A TIME-VARYING WAVELET ESTIMATION METHOD BASED ON MODIFIED SPECTRAL MODELING IN THE T-F DOMAIN

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(Received August 9, 2019; revised version accepted December 3, 2020)

ABSTRACT

Dai, Y.S., Zhang, H.Q., Zhang, Y.H., Wan, Y., Sun, W.F., Han, H.Y. and Wu, S., 2021. A time-varying wavelet estimation method based on modified spectral modeling in the T-F domain. *Journal of Seismic Exploration*, 30: 211-236.

To effectively solve the problem existing in the time-varying seismic wavelet extraction method based on spectral modeling in the time-frequency (T-F) domain, a time-varying seismic wavelet extraction method based on modified spectral modeling in the T-F domain is proposed. For reducing the energy diffusion of the T-F spectra of seismograms, the synchrosqueezing modified S-transform (SSMST) is used to extract the T-F spectra of seismograms. The energy is gathered at the real frequency and the extraction accuracy of the T-F spectra of seismograms is improved. To solve the problem that the polynomial order needs to be determined artificially, an evaluation function describing the quality of estimated wavelet amplitude is established. The polynomial order is determined by comparing the value of the evaluation function. The phase spectra of seismic wavelets are extracted by phase-only filter. The validity of the method is verified by simulation experiments and real seismograms data processing.

KEY WORDS: synchrosqueezing modified S-transform, wavelet extraction, determining polynomial order, phase-only filter.

INTRODUCTION

Seismic wavelets have many applications in seismic data processing. The accuracy of estimated seismic wavelets directly affects the results of deconvolution, wave impedance and forward modeling. In field seismic data, seismic wavelets are scattered from and absorbed by the underground medium during propagation, resulting in energy attenuation and causing the

0963-0651/21/$5.00

phase distortion of the wavelets. In the S-domain, Li et al. (2015) used time-varying wavelet to deconvolution with seismograms, so that the in-phase axis of the seismic profile became thinner and the continuity became better, the shallow, medium and deep energy were compensated, and the deep energy was significantly improved. Chi et al. (2015) pointed out that compared with the non-time-varying wavelet adopted by predecessors, the seismic profile resolution after spectral inversion processing using time-varying wavelet is higher. Therefore, the extraction of time-varying seismic wavelet is one of the most important research methods for high-resolution seismic data processing.

There are two steps in the extraction of amplitude spectra for the seismic wavelet. Firstly, the T-F analysis methods are used to extract the T-F spectra of the seismograms and the amplitude spectra of seismograms corresponding to each moment are obtained. Then, the nature of the seismic wavelet amplitude spectra is used to fit the amplitude spectra of the seismic wavelet from the seismogram amplitude spectra.

In the first step, various time-frequency analysis methods have been applied to extract time-varying wavelet amplitude spectra. According to the time-varying characteristics of seismic wavelet, a method using Gabor transform (GT) for extracting the T-F spectra of seismograms was proposed by Margrave et al. (1988, 2011). To improve the accuracy of the amplitude spectra of seismogram, the generalized S-transform (GST) was proposed to extract the T-F spectra of seismograms by Zhou et al. (2014, 2016), which improved the T-F resolution of the T-F spectra. By analyzing the influence of the truncation effect at the endpoint of the analysis window on the estimated amplitude spectra, Wang et al. (2010, 2013) and Gao et al. (2009) proposed an adaptive segmentation method for seismogram and realized the reasonable division of window functions. In order to achieve a reasonable choice of T-F resolution, a modified S-transform (MST) was proposed by Li et al. (2016), which has a good application in wavelet extraction. However, due to the limitation of uncertainty principle, the T-F resolution of the above-mentioned methods is limited, which leads to the existence of false bandwidth and false frequency components in the T-F spectra of seismograms. Daubechies et al. (2011) proposed synchrosqueezing transform (SST) to solve this problem. The T-F spectra resolution was improved by extrusion time spectra energy and had a wide range of applications in seismic denoising (Wang et al., 2014), spectral decomposition (Shang et al., 2013), oil and gas detection (Thakur et al., 2011). By analyzing the advantages of S-transform over wavelet transform, Huang et al. (2016) and Liu et al. (2017) proposed the synchrosqueezing S-transform (SSST) and applied it to thin layer recognition and achieved good results. However, the function of S-transform window is fixed and cannot be adjusted flexibly according to actual needs. In the second step, autocorrelation method and high-order cumulant method are inappropriate due to the limitation of the length of seismogram. The main methods of wavelet amplitude spectrum extraction are the spectral modeling method (Rosa, 1991) and the quadratic spectra method (Liu et al., 2010). The quadratic spectra method has high computational efficiency, but it is difficult to set the cut-off frequency of the filter. Spectral modeling method has a high extraction accuracy and is widely used in seismogram wavelet amplitude spectra extraction (Dai et al., 2015; Wang et al., 2015, 2016; Li et al., 2015), but polynomial order in this method needs to be set artificially. When the polynomial order is not appropriate, it will seriously affect the accuracy of estimated wavelet amplitude spectra. How to determine the polynomial order is an important problem needed to be solved urgently in spectral modeling method.

