SEISMIC TRACE NOISE REMOVAL BY SMOOTHED SURESHRINK

REGIS NUNES VARGAS and ANTÔNIO CLÁUDIO PASCHOARELLI VEIGA

Faculty of Electrical Engineering, Federal University of Uberlândia, Uberlândia, MG, Brazil. regisnv@yahoo.com.br; acpveiga@ufu.br

(Received September 27, 2018; revised version accepted April 20, 2020)

ABSTRACT

Vargas, R.N. and Veiga, A.C.P., 2020. Seismic trace noise removal by smoothed SureShrink. *Journal of Seismic Exploration*, 29: 363-370.

Seismic traces are usually corrupted by Additive White Gaussian Noise (AWGN). AWGN hinders the evaluation of seismic attributes and can lead to distortions during seismic interpretation. Therefore, the development of methods that can effectively remove the noise and extract the signal from the seismic trace is critical. Here we propose a new seismic trace noise removal method called SureShrinkWin, which evaluates the estimates obtained by the SureShrink method when SureShrink is applied in signal windows. To validate the efficacy of the SureShrinkWin method, we performed a Monte Carlo Simulation that considered sixteen seismic traces that were obtained from the astsa R package.

KEY WORDS: wavelets, Monte Carlo simulation, SureShrink, seismic trace, denoising.

INTRODUCTION

Seismic traces are frequently corrupted by high noise levels during acquisition; therefore, the accuracy of the conclusions obtained from analyzing this signal is closely related to the effectiveness of the noise reduction method applied to the seismic trace during the processing stage. However, the non-stationarity characteristic associated with this type of signal makes its smoothing a challenge.

The discrete wavelet transform (DWT) is an effective tool for reducing noise in non-stationary signals since its coefficients can be localized in time-frequency bands (Donoho and Johnstone, 1995; Percival and Walden, 2006; Condat, 2013); for this reason, numerous methods based on this transform have been applied in seismic traces (Mousavi and Langston, 2016; Vargas and Veiga, 2017; Han and van der Baan, 2015; Gómez and Velis, 2016).

0963-0651/20/$5.00 © 2020 Geophysical Press Ltd.

The SureShrink and Universal Threshold Applied by Hard-thresholding Function (UHTWS) (Percival and Walden, 2006) methods have been applied in various types of research, e.g., for improving wireless system performance (Damati et al., 2016), ECG denoising (Han and Xu, 2016) and seismic denoising (Mohanalin et al., 2016). The Total Variation Denoising (1DTVD) (Condat, 2013) method has been applied not only in seismic traces (Liu et al., 2016), but also for ECG denoising (Ning and Selesnick, 2013) and the recovery of sharp images from blurred images (Perrone and Favaro, 2016).

In this paper, we propose a new noise reduction method, called the SureShrink method by Signal Windows (SureShrinkWin). The SureShrinkWin method consists of the application of applying the SureShrink method in signal windows and the subsequent subsequently processing of the signal estimates from the minimization of the absolute loss function. To verify the efficacy of the proposed method we performed a Monte Carlo simulation (Mooney, 1997) for sixteen seismic traces obtained from the astsa R package in order to compare the performance of the new method SureShrinkWin with three well-know denoising methods: SureShrink (Donoho and Johnstone, 1995), the universal threshold applied by hard-thresholding function UHTWS (Percival and Walden, 2006) and the total variation denoising 1DTVD methods (Condat, 2013).

The structure of the paper is as follows: the second section presents the DWT, the wavelet shrinkage and the SureShrink method. The third section presents the proposed SureShrinkWin method and its applications can be found in the fourth section. The last section reports the study's conclusions.

