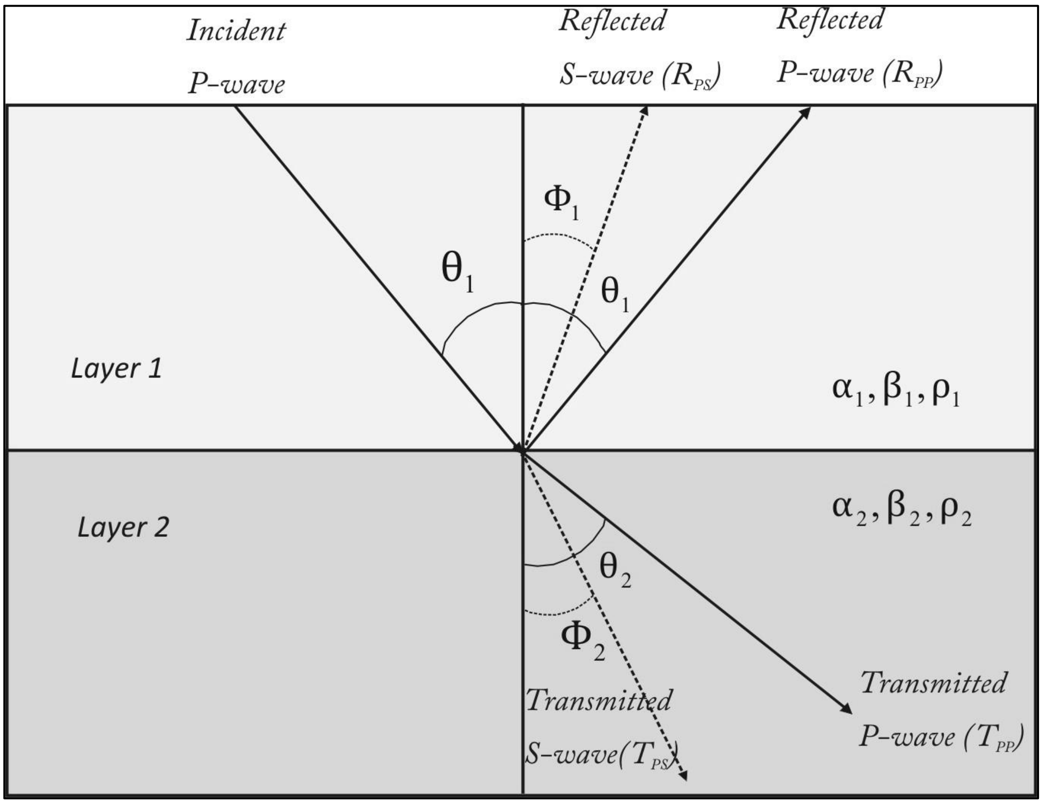

Fig. 1. Partitioning of the amplitude of a P-wave incident of an interface between two elastic solids.

- Amplitude losses by absorption due to traveling across deeper formations with unknown absorption coefficients.
- Amplitude losses due to reflection at deeper interfaces.
- Wavelet non-stationarity by shifting to lower frequencies due to traveling longer (deeper) distances in a subsurface that acts as a low-pass filter.
- Accuracy loss in modeling due to ray bending by velocity heterogeneities in deeper formations.

Aki and Richards (2002) presented a first-order linear approximation of Zoeppritz equations valid for reservoirs exhibiting small velocity and density contrasts and at incidence angles less than the critical angle. Shuey (1985) further simplified Aki and Richards' approximation of the reflected R_{PP} Zoeppritz equation into a three-term equation with each term contributing at a different range of incidence angles. Xu and Bancroft (1997) presented a joint AVO analysis on R_{PP} and R_{PS} seismic data from Blackfoot, Alberta. They derived equations for the R_{PS} coefficient convenient for use in PS-reflected AVO analysis. They utilized the least squares regression analysis to extract elastic parameters from pre-stack seismic data at any incidence angle. Coulombe et al. (1992, 1996) applied AVO on VSP data. They considered the use of multi-offset VSP geometry with a multi-component processing workflow to obtain the reflection coefficients for the purpose of AVO analysis. They mentioned that due to the difference in the frequency of operation between the measurements of borehole log and surface seismic, AVO response using sonic velocities may not effectively match surface seismic observations. Donati and Martin (1998) expressed the R_{PS} coefficient as a polynomial series of sines and cosines. They reported that the approximation with a polynomial of sine series to be more accurate up to a fairly large incidence angle compared to that of cosine.