There are three main methods to extract phase spectrum of seismic wavelet, namely constant phase rotation method (Zhou et al., 1989), high-order cumulant method (Yu et al., 2011) and phase-only filter method (Zhang et al., 2014). The constant phase rotation method assumes the phase of the seismic wavelet is a constant, which is inconsistent with the actual situation. A certain amount of data is required in high-order cumulant method and it is not applicable to the case of short data. So that high-order cumulant method cannot be directly applied to the extraction of time-varying wavelet phase spectra. The phase-only filter method does not require the assumption that the seismic phase is a constant and is suitable for the case of short data. Therefore, this paper uses the phase-only method to extract the phase spectra.

To solve the problems existing in the time-varying wavelet extraction method based on spectral modeling in the T-F domain, SSMST is proposed to extract the time-frequency spectrum of seismogram, which can flexibly select window function according to the data to be processed, and effectively reduce the false frequency components generated by the traditional T-F analysis methods. Meanwhile, the evaluation function describing the quality of the estimated wavelet amplitude spectra is established. By comparing the values of evaluation functions corresponding to different polynomials, the corresponding spectral modeling parameters are determined adaptively, so the estimation accuracy of wavelet amplitude spectrum is improved. Finally, the phase spectra of seismic wavelets are extracted by phase-only filter. Seismogram wavelets in time-domain are obtained by combining the amplitude spectra and phase spectra. The results of numerical simulation and seismic data processing verify, compared with the traditional time-frequency analysis method, the proposed method can obtain more real time-frequency spectrum of seismogram, and can adaptively select spectral modeling parameters, thus effectively improving the extraction accuracy of time-varying wavelet amplitude spectrum.

THEORY

In this paper, the SSMST is used to extract the T-F spectra of seismograms at first. Then, the amplitude spectrum of time-varying wavelet is extracted by adaptive spectral modeling method. Finally, the seismic wavelet amplitude spectra are combined with the phase spectra extracted by the phase-only filter method to extract the time-varying seismic wavelet. The processing flow is as follows:

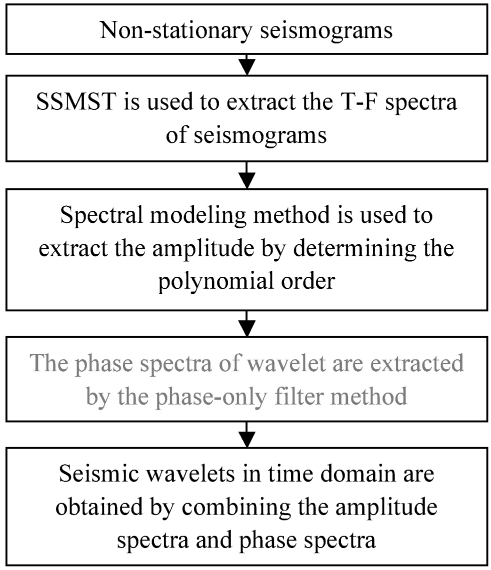

Fig. 1. Flow chart of time-varying wavelet extraction method based on modified spectral modeling in the time-frequency domain.

T-F analysis of seismogram based on SSMST

Due to the wavefront diffusion and absorption attenuation, the energy of the seismic wavelets will be attenuated and the frequency band will be narrowed during propagation. To describe the variation of the seismic wavelet, the T-F analysis methods are used to obtain the amplitude spectra of the seismograms corresponding to each moment. However, the amplitude spectra of the seismograms extracted by the traditional T-F analysis methods have false frequencies because of the limitation of the uncertainty principle. To solve this problem, the SSMST is used to extract the spectra of seismograms in this paper.

SSMST

T-F analysis methods have a wide range of applications in seismic wavelet extraction. Considering the attenuation characteristics of the seismic wavelets, MST was proposed by Li. In this method, the time width and bandwidth of the window function can be adjusted by adjusting two parameters and the desired temporal resolution and frequency resolution can be obtained. The T-F resolution of seismograms is improved by adopting MST. The time domain expression for MST is

$$MST_x(f,\tau) = \int_{-\infty}^{+\infty} x(t) \frac{|Af+B|}{\sqrt{2\pi}} e^{-\frac{(Af+B)^2(t-\tau)^2}{2}} e^{-i2\pi f t} dt \quad , \qquad (1)$$

where $MST_x(f,\tau)$ is the MST of $x(t)$ and A、B are two adjustment parameters.

Let $\varphi(t)=\frac{1}{\sqrt{2\pi}}e^{\frac{t^2}{2}}e^{i2\pi t}$, the expression of the MST in the frequency domain can obtained:

$$MST_x(f,\tau)=\frac{1}{2\pi}e^{-i2\pi f\tau}\int_{-\infty}^{+\infty}\hat{x}(\xi)\overline{\hat{\varphi}(\frac{\xi}{Af+B})}e^{i\tau\xi}d\xi, \tag{2}$$

where $\hat{x}(\xi)$ is the FT of $x(t)$ and $\overline{\varphi(\xi)}$ is the complex conjugate of FT of $\varphi(t)$.

