

SEISMIC NOISE ATTENUATION BASED ON A DIP-SEPARATED FILTERING METHOD

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ABSTRACT

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Mode decomposition and reconstruction is a commonly used denoising algorithm for seismic data. The principle of the decomposition based method is that the signal and noise can be represented by different parts in a mode decomposition process. While the features of useful signals can be captured by the principal components, the noise is separated out by rejecting the less important components during the reconstruction process. The decomposition based method can be optimally applied in the frequency-space domain, where signal and noise are separated by their differences in the wavenumber spectrum. The useful signals are mainly corresponding to the low-wavenumber components, i.e., less oscillating, while the noise corresponds to the highly oscillating components. Such decomposition acts as a dip filter, which can be combined with a spatial coherency based smoothing operator. The overall algorithm is thus a dip-separated structural filtering method. In this paper, we use the variational mode decomposition (VMD) method to decompose the seismic data into several dipping components, which is followed by a low-rank approximation filtering step. We apply the proposed method to both synthetic and field data examples and obtain satisfactory results.

KEY WORDS: noise attenuation, filtering, variational mode decomposition.

INTRODUCTION

Noise attenuation is critical in seismic exploration. Among many classic denoising methods (Gulunay, 2000; Naghizadeh and Sacchi, 2012; Liu et al., 2012; Liu and Chen, 2013; Tian et al., 2014; Liu et al., 2015a,b; Chen and Fomel, 2015; Jiao et al., 2015; Yang et al., 2015; Gan et al., 2016; Zu et al.,

2016; Chen et al., 2016; Xue et al., 2016; Mousavi and Langston, 2017; Liu et al., 2018a), the low-rank approximation method has been studied extensively for seismic noise attenuation. In this paper, we propose a multi-scale dip-separated low-rank approximation method to separate signal and noise in a seismic data. The seismic data is first decompose into several dipping components by means of the variational mode decomposition (VMD) Dragomiretskiy and Zosso (2014); Liu et al. (2017). Then, with more consistent and low ranks for each local processing patch, we are able to apply the low-rank approximation filter more effectively.

There are still three main problems in the low-rank approximation method. The first problem of the low-rank approximation method is the low computational efficiency caused by the singular value decomposition (SVD) (Zhou and Zhang, 2017) in the low-rank approximation method. The SVD decomposition is computationally intensive and is a requirement of the low-rank approximation. The computational complexity of SSA will also make the proposed method suffer from the low computational efficiency problem. The problem of low computational efficiency can be partially solved via some approximated SVD algorithms or some SVD-free matrix completion algorithms. One algorithm would be the randomized SVD method (Oropeza and Sacchi, 2010). The randomized SVD method approximates the exact SVD by means of randomization operation. The randomized SVD can greatly reduce the computational cost by about two times. Some other efficient SVD-like decomposition that can be used for the rank reduction in the low-rank approximation is the online subspace tracking method as introduced in Zhou et al. (2018). The online algorithm introduced in Zhou et al. (2018) used incremental gradient descent algorithm to Grassmannian manifold of subspace and obtain more efficient performance than the classic SVD based singular spectrum analysis (SSA) method. It is interesting to note that in the online subspace tracking method, there is also a randomization step involved to improve the noise rejection effect for spatially coherent signals. Other choices to substitute the SVD operation is also possible, and deserves future investigation.

The second problem of the classic low-rank approximation method is the rank to be defined before the filtering process. Theoretically, the rank in the low-rank approximation method is equal to number of linear seismic events. For synthetic example, where we know exactly how many linear events are shown in the data, we can easily define a correct rank. This makes all synthetic example processed by the low-rank approximation method perform almost perfect. However, the real seismic data is complicated and cannot be easily characterized by a combination of some simple linear events. In this case, the selection of the optimal rank for real data is not trivial. A smaller rank tends to damage most of the useful energy while a larger rank is easy to preserve to most of the unwanted random noise. Useful energy and noise energy is compromise in the SSA based rank reduction method. One possible strategy to alleviate the problem caused by the input parameter rank is to use localized windows. In localized windows, the seismic data become structurally simple and can be easier to be

characterized as linear events. Besides, in localized windows, the rank can be much smaller and the uncertainties caused by the ranks are much smaller than the situation without localized windows. Nevertheless, localized windows caused another undefined parameter, that is, the window size. Defining the window size, again, requires a prior knowledge and human interference. From this perspective, it is almost not possible to achieve satisfactory noise suppression performance and easy parameterization process in the same time. In the proposed dip-separated filtering framework, we also propose an automatic rank selection strategy. We calculate the singular value difference spectrum, which is formed to describe the variation of every adjacent singular values and then we propose to use the sharp peak $\max(b_i) = b_r$, $i = 1, 2, \dots, d-1$, to represent the boundary between signal and noise. Although not possible to obtain the perfect performance, it serves as the most appropriate way to balance the noise suppression performance and parameterization.