In this study, we propose the application of AVO analysis on transmitted rather than reflected amplitudes using conventionally recorded VSP data. Particularly, appropriate approximations will be derived and expressed in a form similar to the conventional AVO analysis in order to facilitate their incorporation within existing AVO technologies. The approximate equations relating transmission amplitudes to incidence angle will then be inverted for important reservoir parameters. We call the new method: transmission amplitude variation with offset (TAVO).

METHODOLOGY

Starting from the approximate expressions given by Aki and Richards (2002) T_{PP} and T_{PS}:

$$T_{PP} = 1 - \frac{1}{2}\frac{\Delta\rho}{\rho} + \left(\frac{1}{2\cos^2\theta} - 1\right)\frac{\Delta\alpha}{\alpha} \tag{1}$$

$$T_{PS} = \frac{p\alpha}{2\cos\phi}\left[\left(1 - 2\beta^2p^2 - 2\beta^2\frac{\cos\theta}{\alpha}\frac{\cos\phi}{\beta}\right)\frac{\Delta\rho}{\rho} - 4\beta^2\left(p^2 + \frac{\cos\theta}{\alpha}\frac{\cos\phi}{\beta}\right)\frac{\Delta\beta}{\beta}\right]. \tag{2}$$

where p in eq. (2) is the ray parameter given as:

$$p = \frac{\sin\theta}{\alpha} = \frac{\sin\phi}{\beta} \quad . \tag{3}$$

In the above and subsequent equations, α , β and ρ indicate the average values of the P-wave velocity $(\alpha_1 + \alpha_2)/2$, S-wave velocity $(\beta_1+\beta_2)/2$ and density $(\rho_1+\rho_2)/2$ across the interface respectfully; while $\Delta\alpha = (\alpha_2 - \alpha_1)$, $\Delta\beta = (\beta_2 - \beta_1)$, and $\Delta\rho = (\rho_2-\rho_1)$ indicate the differences in these parameters across the interface and the angle θ is also the average between the incidence and transmission angles of the P-wave (Shuey, 1985).

Eq. (1) can be expressed as

$$T_{PP}(\theta) = \left(1 - \frac{\Delta\rho}{2\rho} - \frac{\Delta\alpha}{2\alpha}\right) + \frac{\Delta\alpha}{2\alpha}\tan^2\theta. \tag{4}$$

Eq. (4) is already in an AVO-convenient form;

$$T_{PP}(\theta) = A + B\tan^2\theta \quad , \tag{5}$$

where

$$A = 1 - \left(\frac{\Delta\rho}{2\rho} + \frac{\Delta\alpha}{2\alpha}\right) = 1 - R_{PP0} = T_{PP0} \tag{6}$$

$$B = \frac{\Delta\alpha}{2\alpha}. \tag{7}$$

where R_{PP0} and T_{PP0} are the zero-offset PP-reflection and transmission coefficients, respectively. Fig. 2(a) shows a plot of T_{PP} as a function of incidence angle for Zoeppritz, Aki-Richards and eq. (4) using the model parameters shown in Table 1 reported by Donati and Martins (1998) for an oil sand reservoir.

Terms involving angle φ in eq. (2) can be expressed in terms of angle θ as:

$$\sin\phi = \frac{\beta}{\alpha}\sin\theta. \tag{8}$$

$$\cos\phi = \sqrt{1 - \left(\frac{\beta}{\alpha}\right)^2 \sin^2\theta}. \tag{9a}$$