According to eq. (2), the MST of the signal $y(t)=acos(2*\pi*f_0)$ can be obtained and the mathematical expression is

$$GST_y(f,\tau)=\frac{a}{2}e^{-i2\pi(f-f_0)\tau}\overline{\hat{\varphi}(\frac{2\pi f_0}{Af+B})}. \tag{3}$$

However, the Fourier Transform (FT) of the signal is

$$FT_y(f)=\frac{a}{2}[\delta(f+2\pi f_0)+\delta(f-2\pi f_0)]. \tag{4}$$

It can be known from eq. (4) that the energy of the amplitude spectra of the signal $y(t)$ should be concentrated at f_0, but the T-F spectra obtained by MST are within a certain range near f_0. There are false frequencies in the T-F amplitude spectra extracted by the MST.

Fig. 2 is the amplitude spectra comparison of the FT and MST for $f(t)=\sin(100\pi t)+\sin(220\pi t)$. Fig. 2a is the amplitude spectrum of FT and Fig. 2b is the amplitude spectrum of MST.

It can be seen from Fig. 2 that the energy of amplitude spectrum corresponding to the FT is concentrated at 50 Hz and 110 Hz, which is match the true situation. The energy of the amplitude spectrum corresponding to the MST is outwardly diffused around 50 Hz and 110 Hz and there are false frequencies. Similarly, T-F amplitude spectra extracted by the MST have false frequencies.

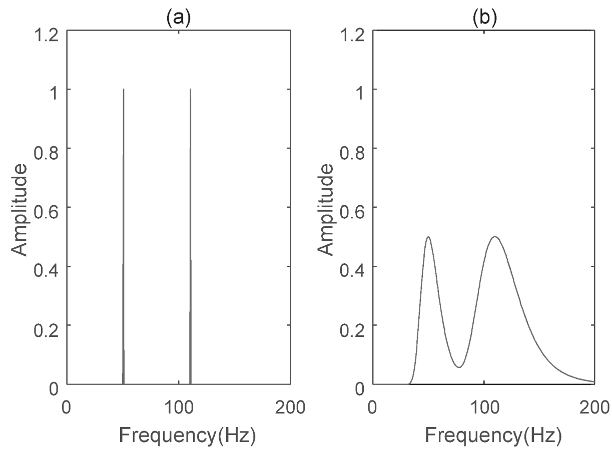

Fig. 2. Comparison of amplitude spectra. (a) FT; (b) MST.

For effectively reducing false frequencies existing in the amplitude spectrum, it is necessary to convert the false frequencies into its corresponding instantaneous frequency. For example, convert the false frequencies in Fig. 2(b) to 50 Hz and 110 Hz. To achieve this goal, firstly, calculate the derivative of the T-F spectrum of MST. By deriving eq. (4), the following equation can be obtained:

$$\frac{\partial MST_y(f,\tau)}{\partial \tau} = -i\pi a(f-f_0)e^{-i2\pi(f-f_0)\tau}\overline{\hat{\varphi}(\frac{2\pi f_0}{Af+B})}. \tag{5}$$

The energy of the amplitude spectrum of $f(t)$ should be concentrated at f_0. The expression of the instantaneous frequency of the signal is further derived by using this feature. The specific equation is as follows:

$$\hat{f}(f,\tau) = f + [i2\pi MST_y(f,\tau)]^{-1}\frac{\partial MST_y(f,\tau)}{\partial \tau}. \tag{6}$$

For the signal $y(t)$, $f(f,\tau)=f_0$ and eq. (7) can convert the false frequency to its corresponding instantaneous frequency.

After calculating the instantaneous frequency, the energy concentrated on false frequencies is re-superimposed onto the instantaneous frequency. The specific energy superposition equation is as follows:

$$SSMST_x(\hat{f}_l,\tau) = (\Delta\hat{f}_l)^{-1} \times \sum_{f_k:|\hat{f}_l(f_k,\tau)-\hat{f}_l|\leq\Delta\hat{f}/2} |MST_x(f,\tau)| f_k \Delta f_k, \tag{7}$$

where f_k and Δf_k are the discrete frequency and discrete frequency interval corresponding to the MST, respectively, $\Delta f_k = f_k - f_{k-1}$. $\hat{f}_l$ and $\Delta\hat{f}_l$ are the discrete frequency and discrete frequency interval corresponding to the *SSMST*, respectively.

Eq. (7) is the equation of *SSMST*. The spectrum is compressed by superimposing the energy corresponding to the false frequencies to the instantaneous frequencies. False frequencies are reduced and the T-F resolution is improved.