Another problem of the low-rank approximation method is the assumption of plane waves (Siahsar et al., 2017). It is required by the low-rank approximation method that the data to be filtered needs to be only containing plane waves. This requirement makes the traditional low-rank approximation method only valid in localized windows, where the seismic data are most similar to a superposition of linear events. As explained above, the localized windows require a pre-defined optimal window size, which is usually difficult to choose. The proposed dip-separated processing algorithm framework serves as a multi-spectral processing strategy for the low-rank approximation method. Because of the VMD decomposition, the seismic data corresponding to different spatially oscillating components are separated. The different spatially oscillating components are also referring to different dip bands of the seismic events. Simply speaking, because of the VMD decomposition, the seismic data are decomposed into different dipping components, where for each dipping component the seismic events are approximating to plane waves. Thus, the different dipping components after the VMD decomposition are appropriate for SSA filtering. From this perspective, the VMD process prepares a seismic dataset that is better suited by the low-rank approximation assumption. In summary, the proposed dip-separated processing algorithm framework not only relieves the dependency of the VMD method on the accurate selection of number of components (K), compensating for the mode-mixing weakness of the VMD process, but also makes the low-rank approximation perform better in preparing more spatially stationary, that is to say, more linear seismic events, for the subsequent SSA. From this perspective, the proposed method is less sensitive to additional noise than the traditional SSA method.

The rest of the paper is organized as follows: in the first section, we introduced the SSA based low-rank approximation algorithm; then we introduce the dip-separated low-rank approximation method; in the next section, we use both synthetic and real seismic data examples to demonstrate the performance of the proposed denoising method, finally, we draw some key conclusions.

THEORY

Variational mode decomposition

VMD algorithm attempts to non-recursively decompose a real-valued multi-component signal into a set of quasi-orthogonal band-limited modes where each mode is considered as being nearly compact around their respective center frequencies, simultaneously, every mode is characterized by the sparsity within the bandwidth. In addition, the sum of these decomposed modes is able to recover the input signal well (Dragomiretskiy and Zosso, 2014). The bandwidth of each mode is estimated by the following scheme:

- The analytic signal of every mode is obtained by Hilbert transform, which aims at a one-sided frequency spectrum with only positive frequencies:

$$\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \quad (1)$$

- Shift the frequency spectrum of each mode to baseband using the exponential term $e^{-jw_k t}$:

$$\left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-jw_k t} \quad (2)$$

- The bandwidth of each mode is calculated by the squared norm of the gradient:

$$\left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-jw_k t} \right\|_2^2 \quad (3)$$

The resulting constrained variational problem is written as (Dragomiretskiy and Zosso, 2014):

$$\begin{aligned} \min_{\{u_k\}, \{w_k\}} & \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-jw_k t} \right\|_2^2 \right\} \\ \text{s.t.} & \sum_k u_k(t) = f(t) \end{aligned} \quad (4)$$

where u_k and w_k denote the k -th decomposed mode of $f(t)$ and its center frequency, respectively. $\{u_k\}$ and $\{w_k\}$ represent the ensemble of modes and their corresponding center frequencies after decomposition for the signal $f(t)$, respectively. $f(t)$ is the given signal and $\delta(t)$ is the Dirac function. The unconstrained formula can be expressed as eq. (5) using a quadratic

penalty factor α and Lagrangian multipliers λ (Dragomiretskiy and Zosso, 2014; Liu et al., 2016, 2018b):

$$L(\{u_k\}, \{w_k\}, \lambda) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-jw_k t} \right\|_2^2 + \left\| f(t) - \sum_k u_k(t) \right\|_2^2 + \langle \lambda(t), f(t) - \sum_k u_k(t) \rangle, \quad (5)$$

where α is utilized to constrain the data-fidelity. The ADMM (Alternate Direction Method of Multipliers) (Hestenes, 1969) algorithm can be employed to solve eq. (5), which produces an ensemble of mode functions and their corresponding center frequencies. Each mode, obtained from solution in spectral domain, is formulated as:

