PHASE VELOCITY DEPENDENT P-WAVE REFLECTION COEFFICIENT EQUATION FOR VTI/VTI MEDIA

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(Received March 13, 2019; revised version accepted March 17, 2020)

ABSTRACT

Liu, J.-W., Chang, Y.-F. and Yeh, Y.-L., 2020. Phase velocity dependent P-wave reflection coefficient equation for VTI/VTI media. *Journal of Seismic Exploration*, 29: 389-401.

Amplitude variation with offset (AVO) has become a commonly used seismic attribute in the petroleum exploration to reveal the lithology and estimate pore fluids underground. However, strata usually exhibit velocity anisotropy, thus the effect of anisotropy must be taking into account when applying the AVO analysis. Ruger's approximation of the P-wave reflection coefficient equation at an interface between two vertical transverse isotropic (VTI) layers, in welded contact, is widely used in anisotropic AVO analysis. Based on Ruger's approximate equation, a new anisotropic term of the P-wave reflection coefficient for VTI/VTI media is derived in this study which is only function of the velocity difference between the incident angle dependent phase velocity and the vertical velocity.

Ruger's, Banik's and new derived approximate equations of the P-wave reflection coefficient for three commonly occurring in close in-situ proximity shale and gas sand models are calculated and compared. Study results show that the anisotropic effect is important in the reflection amplitude even though the anisotropies of the two layers are weak. Since the anisotropic effects of the P-wave phase velocity is dominated by the anisotropic parameter δ for VTI media within small incident angles, the anisotropic effect of the reflection P-wave amplitude within intermediate offset is also dominated by δ. All approximate equations fit the exact solution well within the intermediate offset except the anisotropic parameter ε is large. For the far offset, above approximate equations using to analyze the anisotropic reflection amplitude must be avoided except for analyzing the negative high acoustic impedance contrasts data by using the new derived approximate equation. In addition, this new derived approximate equation has a simple and direct physical meaning which is useful in understanding the effect of anisotropy on the AVO response.

KEY WORDS: AVO, anisotropy, VTI.

0963-0651/20/$5.00

INTRODUCTION

The technique of amplitude variation with offset (AVO) (or reflection coefficient variation with incidence angle) of seismic reflections has become an important tool for hydrocarbon prospecting. In conventional AVO analysis it is assumed that the strata are isotropic. However, the strata in the crust usually exhibit velocity anisotropy (Crampin, 1981; Crampin and Lovell, 1991). Therefore, the AVO analysis of seismic wave propagation in the isotropic media is evolving to the wave propagation in the anisotropic media to fulfill the requirements of accurate reservoir characterization. In an undisturbed environment, the natural deposition process tends to produce horizontally layered sedimentary rock. This layered stratum is commonly characterized as a transversely isotropic medium (TI medium) with a vertical symmetry axis (VTI) which is frequently observed in both land and marine environments (Backus, 1962). In addition to the VTI medium, a TI layer with a horizontal symmetry axis (HTI) or a tilted symmetry axis (TTI) is also found in the field. Thus, in the AVO analysis, the effect of anisotropy of the layer must be considered.

Daley and Hron (1977) had derived the solution of the reflection coefficients at the interface of anisotropic media. But, the solution is complex and lack of physical insight to the AVO signature. To make the solution more intuitive, research is conducted to solve the problem. Thomsen (1986) defined three anisotropic parameters and used them to express the phased velocities of P- and S-wave propagation in TI media. Banik (1987) based on Thomsen's conclusion (Thomsen, 1986), which the most anisotropic effects of the P-wave phase velocity is dominated by δ (one of Thomsen's anisotropic parameters) under a weak anisotropy and small incident angles assumptions, found that the anisotropic term of the P-wave reflection coefficient is also dominated by δ. Thomsen (1993) derived a new expression of the P-wave reflection coefficient under an assumption of weak anisotropy, for larger incident angles of the P-wave especially. Ruger (1997) refined Thomsen's (1993) work and obtained an approximate equation of the P-wave reflection coefficient under an assumption of weak anisotropy and small acoustic impedance contrast between media. He also conducted a detailed derivation of the AVO for the HTI media considering the impact of the azimuth angle on AVO, namely azimuthal amplitude variation with offset analysis (AVAZ) (Ruger, 1998, 2002). Because Ruger's approximation for VTI media is a linear equation of anisotropic parameters, full of physical meaning and more accurate at larger incident angles of the P-wave, it is now widely used in hydrocarbon exploration. Ruger and Gray (2014) had highlighted the high sensitivity of the azimuthally changing reflection coefficient to pertinent anisotropy parameters for using AVAZ to characterize fractured reservoirs. Pan et al. (2016) used the P- and PS-wave AVAZ data for reservoir fracture characterization based on P- and PS-reflection coefficients are sensitive to fracture weaknesses. Ehirim and Chikezie (2017) used anisotropic AVO analysis for reservoir characterization in Derby field southeastern Niger delta shows the importance of seismic anisotropy in AVO analysis.