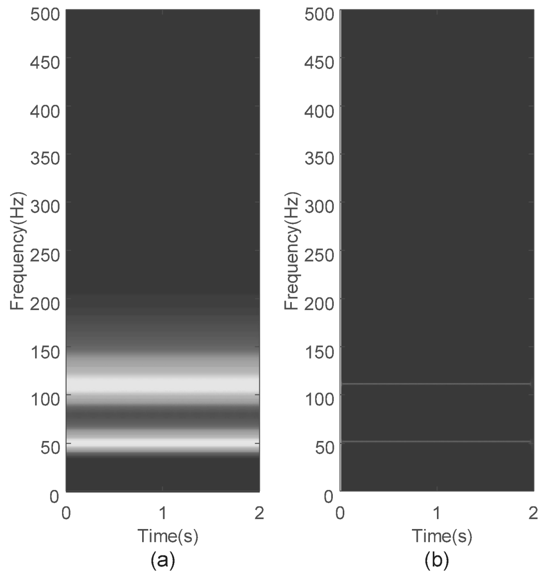

Fig. 3. Comparison of T-F amplitude spectra (a) MFT; (b) SSMST.

Fig. 3 is the T-F amplitude spectra comparison of the MST and SSMST. Fig. 3a shows the T-F amplitude spectrum of MST and Fig. 3b shows the T-F amplitude spectrum of SSMST. It can be seen from the Fig. 3 that the T-F amplitude spectrum of MST has obvious false frequency and the energy diffusion phenomenon is more serious. The T-F spectrum obtained by the SSMST effectively solves the above problem and the spectrum energy can be concentrated on instantaneous frequencies. Therefore, in this paper, SSMST is used to extract the T-F amplitude spectra of seismograms.

T-F analysis of seismogram based on SSMST

Considering the absorption attenuation of the seismic wavelet, the dynamic convolution model of non-stationary seismogram can be expressed as:

$$s(t) = w(t,\tau) * r(t) = \int_{-\infty}^{+\infty} w(t-\tau) r(\tau) d\tau, \tag{8}$$

where $s(t)$ is a non-stationary seismogram, $w(t,\tau)$ is a time-varying seismic wavelet, $r(t)$ is a sequence of reflection coefficients and $*$ represents convolution.

Eq. (8) can be converted into T-F domain by using SSMST to extract the T-F spectrum. The specific equation is as follows:

$$x_{SSMST}(t,f) \approx w(f)\alpha_Q(t,f)r_{SSMST}(t,f) = w_\alpha(t,f)r_{SSMST}(t,f), \qquad (9)$$

where $\alpha_Q(u,f)$ is the attenuation factor describing the seismic wavelet attenuation and it can be expressed as:

$$\alpha_Q(u,f) = \exp(-\frac{\pi ft}{Q}). \qquad (10)$$

In the case where only the amplitude spectrum is considered, eq. (9) can be further expressed as:

$$|x_{SSGST}(t,f)| \approx |w(f)||\alpha(t,f)||r_{SSGST}(t,f)| = |w_\alpha(t,f)||r_{SSGST}(t,f)|, \qquad (11)$$

where $x_{SSGST}(t,f)$ is the SSMST of non-stationary seismogram, $w(f)$ is the FT of the source wavelet, $r_{SSGST}(t,f)$ is the SSMST of reflection coefficients, $w_\alpha(t,f)$ is the dynamic wavelet spectrum, $|\bullet|$ represents the modulating operator.

It can be seen from eqs. (9) and (11) that the T-F spectrum of the non-stationary seismogram is approximately equal to the product of the spectrum of the dynamic wavelet and the reflection coefficient. When the time *t* is fixed, the amplitude spectrum of the non-stationary seismogram is equal to the product of the dynamic wavelet amplitude spectrum and the reflection coefficient amplitude spectrum. The seismic wavelet amplitude spectrum corresponding to time *t* can be extracted by fitting the seismic wavelet amplitude spectrum from the seismogram amplitude spectrum.

Extraction of amplitude spectra by spectral modeling

After extracting the amplitude spectra of the seismogram, the spectral modeling method is used to fit the seismic wavelet amplitude spectra from the known seismogram amplitude spectra. However, the polynomial order needs to be determined artificial in spectral modeling method. When the polynomial order is not appropriate, the accuracy of the extracted wavelet amplitude spectra will reduce. Aiming at this problem, the evaluation function describing the quality of the estimated amplitude spectra is established by analyzing the influence of the polynomial order on the amplitude spectra of the estimated wavelet. The value of the polynomial order is determined by comparing the value of the evaluation function. In this section, the basic principle of the spectral modeling method and its existing problems are introduced at first. Then, the influence of the polynomial order on the amplitude spectra of the estimated wavelet is analyzed and evaluation function is established. At last, the polynomial order is determined.

Spectral modeling method

Through many observations, Rosa considered that the seismic wavelet amplitude spectrum is a unimodal smooth curve and Ricker-like. The model of the wavelet amplitude spectrum is established according to these characteristics. The mathematical expression of amplitude spectrum of seismic wavelet is as follows:

$$w_\alpha(t,f) = |f|^k \exp(\sum_{n=0}^{N} a_n f^n), \tag{12}$$

where k is a constant, the value of k is from 1 to 3, generally takes 2, N is a polynomial order and the range of its values is from 2 to 7 and a_n is a polynomial coefficient.