$$\hat{u}_k^{n+1}(w) = \frac{\hat{f}(w) - \sum_{i=1}^{k-1} \hat{u}_i^{n+1}(w) - \sum_{i=k+1}^K \hat{u}_i^n(w) + \frac{\hat{\lambda}^n(w)}{2}}{1 + 2\alpha(w - w_k^n)^2}. \quad (6)$$

where $\hat{f}(w)$, $\hat{u}_i(w)$, $\hat{\lambda}(w)$, $\hat{u}_k^{n+1}(w)$ are the Fourier transforms of the input signal, i -th decomposed mode, Lagrangian multiplier and k -th mode in the case of $(n + 1)$ -th iteration, respectively, and the n represents iterations.

Low-rank approximation via SSA

Let the time series of a signal to be denoised be $\{h_i, i = 1, 2, \dots, N\}$, and then it is used to calculate a Hankel matrix with the expression:

$$\mathbf{H} = \begin{pmatrix} h_1 & h_2 & \cdots & h_K \\ h_2 & h_3 & \cdots & h_{K+1} \\ \vdots & \vdots & \ddots & \vdots \\ h_L & h_{L+1} & \cdots & h_N \end{pmatrix}, \quad (7)$$

where L represents window length parameter and $1 < L < N$. K is defined a $N - L + 1$. The delay value is 1. The matrix \mathbf{H} is referred to the trajectory

matrix. The resulting trajectory matrix is then decomposed by means of singular value decomposition (SVD) and \mathbf{H} can be rewritten as:

$$\mathbf{H} = \sum_{i=1}^d \mathbf{H}_i, \text{ with, } \mathbf{H}_i = \sqrt{\lambda_i} \mathbf{U}_i \mathbf{V}_i^T, \quad (8)$$

where $d = \text{rank}(\mathbf{H})$ and it is the number of eigen-components or modes with non-zero eigenvalue. λ_i are the singular values sorted in the descending order. \mathbf{U}_i and \mathbf{V}_i are respectively the associated left and right singular vectors. The group denoted by $\{\sqrt{\lambda_i}, \mathbf{U}_i, \mathbf{V}_i\}$ is called the i -th eigentriple. Then the elementary matrices $\mathbf{H}_i (i = 1, 2, 3, \dots, d)$ are split into signal and noise groups. However, the important step is to determine a subset of eigentriple that encompass the dominant variation in \mathbf{H} . This amounts to approximating matrix $\tilde{\mathbf{H}}$ by the summation of the first r elementary

matrices, using the following equation: $\tilde{\mathbf{H}} = \sum_{i=1}^r \mathbf{U}_i \sqrt{\lambda_i} \mathbf{V}_i^T$, where $\tilde{\mathbf{H}}$ is attributed to signal, and r denotes the number of eigentriples selected for

signal reconstruction, and the residual $\mathbf{R} = \sum_{i=r+1}^d \mathbf{U}_i \sqrt{\lambda_i} \mathbf{V}_i^T$ is taken as noise.

Dip separated low-rank approximation method

We are inspired from the decomposition based dip filter proposed in Chen and Ma (2014). The principle of the proposed method is that we use a decomposition method to prepare several dip components based on the dip filter, because of the dip separation, the ranks in each dip band become more consistent and is easier to use a global rank for the low-rank approximation in each processing window.

The VMD method is an advance in the decomposition based seismic data analysis literature. Its traditional counterpart, i.e., the empirical mode decomposition (EMD), method, has been investigated for a long time because of its strong capability in characterize the sharp frequency change in the seismic data (Chen et al., 2017). In these decomposition based methods, a decomposition step is first done to separate the whole seismic signal into a number of components with different oscillating frequency information. This decomposition step is very beneficial since it decomposes a highly non-stationary seismic signal into an ensemble of more stationary

signals, where some classic signal analysis tool, e.g., de-noising, reconstruction, time-frequency analysis, can be applied. Traditional decomposition methods, however, either have strong mode-mixing problem, or fail to reconstruct the original signal. Besides, traditional EMD based methods lacks mathematical implication and thus the decomposition is difficult to control. For example, one can not arbitrarily define a decomposition number and the target frequency range. These reasons make the traditional decomposition based approaches fail, in many situations, in the realistic applications.