In this study, based on Ruger's approximate equation of P-wave reflection coefficients for VTI media, a new, intuitive and simple approximate equation is proposed. Then this new approximate equation will be compared with Ruger and Baniks' approximate equations by using commonly-used rock parameters.

P-WAVE REFLECTION COEFFICIENT FOR VTI/VTI MEDIA

Thomsen proposed three non-dimensional anisotropic parameters (ε, γ and δ) to express the anisotropies of a medium, and now these parameters are commonly used in the seismic exploration for the seismic wave propagation in anisotropic media (Thomsen, 1986). The parameter ε represents the fractional difference between the horizontal and vertical P-wave velocities, γ expresses the same measurement for a SH-wave and δ gives the variation of P-wave velocity near the symmetry axis.

The interleaving thin horizontal sediments on a smaller scale than the seismic wavelength is characterized as the transverse isotropy, which is one of the most common anisotropic models used to study the behaviors of seismic wave propagation in anisotropic media (Thomsen, 1986). A VTI medium is frequently observed in both land and marine environments (Backus, 1962). Therefore, P-wave propagating along the vertical direction of a VTI medium has the slowest velocity. In addition, δ gives a strong effect on P-wave velocity along this propagation direction. In opposite, propagation along the horizontal direction, P-wave velocity is dominated by ε and P-wave has the fastest propagation velocity along this direction (Thomsen, 1986).

Ruger (1997) derived an approximate P-wave reflection coefficient equation for VTI/VTI media under small contrasts of the anisotropy and acoustic impedance across the interface which is:

$$R_P^{VTI}(i) \approx \frac{1}{2}\frac{\Delta Z}{\overline{Z}} + \frac{1}{2}\left\{\frac{\Delta V_{P0}}{\overline{V}_{P0}} - \left(\frac{2\overline{V}_{S0}}{\overline{V}_{P0}}\right)^2 \frac{\Delta G}{\overline{G}} + \Delta\delta\right\}\sin^2 i + \frac{1}{2}\left\{\frac{\Delta V_{P0}}{\overline{V}_{P0}} + \Delta\varepsilon\right\}\sin^2 i \cdot \tan^2 i, \quad (1)$$

where $R_P^{VTI}(i)$ is the P-wave reflection coefficient equation for VTI/VTI media; i is the incident angle; V_{P0} and V_{S0} are the vertical velocities of P- and S-wave, respectively. $Z = \rho V_{P0}$ and $G = \rho V_{S0}^2$; ρ is the density of the layer. The average value and the difference in vertical P-wave velocity are defined as $\overline{V}_{P0} = \frac{1}{2}(V_{P0_1} + V_{P0_2})$ and $\Delta V_{P0} = V_{P0_2} - V_{P0_1}$, and correspondingly for $\overline{V}_{S0}$, ΔV_{S0}, $\overline{Z}$, ΔZ, $\overline{G}$ and ΔG. Subscripts 1 and 2 represent the layers 1 and 2, respectively. The differences in anisotropic coefficients across the boundary are written as $\Delta\delta = (\delta_2 - \delta_1)$ and $\Delta\varepsilon = (\varepsilon_2 - \varepsilon_1)$. This equation is a function of contrasts of the P-wave vertical velocity, vertical acoustic impedances of P- and S-wave as well as anisotropic coefficients δ and ε across the interface, and it is independent of γ. This equation is linear of anisotropic parameters, and currently is widely used in seismic anisotropic AVO analysis.