Under the assumption that the reflection coefficient sequence is white, the seismogram amplitude spectra is approximately equal to the seismic wavelet amplitude spectra. The corresponding mathematical expression is as follows:

$$\left|x_{SSGST}(t,f)\right| \approx \left|w_\alpha(t,f)\right|. \tag{13}$$

The polynomial coefficients in the corresponding wavelet amplitude spectrum can be calculated by letting the difference of the wavelet amplitude spectrum and the seismogram amplitude spectrum minimized. The mathematical expression of the difference is as follows:

$$Q = \sum_{f=f1}^{f2} [\left|x_{SSGST}(t,f)\right| - \left|w_\alpha(t,f)\right|]^2, \tag{14}$$

where Q is the sum of the squares difference, f_1 and f_2 are the upper and lower limits of effective frequency, respectively.

In the spectral modeling method, the polynomial order N needs to be set artificially. However, there are significant differences in the estimated wavelet amplitude spectrum corresponding to different polynomial values. When the N set is not suitable, the accuracy of estimated seismic wavelet amplitude spectrum will be seriously affected (Li et al., 2013). This is a problem that needs to be solved urgently in the spectral modeling method.

Therefore, to improve the accuracy of estimated seismic wavelet amplitude spectrum, automatically select the appropriate polynomial order should be realized.

Automatic determination of polynomial order

The overall idea of determining the polynomial order automatically is to analyze the influence of the polynomial order on the amplitude spectrum of the estimated wavelet at first. Then, the mean square error of estimated wavelet secondary amplitude spectrum and seismogram secondary amplitude spectrum and correlation coefficient between power spectrum and constant are selected as two criteria to evaluate the quality of the estimated wavelet amplitude spectrum. At last, based on the above two criteria, an evaluation function that can represent the quality of the estimated wavelet amplitude spectra is further established. The evaluation function values corresponding to each order can be calculated and the appropriate polynomial order can be chosen according to the size of the value.

The following equation can be obtained by dividing eq. (12) by $|f|^k$:

$$\frac{|w_\alpha(f)|}{|f|^k} = \exp(\sum_{n=0}^{N} a_n f^n). \tag{15}$$

The following equation can be obtained by taking the logarithm of the two sides of eq. (15):

$$\ln(\frac{|w_\alpha(f)|}{|f|^k}) = \sum_{n=0}^{N} a_n f^n. \tag{16}$$

It can be seen from eq. (16) that the process of seeking the parameters in the spectral modeling method is equal to the process of fitting the curve by using a polynomial function. Sheng et al. (2015) pointed out that when the polynomial order value is small, only the approximate shape of the curve can be fitted and in an under-fitting state. It is difficult to describe the detail part. When the polynomial order value is large, the fitted curve is in over-fitting state.

Therefore, when the polynomial order value in the spectral modeling method is small, the estimated wavelet amplitude spectra is in an under-fitting state and it is difficult to describe the detail part of the wavelet amplitude spectrum. The corresponding estimated wavelet secondary amplitude spectrum is small. When the value of the polynomial order is large, the energy of the reflection coefficient amplitude spectrum is doped into the estimated wavelet amplitude spectrum. The secondary amplitude spectrum of estimated seismogram is larger. Considering the characteristics that the secondary amplitude spectrum of the seismogram and seismic wavelet is approximately equal in low-frequency part, the mean square error of estimated wavelet secondary amplitude spectrum and seismogram secondary amplitude spectrum is selected as a criterion to evaluate the quality of the estimated wavelet amplitude spectra. The specific mathematical expression is as follows:

$$MSE_K = \sum_{f=f1}^{f2} [|x_{SSGST}(t,f)|^{(2)} - |w_{\alpha K}(t,f)|^{(2)}]^2, \tag{17}$$

where MSE_K is the mean square error of the estimated wavelet secondary amplitude spectrum and the secondary spectrum of the seismogram corresponding to $N = K$, $\left|x_{SSGST}(t,f)\right|^{(2)}$ is the secondary amplitude spectrum of the seismogram, $\left|w_{\alpha K}(t,f)\right|^{(2)}$ is the secondary amplitude spectrum of estimated wavelet corresponding to $N = K$.

Gao et al. (2016) pointed out that the spectral modeling method assumes that reflection coefficient sequence is white. The power spectra of the reflection coefficient should be roughly constant and strongly correlated with the constant. However, when the value of the polynomial order is not suitable, the amplitude of the power spectra of the estimated reflection coefficient will change significantly, which will decrease the correlation with the constant. Therefore, the correlation coefficient between the power spectrum of reflection coefficient and the constant can be selected as a criterion for evaluating the quality of the estimated wavelet amplitude spectrum. The specific mathematical expression is as follows:

$$\gamma[P_{rK},1] = \frac{n\sum_{i=1}^{n} P_{rK_i}}{\sqrt{n\sum_{n=1}^{n} P_{rK_i}^{\ 2}}}, \tag{18}$$

where $\gamma[P_{rK},1]$ the correlation coefficient of the reflection coefficient power spectrum and the constant 1 P_{rK} is expressed as the power spectrum of reflection coefficient corresponding to N = K, and n is the length of the power spectrum.