THE DISCRETE WAVELET TRANSFORM

In this section, we provide a brief overview of wavelets and DWTs without detailing the underlying mathematics or numerical algorithms. The basic components of a wavelet are the time (or space) and location. Wavelet analysis involves approximating signals using the linear combination of wavelets. There are two important functions in wavelet analysis: the mother wavelet and the father wavelet; these wavelets generate a family of functions that can reconstruct the signal. The mother wavelet $\psi(\cdot)$ is defined as a real function $\psi: \mathbb{R} \to \mathbb{R}$ such that $\int_{-\infty}^{\infty} \psi(t)dt = 0$. The father wavelet (or scale function) $\phi(\cdot)$ is a real function $\phi: \mathbb{R} \to \mathbb{R}$ such that $\int_{-\infty}^{\infty} \phi(t)dt = 1$. Both of these functions satisfy the integrability condition, specifically, $\psi, \phi \in L^2(\mathbb{R}) \cap L^1(\mathbb{R})$. Typically, the mother wavelet is bounded and centered on the origin. It decays to zero when $|t| \to \infty$. If we consider that $j, k \in \mathbb{Z}$, we can relate these two functions with the expansion equations $\psi(t) = \sqrt{2}\sum_k h_k \phi(2t-k)$ and $\phi(t) = \sqrt{2}\sum_k g_k \phi(2t-k)$. From the family (ψ, ϕ) we built the following wavelet sequence:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k),$$
$$\phi_{j,k}(t) = 2^{\frac{j}{2}}\phi(2^j t - k).$$

The values g_k and h_k represent the low-pass filter and high-pass filter coefficients, respectively; they satisfy the function $h_k = (-1)^k\, g_{1-k}$. The DWT maps data from the time domain to the wavelet domain as illustrated in Definition 1.

Definition 1

Let $\boldsymbol{y} = (y_0, y_1, \cdots, y_{N-1})'$ *be an i.i.d. random sample, with* $N = 2^J$, $J \in \mathbb{N}$. *Therefore, the DWT of* $\boldsymbol{y}$, *with respect to the mother wavelet* $\psi(\cdot)$, *is defined as*

$$d_{j,k} = \sum_{t=0}^{N-1} y_t\, \psi_{j,k}(t) \quad , \tag{1}$$

for all $j = 0,1,2,\cdots,J-1$ *and* $k = 0,1,2,\cdots,2^j-1$.

We can write the transform (1) in matrix form by

$$\boldsymbol{d} = \boldsymbol{W}\boldsymbol{Y}, \tag{2}$$

assuming that there are appropriate boundary conditions, the transform is orthogonal, and the Inverse DWT (IDWT) can be obtained as:

$$\boldsymbol{Y}' = \boldsymbol{W}'\boldsymbol{d} \quad .$$

To calculate the DWT, a fast pyramid algorithm with complexity $O(N)$ is applied, which consists of a sequence of high-pass and low-pass filters (Meyer, 1993). Given a noisy signal, $\boldsymbol{y}$, this pyramid algorithm returns two sets of wavelet coefficients. The detail wavelet coefficients set is $\{d_{j,k}\}$ and the smooth wavelet coefficients set is $\{s_{j,k}\}$, where $j \in \{0,\cdots,J-1\}$ and $k \in \{0,\cdots,2^j-1\}$.

Wavelet shrinkage

First, let us formalize the concept of Additive White Gaussian Noise (AWGN), in Definition 2.

Definition 2

Let $\boldsymbol{x}$ represent an N-length signal. If $e_1, e_2, \cdots, e_N$ are iid with normal distribution $\mathcal{N}(0, \sigma)$, then $\boldsymbol{e} = (e_1, e_2, \cdots, e_N)$ is an AWGN and $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{e}$ is a noisy signal.

Wavelet shrinkage usually refers to the reconstructions that are obtained from the shrunken wavelet coefficients. This reduces or even removes the noise mixed in with the real signal. It can be formally defined as follows.