Eq. (1) also shows that the approximate reflection coefficient equation of a VTI/VTI interface is equal to those of an ISO/VTI interface which the top stratum is isotropic ($\delta_1 = 0$ and $\varepsilon_1 = 0$) but the bottom stratum has anisotropic coefficients δ and ε. In other words, they have the same $\Delta\delta$ and $\Delta\varepsilon$ for both interfaces, which leads to $\Delta\delta = (\delta_2 - \delta_1) = \delta$ and $\Delta\varepsilon = (\varepsilon_2 - \varepsilon_1) = \varepsilon$. Furthermore, the P-wave reflection coefficient of a VTI/VTI interface is also equal to those of a VTI/ISO interface which the bottom stratum is isotropic ($\delta_2 = 0$ and $\varepsilon_2 = 0$), but the top stratum has anisotropic coefficients $-\delta$ and $-\varepsilon$. This result is consistent with previous study by Ruger (2002), who stated that if there is no contrast in δ and ε across the interface, the approximate reflection coefficient coincides with that for purely isotropic media even though both layers may be VTI. Therefore, the theoretical solution of the P-wave reflection coefficient for VTI/VTI media ($R_P^{VTI/VTI}$) can be expressed as:

$$R_P^{VTI}(i) = R_P^{VTI/VTI}(i) = R_P^{ISO/VTI}(i) = R_P^{-VTI/ISO}(i) \quad , \tag{2}$$

which is a function of contrasts of the P-wave vertical velocity, vertical acoustic impedances of P- and S-wave and anisotropic coefficients δ and ε between strata.

Ruger (2002) also shows that the P-wave reflection coefficient can be decomposed as the isotropic and anisotropic reflection coefficients which is:

$$R_P^{VTI}(i) \approx R_P^{ISO}(i) + R_P^{ANI}(i) = R_P^{Ruger} \quad , \tag{3}$$

where $R_P^{ISO}(i)$ is the reflection coefficient in the absence of anisotropy ($\Delta\varepsilon = 0$ and $\Delta\delta = 0$), and $R_P^{ANI}(i)$ is the anisotropic term given by:

$$R_P^{ANI}(i) = \frac{1}{2}\Delta\delta \cdot \sin^2 i + \frac{1}{2}\Delta\varepsilon \cdot \sin^2 i \cdot tan^2 i = \frac{1}{2}\Delta\sigma \cdot \sin^2 i + \frac{1}{2}\Delta\varepsilon \cdot tan^2 i \quad , \qquad (4)$$

where $\Delta\sigma = (\sigma_2 - \sigma_1)$ and $\sigma = (\delta - \varepsilon)$ which is a measure of the deviation of the P-wave phase velocity surface from the elliptical. This approximate P-wave reflection coefficient is named as R_P^{Ruger} used in this study.

Focusing on the contribution of anisotropy on P-wave reflection coefficient for small incident angle (i is small), then $\tan^2 i \approx i^2$, $\cos i \approx 1$ and $\sin i \approx i$, and eq. (3) can be expressed as:

$$R_P^{VTI}(i) \approx R_P^{ISO}(i) + R_P^{ANI}(i) = R_P^{ISO}(i) + R_P^{ISO/VTI}(i)$$

$$\approx R_P^{ISO}(i) + \frac{1}{2}(\delta + \varepsilon \cdot i^2) \cdot i^2 = R_P^{ISO}(i) + \frac{1}{2}(\sigma + \varepsilon) \cdot i^2 \quad . \qquad (5)$$

where the anisotropic term in eq. (5) is:

$$R_P^{ANI}(i) \approx \frac{1}{2}(\delta + \varepsilon \cdot i^2) \cdot i^2 = \frac{1}{2}(\sigma + \varepsilon) \cdot i^2 \quad . \qquad (6)$$

For weak anisotropy, the phase velocity of P-wave in a VTI medium can be approximated as (Thomsen, 1986):

$$V_p(i) \approx V_{P0}(1 + \delta \cdot \sin^2 i \cdot \cos^2 i + \varepsilon \cdot \sin^4 i) = V_{P0}(1 + \sigma \cdot \sin^2 i \cdot \cos^2 i + \varepsilon \cdot \sin^2 i) . \qquad (7)$$

For small incident angle, $\cos i \approx 1$ and $\sin i \approx i$, then eq. (7) becomes:

$$V_p(i) \approx V_{P0} + V_{P0}(\delta + \varepsilon \cdot i^2) \cdot i^2 = V_{P0} + V_{P0}(\sigma + \varepsilon) \cdot i^2 \quad . \qquad (8)$$

Eq. (6) and the anisotropic term in eq. (8) are very similar, except the constant before the term $(\delta + \varepsilon \cdot i^2) \cdot i^2$ (or $(\sigma + \varepsilon) \cdot i^2$). Replacing the anisotropic term of eq. (5) with eq. (8), a new form of P-wave reflection coefficient equation is obtained and represented by the following equation:

$$\mathrm{R}_{\mathrm{P}}^{\mathrm{VTI}}(i) \approx \mathrm{R}_{\mathrm{P}}^{\mathrm{ISO}}(i) + \frac{1}{2}\frac{V_p(i) - V_{P0}}{V_{P0}} = \mathrm{R}_{\mathrm{P}}^{\mathrm{New}} \quad . \qquad (9)$$

Eq. (9) indicates that the anisotropic term of the P-wave reflection coefficient is only function of the velocity difference between the incident angle dependent phase velocity and the vertical velocity and named as $\mathrm{R}_{\mathrm{P}}^{\mathrm{New}}$ used in this study.