Based on the two criteria, an evaluation function that can represent the quality of the estimated amplitude spectrum of seismic wavelet is established. The expression is as follow:

$$F_K = \frac{\lambda}{|\gamma[P_{rK},1]|} + \rho MSE_K, \tag{19}$$

where F_K is the value of the evaluation function corresponding to N = K, the value of $\gamma[P_{rK},1]$ ranges from 0 to 1, which is relatively small, the value of MSE_K depends on the actual situation and is not fixed. In order to reduce the influence of the value range of the two evaluation criteria on the evaluation function value and let the two evaluation criteria occupy the same proportion in the evaluation function, two parameters λ、ρ ranging from 0 to 1 are introduced. The proportion of the two evaluation criteria in the evaluation function can be adjusted by adjusting two parameters.

It can be seen from the above analysis that the smaller the value of evaluation function, the better the quality of the corresponding estimated wavelet amplitude spectrum. Therefore, the evaluation function values corresponding to different polynomial orders are sequentially calculated at first. Then, the estimated wavelet amplitude spectrum corresponding to the minimum value of the evaluation function is selected and the estimated wavelet amplitude spectrum is the optimal wavelet amplitude spectra.

The specific steps to determine the order of the polynomial are:

(1) Set the value range of the polynomial order, $2 < N < 8$.
(2) Select different N values to fit the seismogram wavelet amplitude spectra.
(3) Calculate the evaluation function of estimated amplitude spectrum corresponding to different polynomial order.
(4) The estimated wavelet amplitude spectrum with the smallest evaluation function value is selected as the optimal extracted wavelet amplitude spectra.

At each moment, the wavelet spectrum is estimated by the spectral modeling method, which realizes the extraction of the time-varying wavelet amplitude spectrum.

Extraction of wavelet phase by phase-only filter

To describe the time-varying characteristics of the wavelet phase, the non-stationary seismogram is segmented by the average overlapping segmentation method. The segment length is 400 ms, and the overlap rate is 75%. Each segment of the seismogram is regarded as a stationary seismogram, and the wavelet is a time-invariant wavelet.

Wang et al. (2010) proposed that the seismic wavelet can be described by the ARMA model with the parameter:

$$\sum_{i=0}^{p} a_i s(n-i) = \sum_{k=0}^{q} b_k r(n-k), \tag{20}$$

where a_i and b_k both are real numbers, a_i are the parameters autoregressive model (AR) and b_k are the parameters of moving average model, p is the partial order of the wavelet AR model, and q is the order of the wavelet MR model.

In eq. (20), when the zero points in the Z-domain are all located in the unit circle, the minimum phase seismic wavelet is represented. When the zero point in the Z-domain is all on the unit circle or outside the unit circle, maximum phase seismic wavelet is represented. The rest of the situations represent mixed phase seismic wavelet. Therefore, different phase seismic wavelets can be constructed by adjusting the distribution of the zero points. The phase-only filter capable of describing the minimum phase wavelet, the mixed phase wavelet and the maximum phase wavelet can be constructed by the unitizing amplitude spectra of ARMA model.

Eq. (20) is expressed as a frequency domain form and the amplitude spectrum is unitized, which can be expressed as follows:

$$W_p(e^{jw}) = \frac{W(e^{jw})}{|W(e^{jw})|} = \frac{W(w)e^{j\varphi_W(w)}}{W(w)} = e^{j\varphi_W(w)}, \tag{21}$$

where $W_P(\mathrm{e}^{jw})$ is the unitized amplitude spectrum and phase-only filter, $W(e^{jw})$ is the spectrum, $\varphi_W(w)$ is the phase spectrum of $W(e^{jw})$.

The non-stationary seismogram is corrected by using the constructed phase-only filter, which is specifically as follows:

$$s_0(t) = F^{-1}[\frac{r(e^{jw})*w(w)e^{j\varphi_w(w)}}{W_p(e^{jw})}] = F^{-1}[r(e^{jw})*w(w)e^{j[\varphi_w w-\varphi_W w]}], \tag{22}$$

where $s_0(t)$ is the time domain representation of the phase correction result, F^{-1} is an inverse FT operator, $r(e^{jw})$ is the spectrum of the reflection coefficient sequence, $w(w)$ is the amplitude spectrum of the seismic wavelet and $\varphi_w(w)$ is the phase spectrum of the seismic wavelet.

When the constructed phase-only filter is equal to the wavelet phase, the corresponding seismic wavelet is corrected to zero phase. Considering that skewness has a large dynamic range, the zero-phase wavelet is determined by the skewness criterion. The equation for calculating the skewness is as follows:

$$\psi(s(t)) = \frac{\frac{1}{N}\sum_{i=1}^{N} s_0{}^3(t)}{\left[\frac{1}{N}\sum_{i=1}^{N} s_0{}^2(t)\right]^{3/2}}, \tag{23}$$

where N is the length of the phase correction result.

When the wavelet is corrected to zero phase, the maximum value will be obtained. In this paper, the improved particle swarm optimization algorithm is used to optimize the parameters. The process of extracting the wavelet phase spectra is as follows:

(1) The information amount criterion is used to determine the order p and order q of the ARMA model;

(2) ARMA model parameters is used to construct phase-only filter and phase correction result can be obtained;

(3) Calculate the value of the skewness according to eq. (23). If it is greater than the calculated value of the historical parameter, set the parameter of the filter at this time as the best historical parameter;

(4) Determine whether the current parameter value satisfies the termination condition. If yes, turn to (5); if not, modify the particle group according to the particle swarm optimization rule to generate a new ARMA model parameter and go to (2);

(5) Read the historical optimal parameters and complete the wavelet phase extraction.