The dip-separated processing method is detailed as follows:

1. Transform seismic data from $t-x$ domain to $f-x$ domain.
2. For each frequency slice in the $f-x$ domain, use VMD to decompose the seismic data into different IMFs. (a) Construct Hankel matrices for each component. (b) Apply low-rank approximation introduced in the last subsection for each Hankel matrix.
3. Reconstruct the frequency slice using the low-rank approximated IMFs.
4. Transform seismic data from $f-x$ domain to $t-x$ domain.

The proposed dip-separated processing method can be understood as an empirical low-rank approximation, where we first adaptively separate the seismic data into different dip components that have low individual ranks, and we can simply use a rank of 1 or 2 to optimally represent each dip component. Then the rank for the traditional low-rank approximation algorithm can be easily set to be 1 or 2. The time-space domain interpretation of the nonstationary signal decomposition in frequency-space domain is the decomposition of dip components. In other words, each dip components can be viewed as a low-rank component in its spectrum. To demonstrate the difference between EMD and VMD. We implement a numerical test in Figs. 1-4. Fig. 1 shows a demonstration of EMD separation in the $t-x$ domain, where we can see the three dipping components are almost separated. But there are some mode-mixing issues. Fig. 2 shows a demonstration of VMD separation in the $t-x$ domain, where three dipping components are also separated but with less serious mode-mixing issues. Figs. 3 and 4 show the demonstrations of the EMD and VMD separations in the $f-x$ domain.

EXAMPLES

In this part, we will use two realistic examples to demonstrate the potential of the proposed dip-separated filtering method in seismic applications.

The first example is based on a synthetic dataset, as shown in Fig. 5. Fig. 5(a) shows the clean data. In this dataset, we have several hyperbolic events. The first event crosses the second event, creating a challenge for traditional low-rank approximation (Oropeza and Sacchi, 2011; Huang et al.,

2016, 2017b,a). The seismic events have different amplitude. Besides, the zero-offset area shows a large curvature, which also makes the traditional low-rank method fail in obtaining acceptable performance. Fig. 5(b) shows the noisy data by adding some band-limited seismic noise. Figs. 5(c) and 5(d) show two zoomed areas from the data of the original sizes 5(a) and 5(b). The results from three methods, namely, the $f-x$ decon method, the traditional low-rank approximation method, and the proposed method are shown in Fig. 6. The top row of Fig. 6 plots the denoised data and the bottom row of Fig. 6 plots the zoomed areas from the top row. It is clear that both $f-x$ decon and the low-rank fail in suppressing a sufficient amount of noise, while the proposed method obtains a very clean result. The comparison is especially obvious in the zoomed sections on the bottom row. Fig. 7 shows three removed noise sections corresponding to the aforementioned denoising methods. It is salient that both $f-x$ decon and the low-rank methods result in a significant amount of signal leakage while the proposed method almost not remove useful energy. Fig. 7 shows the signal recovery errors for the three methods. The errors measures vividly where we have signal damage. From the error comparison, we further confirm that the proposed method causes the least signal recovery error.

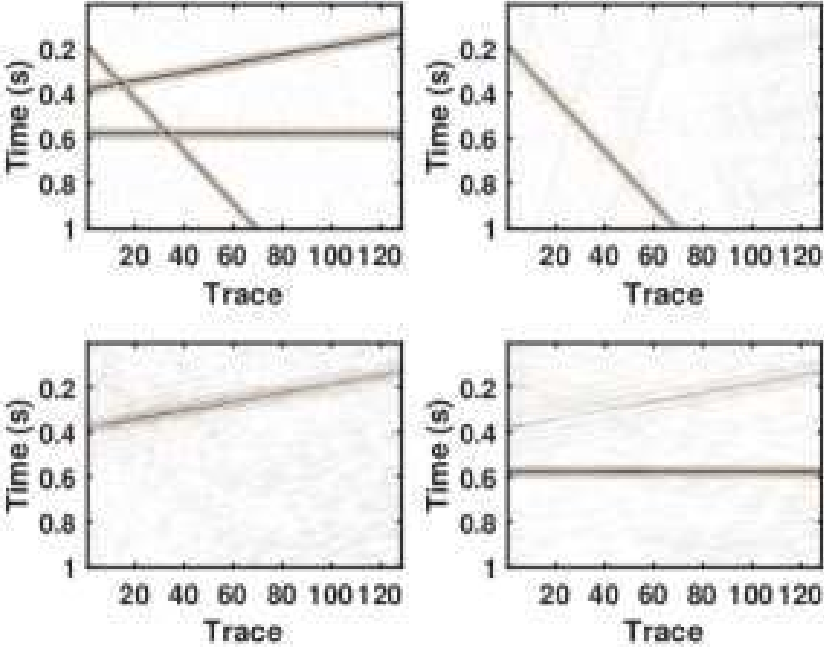


Fig. 1. Demonstration of EMD separation in the $t-x$ domain. (a) Input 2D seismic data. (b) First component. (c) Second component. (d) Third component.