Banik (1987) followed Thomsen's conclusion (Thomsen, 1986) and proposed a two-term and only δ dependent P-wave phase velocity ($V_p^{\delta}(i)$) which is:

$$V_p^{\delta}(i) \approx V_{P0} + V_{P0} \cdot \delta \cdot \sin^2 i \quad .$$

Rewrite this equation as:

$$\delta \cdot \sin^2 i \approx \frac{V_p^{\delta}(i) - V_{P0}}{V_{P0}} \quad . \qquad (10)$$

Then he also showed that the P-wave reflection coefficient equation for VTI media can be approximated as:

$$\mathrm{R}_{\mathrm{P}}^{\mathrm{VTI}}(i) \approx \mathrm{R}_{\mathrm{P}}^{\mathrm{ISO}}(i) + \frac{1}{2}\delta \cdot \sin^2 i \quad . \qquad (11)$$

Replace $\delta \cdot \sin^2 i$ with eq. (10), eq. (11) becomes:

$$\mathrm{R}_{\mathrm{P}}^{\mathrm{VTI}}(i) \approx \mathrm{R}_{\mathrm{P}}^{\mathrm{ISO}}(i) + \frac{1}{2}\frac{V_p^{\delta}(i) - V_{P0}}{V_{P0}} = \mathrm{R}_{\mathrm{P}}^{\mathrm{Banik}} \quad . \qquad (12)$$

The anisotropic term in eq. (12) represents the velocity difference between the δ dependent P-wave phase velocity and the vertical velocity. This equation is named as $\mathrm{R}_{\mathrm{P}}^{\mathrm{Banik}}$ used in this study. If $\delta = \varepsilon$ (or $\sigma = 0$), the angular dependent P-wave phase velocity surface is degenerate elliptical, then $V_p(i) = V_p^{\delta}(i)$ and $\mathrm{R}_{\mathrm{P}}^{\mathrm{New}}$ is equal to $\mathrm{R}_{\mathrm{P}}^{\mathrm{Banik}}$.

NUMERICAL TESTING

Physical parameters of media

Gas and oil are commonly stored in porous sands and overlain by impermeable shale. Gas sand detection is currently the most promising application of AVO analysis (Castagna, 1993). Therefore, the physical parameters of the commonly occurring in close in-situ proximity shales and gas sands are used to calculate the P-wave reflection coefficients. Following Ruger's three-model classifications of shale/sandstone interfaces (Ruger, 2002), which consists positive high (model 1), positive low (model 2) as well as negative high (model 3) acoustic impedance contrasts across the interface. The P- and S-wave velocities and densities of shales and gas sands of models are listed in Table 1. The P-wave reflection coefficient for VTI/ISO media is investigated.

The P-wave reflection coefficient for VTI/VTI media is function of two (δ and ε) of the three anisotropic parameters (Ruger, 1997) and the average values of anisotropies of shales compiled by Thomsen (1986) are ε = 0.12 and δ = 0.13. Therefore, the anisotropic parameters of weak (ε = 0.05; δ = -0.1), moderate (ε = 0.1; δ = 0.1) and strong anisotropies (ε = 0.25; δ = 0.25) are used to test the accuracy of the approximate P-wave reflection coefficient equations. Thus there are three cases of ε values and three cases of δ values, totally 9 cases, will be discussed in this study.

Table 1. P- and S-wave velocities and densities of shales and gas sands used in this study; adopted from Ruger (2002).