After extracting the wavelet phase from each segment, considering the time-varying characteristics of the phase, it is extended to each moment by polynomial fitting method to realize the accurate extraction of the time-varying wavelet phase.

ANALYSIS OF SIMULATION RESULTS

To verify the validity of the method proposed in this paper, the ARMA model is used to generate seismic wavelets. Reflection coefficient sequence with identical and Bernoulli-Gaussian distribution was generated. The length of reflection coefficient sequence is 1000 ms and the sampling interval is 1 ms. A non-stationary seismogram can be obtained by convolve seismic wavelet and reflection coefficient sequence, as shown in Fig. 4 and Fig. 5. The time-varying wavelets amplitude spectra are extracted by spectral modeling method and method proposed in this paper. Then, the estimated seismic wavelet amplitude spectra were compared with the theoretical wavelet amplitude spectra to analyze the effectiveness of the method.

The initial wavelet used in the experiment is the differential form described by ARMA:

$$w(z) = \frac{1 - 1.6^{z-1} + 0.8^{z-2}}{1 - 0.8^{z-1} - 1.2^{z-2}} . \tag{24}$$

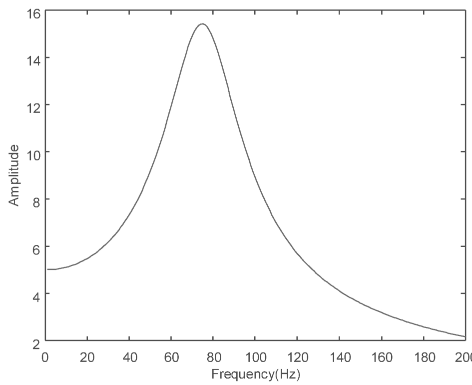

Fig. 4. The amplitude spectra of the seismogram wavelet.

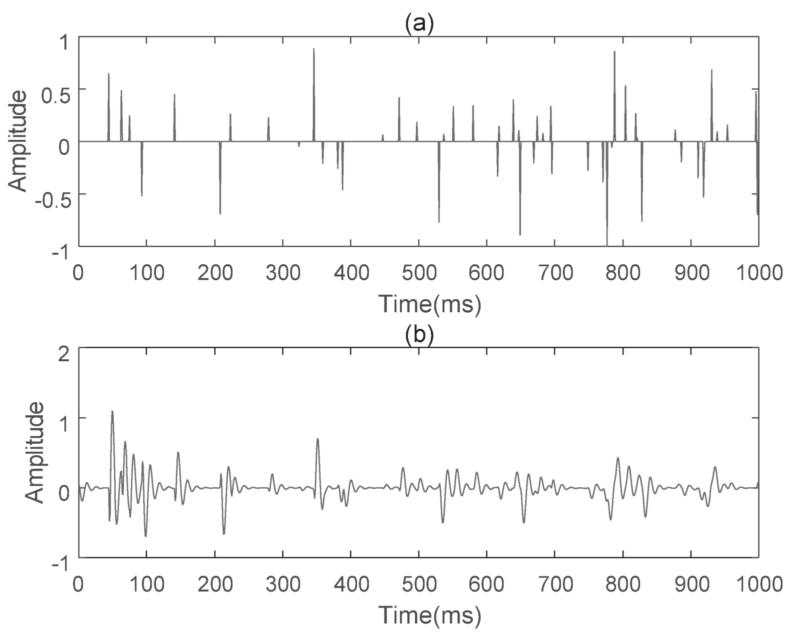

Fig. 5. (a) Reflection coefficient sequence; (b) Non-stationary seismogram.

T-F analysis of seismogram using SSMST

The MST and the SSMST are used to extract the T-F spectra of the non-stationary seismogram. The result is shown in Fig 6. It can be seen from the Fig 6 that there are false frequencies in the amplitude spectrum obtained by the MST. The resolution of T-F amplitude spectrum is improved by using SSMST to extract.

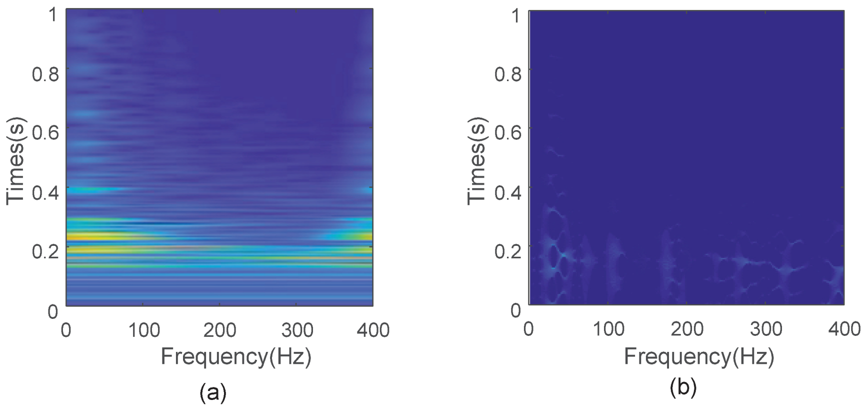

Fig. 6. Comparison of T-F amplitude spectra of seismogram. (a) MST; (b) SSMST.