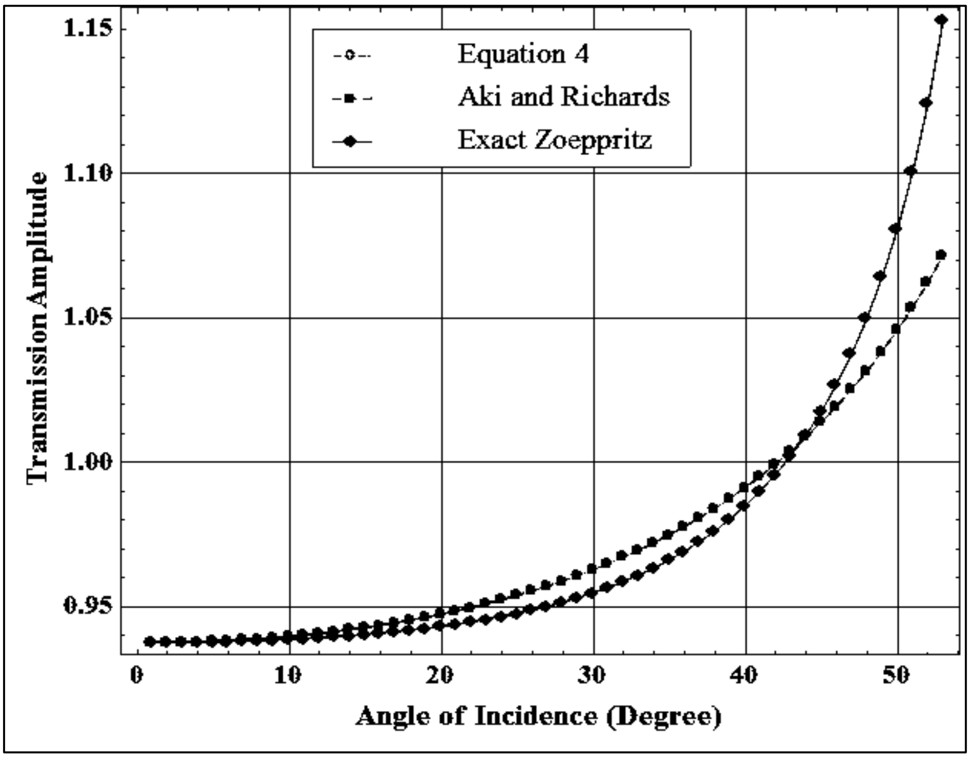

Fig. 2. (a) Plots of the T_{PP} coefficient.

Next, we expand the square root in eq. (9a) using a Maclaurin's series (i.e., about $\theta = 0°$) and truncate at the third term to get:

$$\cos\varphi \approx 1 - \frac{\beta^2 \sin^2\theta}{2\alpha^2} - \frac{\beta^4 \sin^4\theta}{8\alpha^4} \quad . \tag{9b}$$

Substituting eqs. (3) and (9b) into eq. (2), and collecting similar powers of $\sin\theta$ yields:

$$T_{PS}(\theta) \approx \left[-\frac{\beta}{\alpha}\left(\frac{\Delta\rho}{\rho} + \frac{2\Delta\beta}{\beta}\right) + \frac{1}{2}\frac{\Delta\rho}{\rho}\right]\sin\theta + \left[\frac{\beta}{\alpha}\left(\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{2\rho}\right) - \frac{\beta}{\alpha}\left(\frac{2\Delta\beta}{\beta} + \frac{3\Delta\rho}{4\rho}\right)\right)\right]\sin^3\theta + \frac{1}{8}\left(\frac{\beta}{\alpha}\left(\frac{2\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right) - \left(\frac{\beta}{\alpha}\right)^4\left(\frac{8\Delta\beta}{\beta} + \frac{5}{2}\frac{\Delta\rho}{\rho}\right)\right)\sin^5\theta. \tag{10}$$

Eq. (10) is already in an AVO-convenient form:

$$T_{PS}(\theta) \approx C\sin\theta + D\sin^3\theta + E\sin^5\theta \quad , \tag{11}$$

where

$$C = -\frac{\beta}{\alpha}\left(\frac{\Delta\rho}{\rho} + \frac{2\Delta\beta}{\beta}\right) + \frac{1}{2}\frac{\Delta\rho}{\rho}. \tag{12}$$

$$D = \frac{\beta}{\alpha}\left(\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{2\rho}\right) - \frac{\beta}{\alpha}\left(\frac{2\Delta\beta}{\beta} + \frac{3\Delta\rho}{4\rho}\right)\right). \tag{13}$$

$$E = \frac{1}{8}\left(\frac{\beta}{\alpha}\left(\frac{2\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right) - \left(\frac{\beta}{\alpha}\right)^4\left(\frac{8\Delta\beta}{\beta} + \frac{5}{2}\frac{\Delta\rho}{\rho}\right)\right). \tag{14}$$

Fig. 2(b) shows a plot of T_{PS} as a function of incidence angle for Zoeppritz, Aki-Richards and eq. (10) with one, two and three terms using the oil reservoir model shown in Table 1. Eqs. (5) and (11) are the approximations that will be used for the estimation of elastic properties from the T_{PP} and T_{PS} amplitudes.