	Shale			Gas Sand			Contrast			
	Vp (km/s)	Vs (km/s)	ρ (g/cm^3)	Vp (km/s)	Vs (km/s)	ρ (g/cm^3)	$\frac{\Delta Vp}{\overline{Vp}}$	$\frac{\Delta Vs}{\overline{Vs}}$	$\frac{\Delta \rho}{\overline{\rho}}$	$\frac{\Delta Z}{\overline{Z}}$
Model 1	3.3	1.7	2.35	4.2	2.7	2.49	0.240	0.454	0.058	0.297
Model 2	2.96	1.38	2.43	3.49	2.29	2.14	0.164	0.496	-0.127	0.038
Model 3	2.73	1.24	2.35	2.02	1.23	2.13	-0.298	-0.008	-0.098	-0.394

$Z = \rho Vp$

Test results

The P-wave reflection coefficient for isotropic media can be calculated by Aki and Richards's (1980) matrix form of Zoeppritz equations. The exact solution of P-wave reflection coefficient at a VTI/VTI interface can be found in Ruger's (2002) results. The reflection amplitude of the P-wave could dramatically change when the incident angle is close to the critical angle, thus the wide-angle data is seldom used in the AVO analyzing. Therefore, the maximum incident angle is set as 40 degrees in this study. The exact solutions (red lines; Exact R_p), Ruger's (green lines; R_P^{Ruger}), Banik's (magenta lines; R_P^{Banik}) and the new derived (blue lines; R_P^{New}) approximate equations of the P-wave reflection coefficient and isotropic solutions (black lines; $R_P^{Isotropic}$) for ε = 0.05, 0.1 and 0.25 are shown in Figs. 1, 2 and 3, respectively. In Figs. 1-3, δ = -0.1, 0.1 and 0.25 are shown in Figs. (a), (b) and (c), respectively. For cases of $\delta = \varepsilon$ [Figs. 2(b) and 3(c)], the P-wave phase velocity surface is an ellipse, R_P^{New} is equal to R_P^{Banik}, therefore, the blue and magenta lines are overlapping each other.

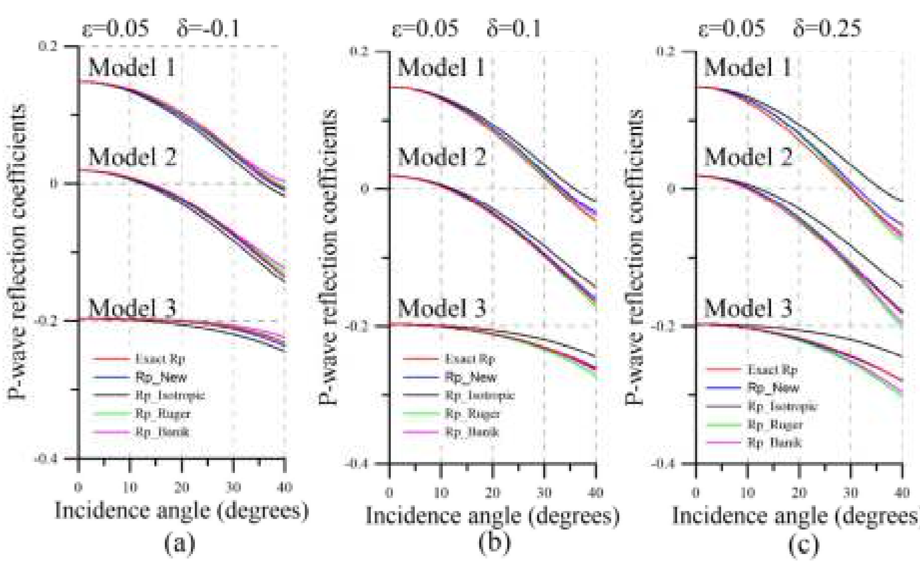

Fig. 1. Incident angle dependent P-wave reflection coefficients for three commonly encountered models (Ruger, 2002) with weak anisotropy (ε = 0.05) and (a) δ = -0.10 (or σ = -0.15), (b) δ = 0.10 (or σ = 0.05), (c) δ = 0.25 (or σ = 0.2).