Definition 3

Let $\boldsymbol{x}$ represent an N-length signal, $\boldsymbol{e}$ an AWGN and $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{e}$ a noisy signal; Let $\{d_{j,k}\}$ represent the detail wavelet coefficients obtained by application of the DWT to the noisy signal $\boldsymbol{y}$ and let $\lambda \in \mathbb{R}$ represent a threshold. The estimated signal of $\boldsymbol{x}$ (denoted by $\hat{\boldsymbol{x}}$) is obtained by the wavelet shrinkage method, applying the IDWT with the soft-thresholding given by the following equation:

$$\eta_s(d_{j,k}, \lambda) = \begin{cases} sgn(d_{j,k})(|d_{j,k}| - \lambda), if \ |d_{j,k}| > \lambda, \\ 0, \quad otherwise, \end{cases} \tag{3}$$

instead of $\{d_{j,k}\}$ for all values of j and k as shown in Definition 1. Therefore, the main problem associated with estimating an appropriate threshold λ, can be solved using the SURE Method.

The SURE method

The SURE method involves estimating the threshold λ. It is based on the following theorem proved by Stein (1981).

Theorem 1. *Let:*

1. *$\boldsymbol{\mu} \in \mathbb{R}^N$ be a parameter to be estimated;*
2. *$\boldsymbol{x}$ be an N-length normal random realization such that $x_i \sim \mathcal{N}(\mu_i, \sigma^2)$, $\sigma \in \mathbb{R}$ $i \in \{1, 2, 3, \cdots, N\}$;*
3. *$\boldsymbol{g}: \mathbb{R}^N \rightarrow \mathbb{R}^N$ such that $\boldsymbol{g}(x)$ estimates $\boldsymbol{\mu} \in \mathbb{R}^N$;*
4. *$\boldsymbol{h}: \mathbb{R}^N \rightarrow \mathbb{R}^N$ where $\boldsymbol{h}(x)$ is weakly differentiable such that $\boldsymbol{h}(x) = \boldsymbol{g}(x) - x$.*

Then,

$$E_{\boldsymbol{\mu}}\|\boldsymbol{g}-\boldsymbol{\mu}\|^2 = N\sigma^2 + \|\boldsymbol{h}(\boldsymbol{x})\|^2 + 2\sigma^2 \sum_{i=1}^{N} \frac{\partial h_i}{\partial x_i} \tag{4}$$

The SURE method (Donoho and Johnstone, 1995) selects a threshold λ that minimizes eq. (4) and considers $\boldsymbol{g}(\boldsymbol{x})$ to be the soft-thresholding given by eq. (3).

THE SURESHRINKWIN METHOD

Here we propose a new noise reduction method called the SureShrinkWin method. It is based on the several estimated values of the signal obtained by an iterative application of the SureShrink method to the signal of various lengths.

A N-length noisy signal, $\boldsymbol{y}$, is obtained from the addition of a white Gaussian noise $\boldsymbol{e}$ to a clean signal $\boldsymbol{x} = (x[1], \cdots, x[N])$, where $N = 2^J, J \in \mathbb{N}, N \geq 16$. To apply the SureShrinkWin method to the noisy signal $\boldsymbol{y}$, the following steps are necessary.

For each $h \in \{1,2,3 \cdots, J-3\}$, we define $n = 16\times 2^{(h-1)}$ and divide the noisy signal $\mathbf{y}$ in N/n non-overlapping windows, where each window has n observations.

Next, we apply the SureShrink method to each window, represented by the interval $I_s = [(s-1)n+1, sn]$ with $s \in \{1,2,3, \cdots, N/n\}$. For all $k \in I_s \cap N^*$, we denote by $\hat{x}_h[k]$ the adjusted fit of the value $x[k]$ on the interval I_s. Then, the estimation without noise $\hat{\boldsymbol{x}} = (\hat{x}[1], \cdots \hat{x}[N])$ of the signal $\boldsymbol{x}$ is obtained by $\hat{x}[k] = \underset{s\in\mathbb{R}}{\operatorname{argmin}} \sum_{h=1}^{J-3} |\hat{x}_h[k] - s|$, for all $k \in \{1,2,3, \cdots, N\}$.