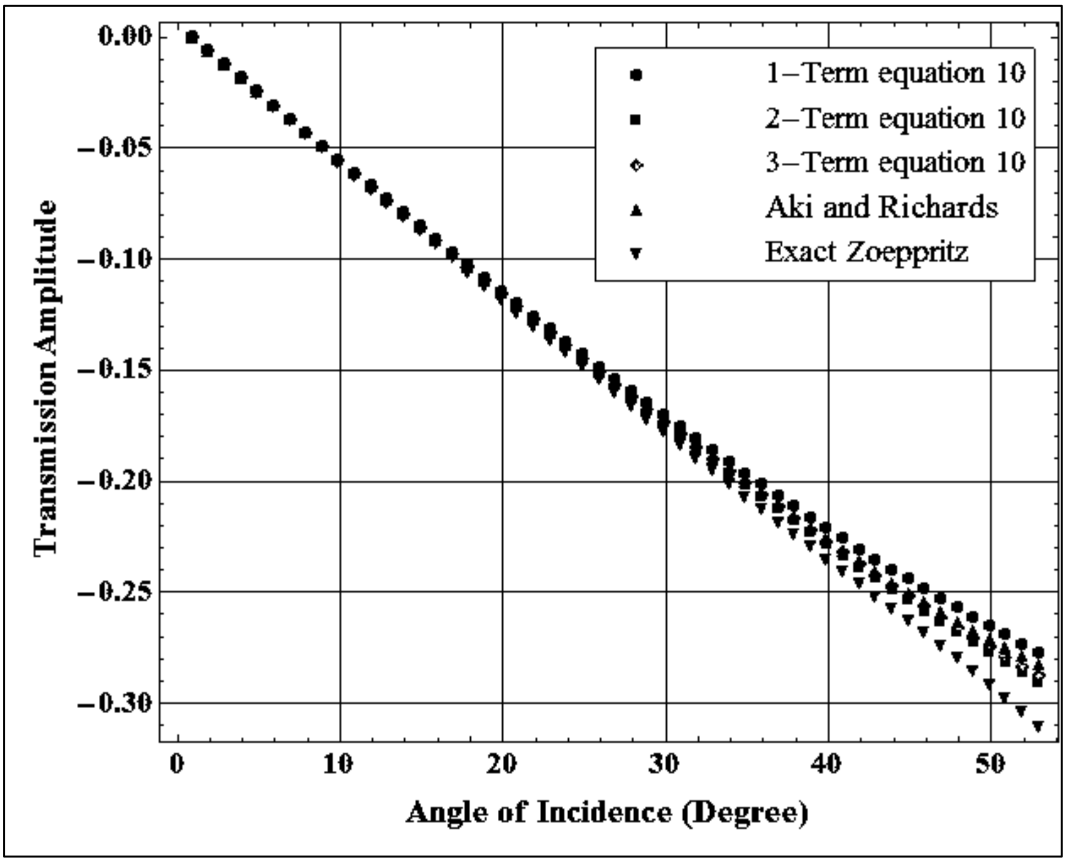

Fig. 2. (b) Plots of the T_{PS} coefficient.

Table 1. Properties of the oil reservoir used in this study (from Donati and Martin, 1998).

Layer 1			Layer 2		
α_1 (*m/s*)	β_1 (*m/s*)	ρ_1 (kg/m^3)	α_2 (*m/s*)	β_2 (*m/s*)	ρ_2 (kg/m^3)
3170	1698	2360	3734	2280	2270

We assume that the model studied is only vertically heterogeneous (no lateral changes in the elastic parameters) and isotropic with no dipping interfaces and we will suggest solutions to some of these limitations later. The plane wave assumption holds for source - receiver distances that are much longer than the wavelength of the incident wave. Expansion of the T_{PS} equation was made using a Maclaurin's series (i.e., around $\theta = 0^\circ$) and might deviate from the exact Zoeppritz expression at large angles of incidence. Finally, in order to obtain the data needed for this analysis, the recorded traces have to be sorted into common receiver gathers (CRG) and assume that T_{PP} is dominant on the Z-component while T_{PS} is dominant on the X-component of the CRG.