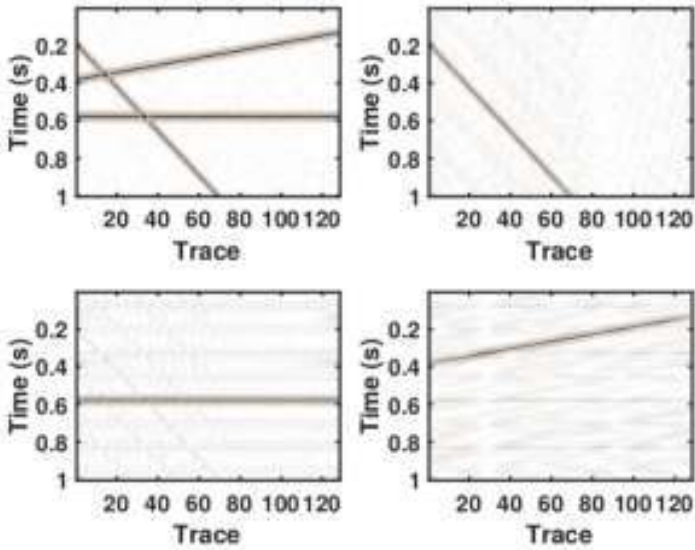


Fig. 2. Demonstration of VMD separation in the t - x domain. (a) Input 2D seismic data. (b) First component. (c) Second component. (d) Third component.

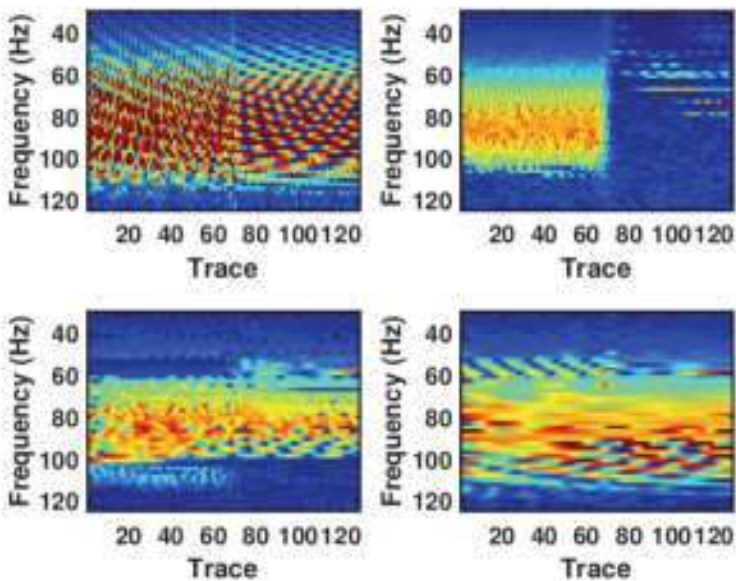


Fig. 3. Demonstration of EMD separation in the f - x domain. (a) Input 2D seismic data. (b) First component. (c) Second component. (d) Third component.

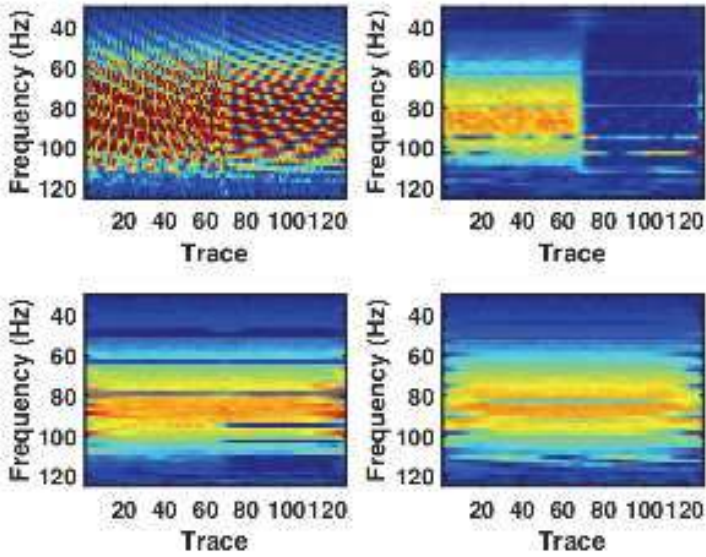


Fig. 4. Demonstration of VMD separation in the f - x domain. (a) Input 2D seismic data. (b) First component. (c) Second component. (d) Third component.