APPLICATIONS

In this section, we perform Monte Carlo simulations (Mooney, 1997) for all the seismic traces obtained from the astsa R package, considering 100 replications of white Gaussian noise addition to each considered signal, in order to compare the performance of the proposed SureShrinkWin method with that of three well well-known noise reduction methods: SureShrink (Donoho and Johnstone, 1995), the universal threshold applied by hard-thresholding function (UHTWS) (Percival and Walden, 2006) and the total variation denoising (1DTVD) (Condat, 2013).

Let $\boldsymbol{x}$ be a seismic trace signal obtained from the astsa R package with length N, and let $\boldsymbol{y}$ be the N-length noisy signal obtained from addition of a white Gaussian noise $\boldsymbol{e}$ to the $\boldsymbol{x}$. To compare the performance of the proposed SureShrinkWin method with that of the others, we consider the Percentage Root Mean Square Difference (PRD) given by

$$\mathrm{PRD}(\boldsymbol{x}, \widehat{\boldsymbol{x}}) = 100\sqrt{\frac{\sum_{i=1}^{N}(x_i - \widehat{x}_i)^2}{\sum_{i=1}^{N} x_i^2}} \quad , \tag{5}$$

where $\widehat{\boldsymbol{x}}$ is the estimation for the clean signal $\boldsymbol{x}$. The lower the PRD value, the better is the method performed.

For each seismic trace signal $\boldsymbol{x}$, we generated 100 replications of time series from the white Gaussian noise model, where each replication was added to the signal $\boldsymbol{x}$, in order to obtain one hundred noisy signals. For each considered method, we applied the method to each noisy signal $\boldsymbol{y}$, calculating the Percentage Root Mean Square Difference PRD). Finally, we obtained the average of the PRD value from all 100 noisy signals that we obtained from the replications. The experiment's results are presented in Table 1. Notably, the SureShrinkWin method outperformed the method SureShrink, UHTWS and 1DTVD methods in, respectively, 100%, 97.9%, and 93.9% of the cases present in Table 1. Fig. 1 represents graphically the experimental results of Table 1. Fig. 2 shows an example of the seismic trace noise reduction accomplished by the SureShrinkWin method.

Table 1. Performance results of the denoising methods for each seismic trace obtained from astsa R package. The bold value represents the best performance.

SNR INPUT	Method	Seismic traces															
		EQ1	EQ2	EQ3	EQ4	EQ5	EQ6	EQ7	EQ8	EX1	EX2	EX3	EX4	EX5	EX6	EX7	EX8
-5	SureShrinkWin	**87.4**	**77.4**	**79.3**	**79.5**	**71.2**	**75.1**	**78.6**	**77.5**	**83.41**	**86.4**	**81.3**	**93.9**	**96.5**	**86.2**	**88.9**	**76.6**
	SureShrink	96.6	91.2	95.2	97.7	95.2	94.9	90.1	94.5	96.2	93.8	92.8	99.7	100	91.0	98.1	95.1
	UHTWS	92.4	84.1	88.0	93.8	88.0	88.2	83.9	85.9	89.4	88.9	86.6	98.5	100	84.7	94.6	90.8
	1DTVD	108	105	105	106	103	104	105	104	107	109	108	113	115	108	109	106
-3	SureShrinkWin	**79.4**	**66.8**	**69.5**	**70.3**	**61.0**	**64.9**	**68.0**	**67.8**	**73.9**	**78.0**	**71.7**	**86.2**	**89.6**	**77.3**	**80.1**	**67.4**
	SureShrink	94.2	87.4	91.0	95.2	91.0	90.8	86.0	89.8	92.4	91.1	89.4	98.5	100.	88.0	95.7	92.6
	UHTWS	88.5	77.6	79.8	88.3	81.0	81.1	77.2	78.7	82.1	84.9	81.6	95.8	99.9	81.7	89.3	86.5
	1DTVD	84.0	79.0	79.0	80.8	76.2	77.4	78.5	78.9	81.9	85.0	82.3	90.1	92.8	83.2	85.4	80.1
-2	SureShrinkWin	**76.1**	**61.7**	**65.0**	**66.4**	**56.7**	**61.2**	**63.5**	**63.7**	**69.8**	**74.4**	**67.0**	83.2	87.0	**73.1**	**76.3**	**63.8**
	SureShrink	92.8	85.0	88.2	93.4	88.6	88.6	84.1	87.3	90.0	89.7	87.6	97.8	99.9	86.3	94.1	91.2
	UHTWS	86.3	73.5	75.3	85.4	77.0	77.9	74.2	75.6	78.3	82.5	78.3	94.0	99.4	80.1	86.4	84.2
	1DTVD	74.8	69.2	69.2	71.6	66.1	67.6	68.7	69.1	72.6	75.8	72.6	**82.1**	**85.5**	74.4	76.4	71.1