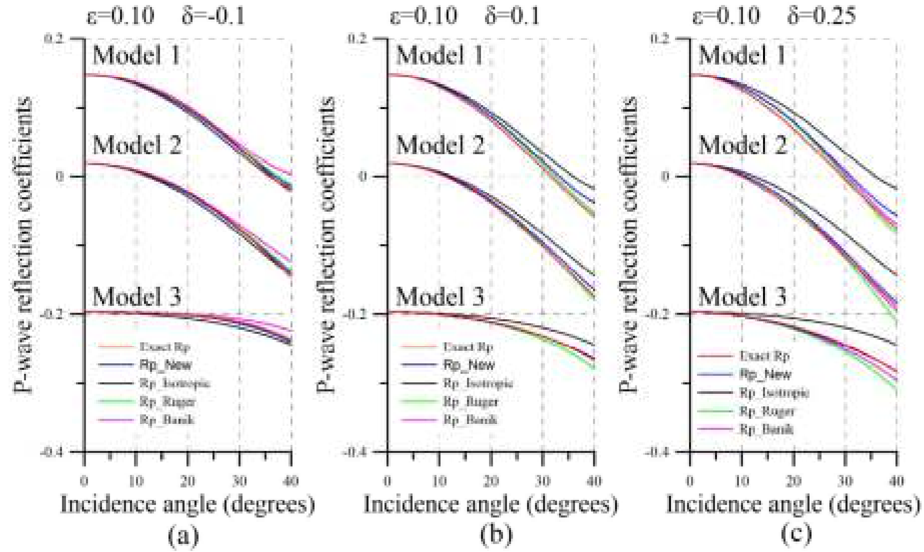

Fig. 2. Incident angle dependent P-wave reflection coefficients for three commonly encountered models (Ruger, 2002) with moderate anisotropy (ε = 0.10) and (a) δ = -0.10 (or σ = -0.2), (b) δ = 0.10 (or σ = 0.), (c) δ = 0.25 (or σ = 0.15).

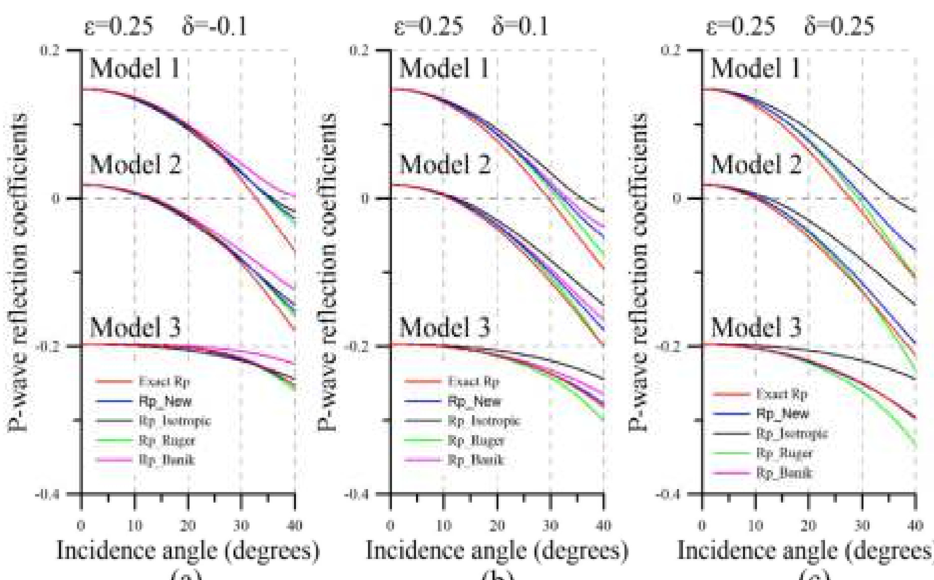

Fig. 3. Incident angle dependent P-wave reflection coefficients for three commonly encountered models (Ruger, 2002) with strong anisotropy (ε = 0.25) and (a) δ = -0.10 (or σ = -0.35), (b) δ = 0.10 (or σ = -0.15), (c) δ = 0.25 (or σ = 0.).

In Thomsen's (1986) notation, anisotropy does not have any influence on the reflection coefficient for vertically incident waves. Thus, in Figs. 1, 2 and 3, the P-wave reflection coefficients of all approximations at the normal incidence (i = 0°) are the same for the same models even though media are anisotropic. In addition, all approximations fit the exact solutions well between 0-10 degrees of P-wave incident angles except for the isotropic solutions with δ = 0.25. For 10-40 degrees of P-wave incident angles, even though the anisotropy is weak, the isotropic solutions deviate largely from the exact solutions except for ε = 0.1, 0.25 and δ = -0.1. This shows the importance of considering the anisotropic effect in the anisotropic reflection amplitude. This is consistent with Ehirim and Chikezie's conclusions by using anisotropic AVO to analyze the real data (2017).