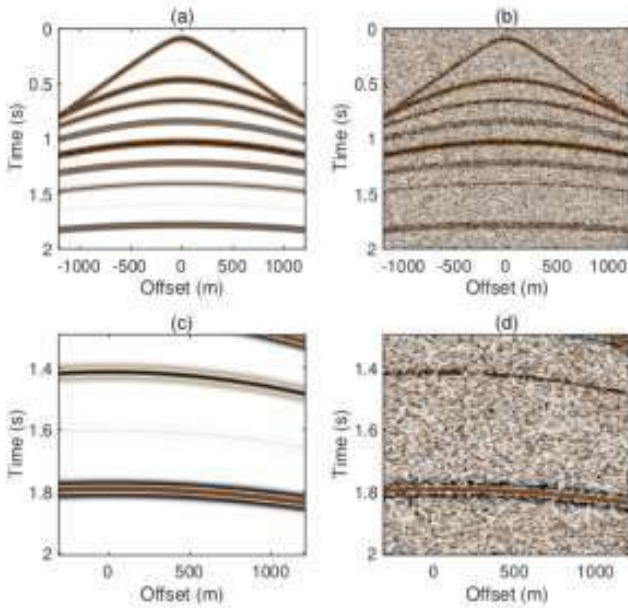


Fig. 5. Synthetic example. (a) Clean data. (b) Noisy data. (c) and (d) Zoomed areas from (a) and (b).

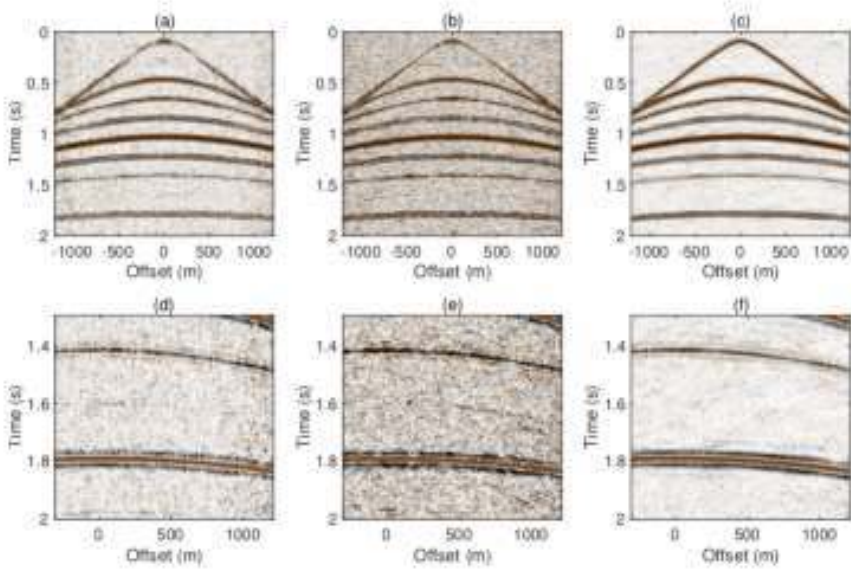


Fig. 6. Synthetic example. (a) Data after noise suppression using the f - x decon method. (b) Data after noise suppression using the low-rank approximation method. (c) Data after noise suppression using the proposed method. (d)-(f) Zoomed areas from (a)-(c).

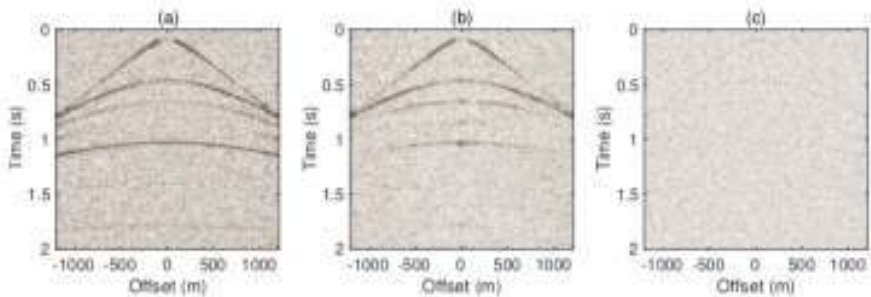


Fig. 7. Synthetic example. (a) Removed noise using the f - x decon method. (b) Removed noise using the low-rank approximation method. (c) Removed noise using the proposed method.