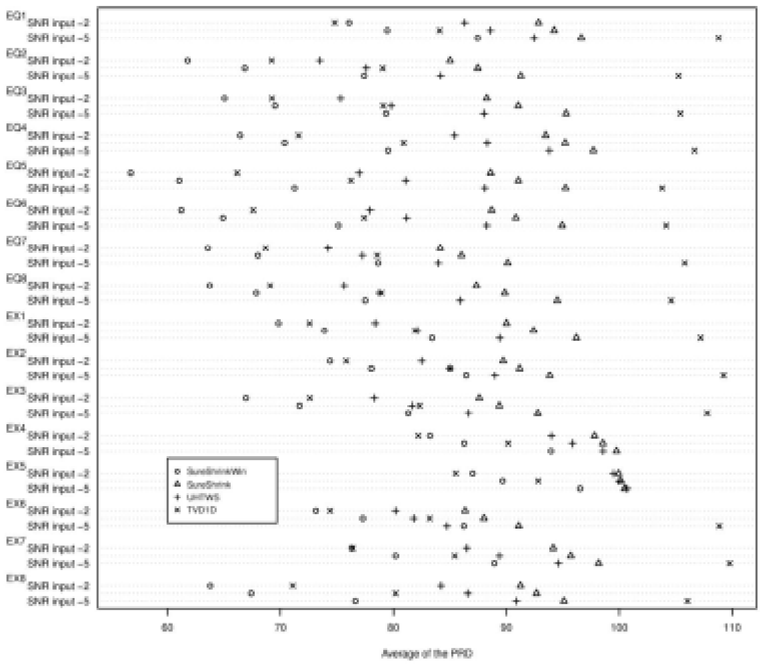

Fig. 1. Averages of the PRD of the Table 1, as lower the PRD value, the better is the performance of the method.

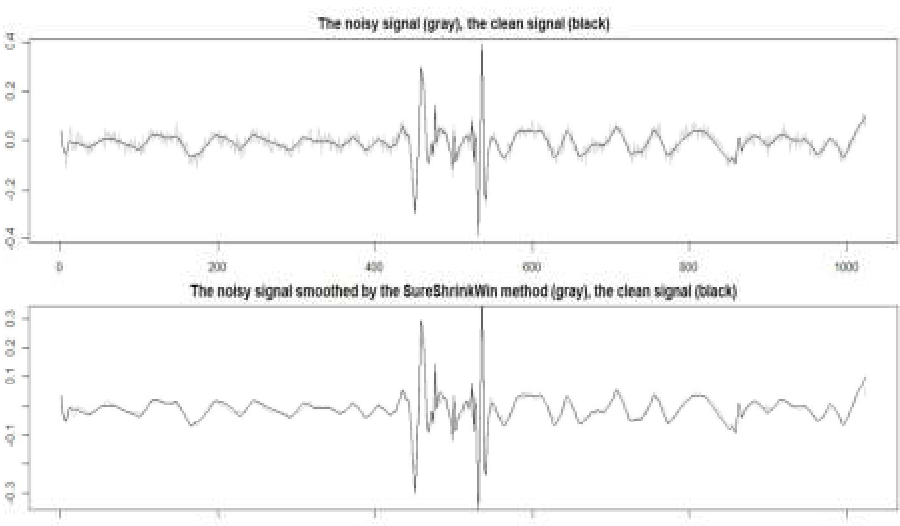

Fig. 2. Noise reduction by the method SureShrinkWin for the P-wave of the EQ1 signal.