The variations of the P-wave reflection coefficient between 0-40 degrees of P-wave incident angles increase with the values of δ and ε for all models. Both of the values of ε and δ can affect the variation between 20-40 degrees of P-wave incident angles, especially for the large value of ε or δ. Which shows that both of the values of ε and δ can affect the anisotropic reflection amplitude between 20-40 degrees of P-wave incident angles. For ε = 0.05, 0.1 and δ = -0.1, 0.1 and 0.25, all approximations fit the exact solution well between 10-30 degrees of P-wave incident angles. This shows that δ dominates the anisotropic effect in the anisotropic reflection amplitude from near to intermediate offsets except the ε is very large. In other words, δ is more sensitive to the P-wave reflection coefficient than ε within the intermediate offset. Which is similar to the role of δ playing in the phase velocity (Thomsen, 1986). Banik's approximate equation is only depend on δ, thus his approximate error is large for cases of which ε is far greater than δ and the incident angles of P-wave from 20 to 40 degrees. For model 3, the new derived approximate equation fit the exact solutions well for all cases from 0 to 40 degrees of P-wave incident angles. On the contrary, for models 1 and 2 at far offset, 30 to 40 degrees of P-wave incident angles, all approximations fit some cases well but some cases do not fit well. Which indicates that using these approximations to analyze the anisotropic reflection amplitude between 30 to 40 degrees of P-wave incident angles must be avoided except for analyzing the model 3 data by using the new derived approximate equation.

DISCUSSIONS

Ruger (1997) modified the Thomsen's (1993) approximate equation for the P-wave reflected from VTI/VTI media and obtained a more accurate, intuitive and linear approximate equation of the P-wave reflection coefficient. Ruger's approximation is consistent with the observations that $\Delta\delta$ and $\Delta\varepsilon$ dominates the behavior of the P-wave AVO for small and large incident angles of P-wave, respectively (Kim et al., 1993). The trigonometric functions in Ruger's anisotropic term [eq. (4)] are shown in Fig. 4. The red solid line represents $\sin^2 i$ and the blue solid line denotes $\sin^2 i \cdot \tan^2 i$. If the incident angle of P-wave is smaller than 40 degrees, $\sin^2 i$ is greater than $\sin^2 i \cdot \tan^2 i$. In other words, the anisotropic parameter δ dominates the anisotropic term from 0 to 40 degrees of P-wave incident angles except ε is far greater than δ. However, when comparing with the exact solutions, Ruger's approximate equation is only correct for 0-30 degrees of P-wave incident angles. These indicate the anisotropic term cannot be express by a simple linear term of anisotropic parameters δ and ε for 30-40 degrees of P-wave incident angles.

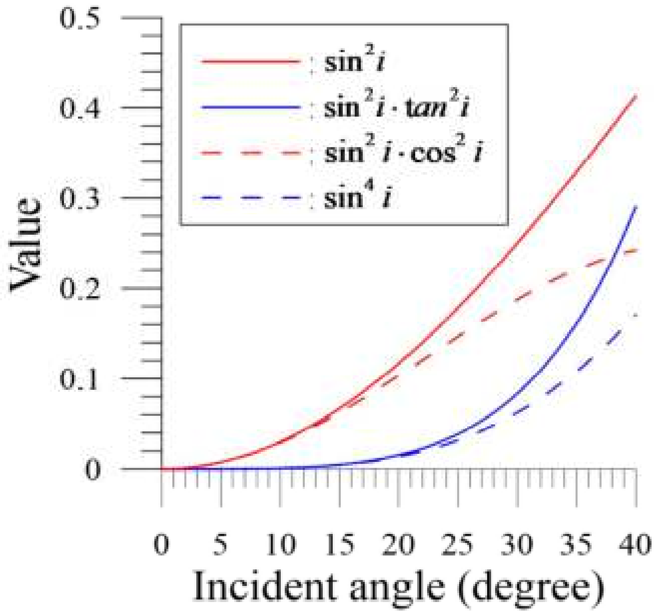

Fig. 4. Values of trigonometric functions in Ruger's and new derived anisotropic terms.

For the new derived approximate equation of the P-wave reflection coefficient at a VTI/VTI interface, the anisotropic term [eq. (9)] is only function of the velocity difference between the incident angle dependent phase velocity and vertical velocity. The trigonometric functions in the phase velocity [eq. (7)] are also shown in Fig. 4. The red dashed line indicates $\sin^2 i \cdot \cos^2 i$ and the blue dashed line represents $\sin^4 i$. In this figure, $\sin^2 i \cdot \cos^2 i$ is greater than $\sin^4 i$ with incident angle of P-wave within 40 degrees. This means δ also dominates the anisotropic term in the new derived anisotropic term except ε is far greater than δ. In addition, $\sin^2 i \cdot \cos^2 i$ is smaller than $\sin^2 i$ and $\sin^4 i$ is also smaller than

$\sin^2 i \cdot \tan^2 i$, which indicates the influences of the anisotropic parameters, δ and ε, in the P-wave reflection coefficient are weakened in the new derived anisotropic term when compared with Ruger's approximation. This weakness can partially compensate the overestimation of the P-wave reflection coefficient in the Ruger's approximation for the negative high acoustic impedance contrasts (model 3) across the interface. Thus the new derived approximation is better than Ruger's approximation for model 3. However, this effect has no help for the positive high and positive low acoustic impedance contrasts (models 1 and 2) across the interface since Ruger's approximation does not overestimate the solution.