Offset-angle transformation

This is a basic step in AVO analysis because seismic data are often recorded in offset-time *(x,t)* domain hence there is a need to transform offset to incidence angle. Using Fig. 3:

$$X2 = (Z - H)\tan\theta_2 \quad . \tag{15}$$

We then use Snell's Law to express θ_2 in terms of $\theta_1$as follows:

$$\sin\theta_2 = \frac{\alpha_2}{\alpha_1}\sin\theta_1 \quad . \tag{16}$$

Hence, $\tan\theta_2$ in eq. (15) becomes:

$$\tan\theta_2 = \frac{\sin\theta_2}{\sqrt{1-\sin^2\theta_2}} = \frac{\frac{\alpha_2}{\alpha_1}\sin\theta_1}{\sqrt{1-\left(\frac{\alpha_2}{\alpha_1}\right)^2\sin^2\theta_1}} . \tag{17}$$

By substituting eq. (17) into eq. (15), $X2$ becomes:

$$X2 = (Z - H)\left[\frac{\frac{\alpha_2}{\alpha_1}\sin\theta_1}{\sqrt{1-\left(\frac{\alpha_2}{\alpha_1}\right)^2\sin^2\theta_1}}\right]. \tag{18}$$

$$X1 = X - X2 = H\tan\theta_1, \Rightarrow X = H\tan\theta_1 + X2. \tag{19}$$

If we substitute eq. (18) into eq. (19), we get a relationship between offset X and incidence angle θ_1 as:

$$X = H\tan\theta_1 + (Z - H)\left[\frac{\left(\frac{\alpha_2}{\alpha_1}\right)\sin\theta_1}{\sqrt{1-\left(\frac{\alpha_2}{\alpha_1}\right)^2\sin^2\theta_1}}\right]. \tag{20}$$

Eq. (20) can be solved numerically for θ_1, provided α_1, α_2, X, Z and H are known, which are usually available in most VSP surveys.

PARAMETER ESTIMATION

An important goal of AVO analysis is to estimate subsurface parameters such as the P-wave velocity contrast ($\Delta\alpha/\alpha$), density contrast ($\Delta\rho/\rho$), S-wave velocity contrast ($\Delta\beta/\beta$) and S-wave/P-wave velocity ratio (β/α) across an interface. These parameters can be estimated from the TAVO approximations [i.e., eqs. (5) and (11)] using the following approach:

1. Sort the data into common receiver gathers.
2. At every receiver gather, use the vertical component to do the following:
   a. Estimate the amplitude of the exact T_{PP} at every shot offset.
   b. Calculate the incidence angle corresponding to each shot offset from eq. (20).
   c. Fit eq. (5) to the (incidence angle, exact T_{PP} amplitude) data set to estimate the fitting parameters A and B.
3. At every receiver gather, use the horizontal component along the x-axis to do the following:
   a. Estimate the amplitude of the exact T_{PS} at every shot offset.
   b. Fit eq. (11) to the (incidence angle, exact T_{PS} amplitude) data set to estimate the fitting parameters C, D and E.
4. Solve eqs. (6) and (7) for $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ as follows:

$$\Delta\alpha/\alpha = 2B \tag{21}$$

$$\frac{\Delta\rho}{\rho} = 2[1-(A+B)] \tag{22}$$

5. Substitute eqs. (21) and (22) into eqs. (12) and (13) to express $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ in terms of A and B as follows:

$$C = 1 + A(-1 + 2\,\beta/\alpha) - B + 2(\beta/\alpha)(-1 + B - \Delta\beta/\beta) \ , \quad (23)$$

$$D = \frac{\beta}{\alpha}[A(\frac{1.5\beta}{\alpha} - 1) + \frac{\Delta\beta}{\beta} + \frac{\beta\left(-\frac{2\Delta\beta}{\beta}+1.5B-1.5\right)}{\alpha} - B + 1] \ . \quad (24)$$