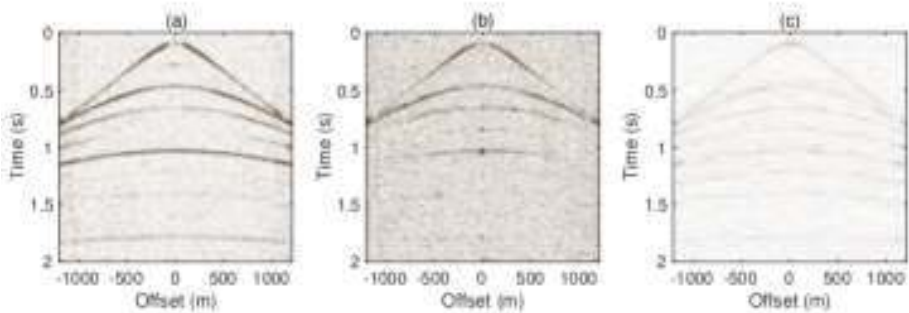


Fig. 8. Synthetic example. (a) Denoising error using the f - x decon method. (b) Denoising error using the low-rank approximation method. (c) Denoising error using the proposed method.

We then apply the proposed method to a very complicated real seismic data. Fig. 9 shows the field dataset in (a) and its zoomed section in (b). The field dataset is complicated in that it contains some weak-amplitude signals and some realistic geological features such as the pinch-out. The complicated structures make a conventional denoising method fail in preserving the useful energy while removing a significant amount of random noise. The proposed method still performs well on this field dataset, as shown in Fig. 10(c). Along with the result from the proposed method are the results from both f - x decon and the low-rank methods, in (a) and (b), respectively. The bottom row of Fig. 10 plots the zoomed areas from the top row of the figure. It is obvious that the proposed method obtains the cleanest result. The low-rank method, however, leaves a lot of residual noise in the data. The strong residual noise is because the data structure is extremely complicated and thus the rank defined for the truncated SVD should be large enough (Zhang et al., 2017; Zu et al., 2017; Chen et al., 2019). In this dataset, because we apply a dip-separation operation, the resulted components have a small rank for each component, which makes the subsequent low-rank method effective. Fig. 11 plots the removed noise sections for the three comparing methods, which confirm the previous observations.

CONCLUSIONS

We have introduced a novel dip-separated structural filtering method for suppressing strong random noise existing in multi-channel seismic data. The dip filter is achieved by applying a variational mode decomposition (VMD) operation to each frequency slice of the seismic data in the frequency-space domain. The VMD method is superior to the previous empirical mode decomposition (EMD) method in better separating the seismic data into

several dipping components without mode mixtures. The VMD based dip separator offers a better domain for applying the low-rank approximation method because in the dip separated domain, the rank is easier to choose for the low-rank method. The VMD based structural filtering is effective for field applications and its potential has been verified via both synthetic and field data examples.