CONCLUSIONS

Based on Ruger's approximate equation of the P-wave reflection coefficient for VTI/VTI media, the new approximate equation is derived in this study which the anisotropic term of the P-wave reflection coefficient is only function of the velocity difference between the incident angle dependent phase velocity and vertical velocity. Testing Ruger's, Banik's and the newly derived approximate equations for three commonly encountered models show that considering the anisotropic effect is important in the anisotropic reflection amplitude analysis even though the anisotropies of the two layers are weak. All approximate equations fit the exact solution well between 0-30 degrees of P-wave incident angles except when ε is large. This shows that δ dominates the anisotropic effect in the anisotropic reflection amplitude from near to intermediate offsets. For the far offset, P-wave incident angles from 30-40 degrees, the approximate equations using to analyze the anisotropic reflection amplitude must be avoided except for analyzing the negative high acoustic impedance contrasts data by using the new derived approximate equation. In addition, this new derived approximate equation has a simple and direct physical meaning. Thus, a new, effective, simple and direct approximate equation for calculating the P-wave reflection coefficient for VTI/VTI media is proposed.

REFERENCES

Aki, K. and Richards, P.G., 1980. Quantitative Seismology: Theory and Methods. W.H. Freeman and Co., San Francisco.

Backus, G. E., 1962. Long-wave elastic anisotropic produced by horizontal layering. J. Geophys. Res., 67: 4427-4441.

Banik, N.C., 1987. An effective anisotropy parameter in transversely isotropic media. Geophysics, 52: 1654-1664.

Castagna, J.P., 1993. AVO analysis-tutorial and review. In: Castagna, J.P. and Backus, M.M. (Eds.), Offset-dependent Reflectivity - Theory and Practice of AVO Analysis. SEG, Tulsa, OK: 3-37.

Crampin, S., 1981. A review of wave motion in anisotropic and cracked elastic-media. Wave Motion, 3: 343-391.

Crampin, S. and Lovell, J.H., 1991. A decade of shear-wave splitting in the earth's crust: What does it mean? What use can we make of it? And what should we do next? Geophys. J. Internat., 107: 387-407.

Daley, P.F. and Hron, F., 1977. Reflection and transmission coefficients for transversely isotropic media. Bull. Seismol. Soc. Am., 67: 661-675.

Ehirim, C.N. and Chikezie, N.O., 2017. Anisotropic AVO analysis for reservoir characterization in Derby Field Southeastern Niger Delta. J. Appl. Phys., 9: 67-73.

Kim, K.Y., Wrolstad, K.H. and Aminzadeh, F., 1993. Effects of transverse isotropy on P-wave AVO for gas sands. Geophysics, 58: 883-888.

Pan, B., Sen, M.K. and Hanming, G.H., 2016. Joint inversion of PP and PS AVAZ data to estimate the fluid indicator in HTI medium: a case study in Western Sichuan Basin, China. J. Geophys. Engineer., 13: 690-703.

Ruger, A., 1997. P-wave reflection coefficients for transversely isotropic models with vertical and horizontal axis of symmetry. Geophysics, 62: 713-722.

Ruger, A., 1998. Variation of P-wave reflectivity with offset and azimuth in anisotropic media. Geophysics, 63: 935-947.

Ruger, A., 2002. Reflection coefficients and azimuthal AVO analysis in anisotropic media. SEG: 22.

Ruger, A. and Gray, D., 2014. Wide-azimuth amplitude-variation-with-offset analysis of anisotropic fractured reservoirs. In: Grechka, V. and Wapenaar, K. (Eds.), Encyclopedia of Exploration Geophysics. SEG: N1, 1-14.

Thomsen, L., 1986. Weak elastic anisotropy. Geophysics, 51: 1954-1966.

Thomsen, L., 1993. Weak anisotropic reflections. In: Castagna, J.P. and Backus, M.M. (Eds.), Offset-dependent Reflectivity - Theory and Practice of AVO Analysis. SEG, Tulsa, OK: 103-114.