6. Solve eqs. (23) and (13) for $\Delta\beta/\beta$ and β/α, which results in two solutions. Testing these solutions on several published reservoir models showed that only the following solution is valid:

$$\frac{\beta}{\alpha} = \frac{\sqrt{C(A+B+C-1)-2D(A+B-1)}+A+B+C-1}{A+B-1} \ , \quad (25)$$

$$\frac{\Delta\beta}{\beta} = \frac{(A+B-1)(2\sqrt{C(A+B+C-1)-2D(A+B-1)}+A+B+C-1)}{2(\sqrt{C(A+B+C-1)-2D(A+B-1)}+A+B+C-1)} \ . \quad (26)$$

Although it was possible to use other combinations of the *C, D* and *E* fitting parameters, testing has shown that combinations involving the *E* fitting parameter result in larger errors than using only the *C* and *D* fitting parameters. This might be due to the fact that the fitting parameter *E* [eq. (14)] involves higher powers of β/α than the *C* and *D* fitting parameters (eqs. (12) and (13)], which will propagate errors in fitting parameters to estimated $\Delta\beta/\beta$ and β/α much faster.

The accuracy of the derived approximations in retrieving key subsurface parameters is tested using the oil-reservoir model described in Table 1 and the VSP geometry detailed in Fig. 3 and Table 2. The maximum shot offset (3000 m) corresponds to an incidence angle of 53°, which is about 90% of the critical angle (about 58°) for this model. The exact $T_{PP}(\theta)$ and $T_{PS}(\theta)$ coefficients are calculated at intervals of 1° and the resulting data sets are fitted to eqs. (5) and (11) in order to estimate the values of fitting parameters *A, B, C, D* and *E* shown in Table 3.

Table 2. Parameters of the VSP survey used in this study.

Parameter	Value
Top of oil reservoir	800 m
Location of first shot (x,z)	(0,0)
Shot spacing	50 m
Receiver depth	1500 m
Number of shots	61
Maximum offset	3000 m