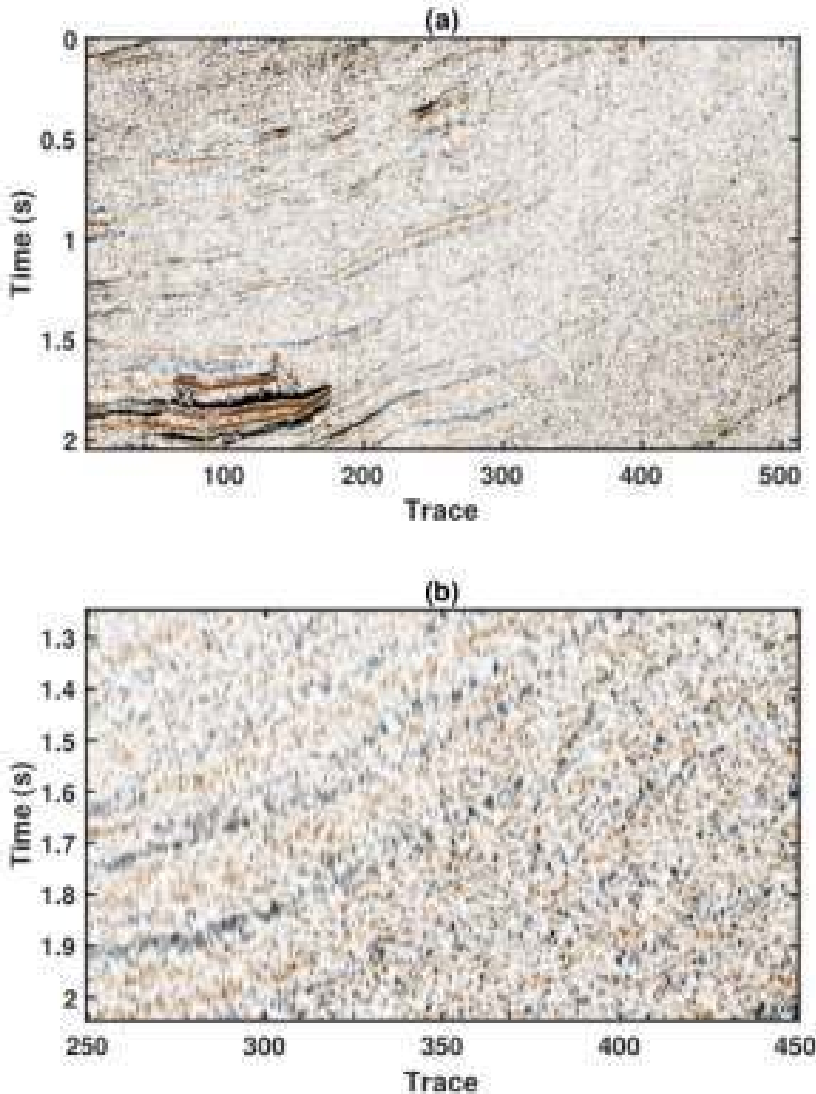


Fig. 9. Real data example. (a) Raw noisy data. (b) Zoomed data.

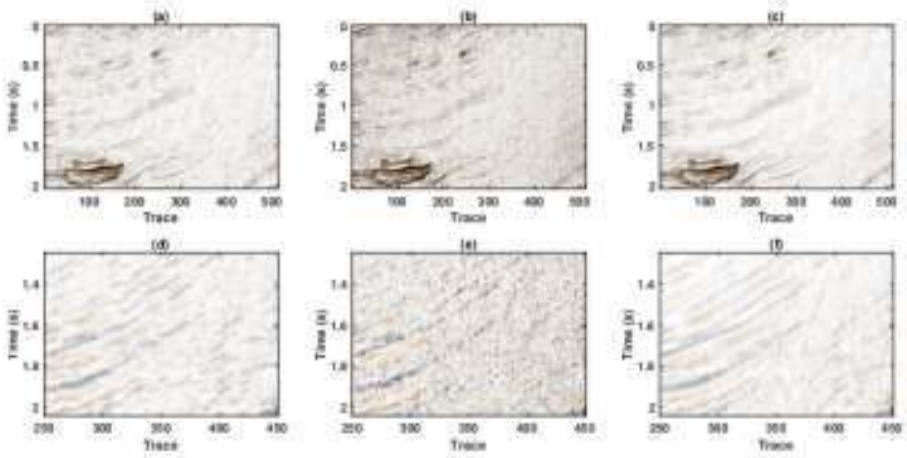


Fig. 10. Real data example. (a) Data after noise suppression using the f - x decon method. (b) Data after noise suppression using the low-rank approximation method. (c) Data after noise suppression using the proposed method. (d)-(f) Zoomed areas from the (a)-(c).

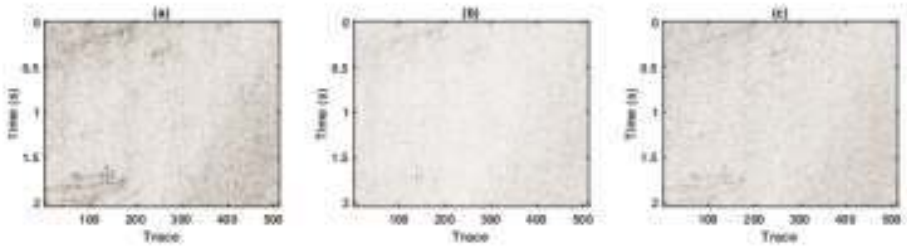


Fig. 11. Real data example. (a) Removed noise using the f - x decon method. (b) Removed noise using the low-rank approximation method. (c) Removed noise using the proposed method.

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