CONCLUSIONS

Here we propose a new noise reduction method called SureShrinkWin that utilizes the effectiveness of the SureShrink method, while taking into account the inherent characteristics of the noise. The application of the SureShrinkWin method is made iteratively in windows of different lengths and returns a set of estimators that allow a better identification of the noise. This feature of the SureShrinkWin method was empirically validated by Monte Carlo simulations that ensured the effectiveness of the SureShrinkWin method when compared the results of the SureShrinkWin method with those of three well-known noise reduction methods.

ACKNOWLEDGEMENTS

This work was supported in part by National Counsel and Technological Scientific Development (CNPq).

REFERENCES

Condat, L., 2013. A direct algorithm for 1D total variation denoising. IEEE Sign. Process. Lett., 20: 1054-1057.

Damati, A., Daoud, O. and Hamarsheh, Q., 2016. Enhancing the odd peaks detection in OFDM systems using wavelet transforms. Internat. J. Communic., Netw. Syst. Sci., 9: 295-303.

Donoho, D.L. and Johnstone, I.M., 1995. Adapting to unknown smoothness via wavelet shrinkage. J. Am. Statist. Assoc., 90: 432, 1200-1224.

Gómez, J.L. and Velis, D.R., 2016. A simple method inspired by empirical mode decomposition for denoising seismic data. Geophysics, 81(6): V403-V413.

Han, J. and van der Baan, M., 2015. Microseismic and seismic denoising via ensemble empirical mode decomposition and adaptive thresholding. Geophysics, 80(6): KS69-KS80.

Han, G. and Xu, Z., 2016. Electrocardiogram signal denoising based on a new improved wavelet thresholding. Rev. Sci. Instrum., 87(8): 084303.

Liu, Y., Dang, B., Li, Y., Lin, H. and Ma, H., 2016. Applications of Savitzky-Golay filter for seismic random noise reduction. Acta Geophys., 64: 101-124.

Meyer, Y., 1993. Wavelets: Algorithms and Applications. Soc. Industr. Appl. Mathemat, Philadelphia, PA.

Mohanalin, J., Prabavathy, S., Torrents-Barrena, J., Puig, D. and Beena, M., 2016. A novel wavelet seismic denoising method using type ii fuzzy. Appl. Soft Comput., 48: S507-S521.

Mooney, C.Z., 1997. Monte Carlo Simulation. Sage Publications, Vol. 116. Thousand Oaks, CA.

Mousavi, S.M. and Langston, C.A., 2016. Hybrid seismic denoising using higher-order statistics and improved wavelet block thresholding. Bull. Seismol. Soc. Am., 106: 1380-1393.

Ning, X. and Selesnick, I.W., 2013. ECG enhancement and QRS detection based on sparse derivatives. Biomed. Sign. Process. Contr., 8: 713-723.

Percival, D.B. and Walden, A.T., 2006. Wavelet Methods for Time Series Analysis. Cambridge University Press, Cambridge.

Perrone, D. and Favaro, P., 2016. A clearer picture of total variation blind deconvolution. IEEE Transact. Patt. Analys. Mach. Intell., 38: 1041-1055.

Stein, C.M., 1981. Estimation of the mean of a multivariate normal distribution. Ann. Statist., 9: 1135-1151.

Vargas, R.N. and Veiga, A.C.P., 2017. Seismic trace noise reduction by wavelets and double threshold estimation. IET Signal Process., 1069-1075.