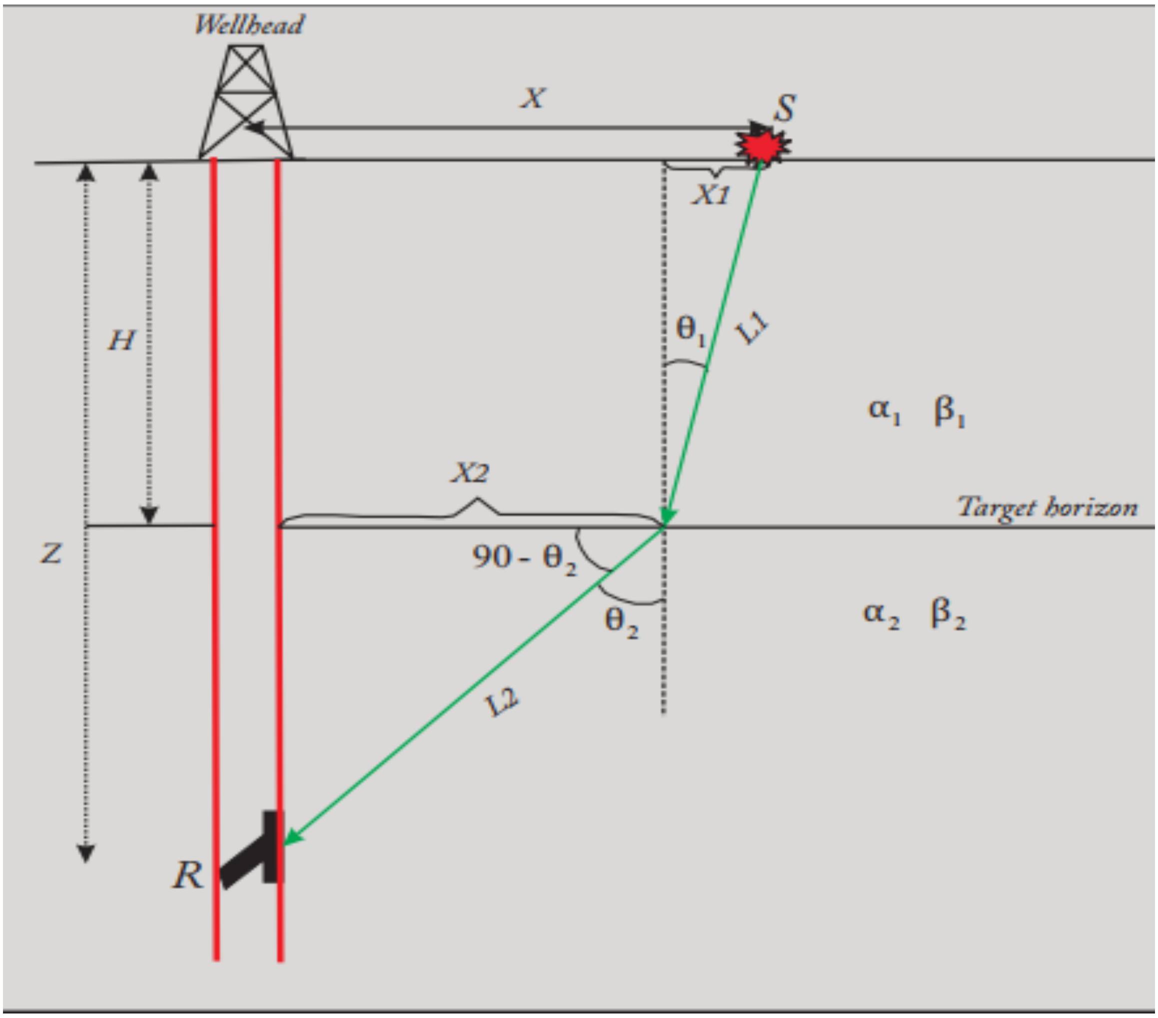

Fig. 3. Geometry of a VSP survey used to derive the offset-angle relationship and test the TAVO method.

Table 3. Values of fitting parameters estimated using eqs. (5) and (11).

Fitting parameter	Value
A	0.937747
B	0.0816918
C	-0.333339
D	-0.0381314
E	0.0377877

Results of applying this approach on this model and geometry setup are shown in Table 4. The errors in $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ are both zeros because eq. (4) is trigonometrically equivalent to eq. (1) and no further approximation was used to go from eqs. (1) to (4). Errors in β/α and $\Delta\beta/\beta$ are about 6.5% and 6%, respectively, which are relatively small considering the large incidence angles involved. These errors are mostly due to approximations used to go from eqs. (2) to (10).

Table 4. Values and absolute errors of subsurface parameters used to test the accuracy of the TAVO method.

Subsurface parameter	True parameter value	Estimated parameter value	Absolute error (%)
$\Delta\alpha/\alpha$	0.163384	0.163384	0
$\Delta\rho/\rho$	-0.0388769	-0.0388769	0
$\Delta\beta/\beta$	0.292609	0.275111	5.98002
β/α	0.576188	0.613619	6.4964

SUMMARY AND CONCLUSIONS

New approximations of the exact Zoeppritz expressions for the $T_{PP}(\theta)$ and $T_{PS}(\theta)$ coefficients were derived. These approximations gave relatively small deviations (less than 7%) from the exact expressions even at relatively large angles of incidence (up to 90% of the critical angle). We used the new approximations to estimate the subsurface parameters $\Delta\alpha/\alpha$, $\Delta\rho/\rho$, $\Delta\beta/\beta$ and β/α with absolute errors of 0%, 0%, 6% and 6.5%, respectively. These small errors, despite the large incidence angles involved, show that the new approximations can provide an accurate method for estimating these important subsurface parameters from VSP surveys.

We have assumed here that the formations are laterally homogeneous in order to ensure that all analyzed rays were transmitted across formations that have the same properties. This assumption might not be feasible for some geological settings. We are currently working on developing a new sorting domain that is more appropriate for this TAVO analysis, which sorts rays in common transmission gathers (CTG). A CTG is a set of traces that were transmitted through a subsurface interface from the same point.

Although results on the tested model show that the new approximations follow the exact coefficients and inverts important subsurface parameters accurately, further testing is required in order to ascertain this result. In case of discrepancies, we suggest attempting expansions of the square root in eq. (9a) other than Maclaurin's, particularly those that are valid far from $\theta = 0°$.

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