Extraction of wavelet amplitude spectra

After using the T-F analysis method to obtain the amplitude spectra corresponding to the seismogram at each moment, the seismic wavelet amplitude spectra need to be fitted. First, select different polynomial orders and fit a cluster of estimated wavelet amplitude spectra, as shown in Fig 7.

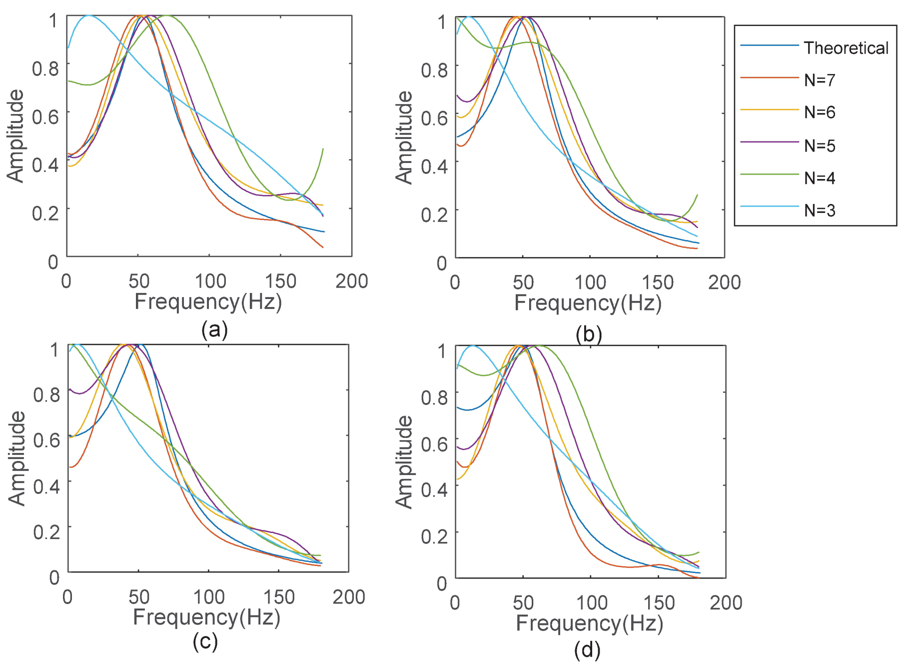

Fig 7. Comparison of estimated wavelet amplitude spectra with different polynomial orders. (a) t = 191ms; (b) t = 400 ms; (c) t = 602 ms; (d) t = 800 ms.

Fig. 7 shows the comparison of the estimated wavelet amplitude spectra with different polynomial orders and the theoretical wavelet amplitude spectra. In Fig. 6 the dark blue solid line corresponds to the theoretical wavelet amplitude spectra, and the red solid line, the orange solid line, the purple solid line, the green solid line, and the light blue solid line corresponds to polynomial order N = 7, N = 6, N = 5, N = 4, N = 3, respectively. Figs. 7a, 7b, 7c and 7d correspond to time 191ms, 400 ms, 602ms and 800 ms, respectively. It can be seen from Fig. 7 that at each moment, the estimated wavelet amplitude spectra corresponding to different polynomial orders have significant differences. How to select the optimal estimated wavelet amplitude spectra from the estimated wavelet amplitude spectra corresponding to different polynomial orders is a problem that needs to be solved now.

After fitting the estimated wavelet amplitude spectra corresponding to different polynomial orders, the evaluation function values of the estimated wavelet amplitude spectra corresponding to different polynomial orders are calculated, as shown in Table 1. As can be seen from the Table 1, when the polynomial orders are equal to 7, the corresponding evaluation function value is the smallest both all the time. The corresponding estimated wavelet amplitude spectra is the optimal estimated wavelet amplitude spectra when polynomial orders are equal to 7.

At the same time, the mean square error of the theoretical wavelet amplitude spectra and the estimated wavelet amplitude spectra is calculated. The results are shown in Table 2. It can be seen from the Table 2 that when polynomial orders are equal to 7, the mean square error of the theoretical wavelet amplitude spectra and the estimated wavelet amplitude spectra is the smallest. Therefore, the purpose that optimal wavelet amplitude spectra can be selected automatically can be realized. Thus, the effectiveness of the method is verified.

Table 1. The value of evaluation function of estimation wavelet amplitude spectra.

Time	polynomial order				
(ms)	3	4	5	6	7
191	6.9	5.5	4.6	2.4	2.2
400	7.5	6.0	4.3	3.3	3.0
602	7.3	5.6	4.9	2.3	2.4
800	7.1	6.4	5.1	3.5	2.9

Table 2. The mean square error of theoretical wavelet amplitude spectra and estimated wavelet amplitude spectra.

Time (ms)	polynomial order				
	3	4	5	6	7
191	12.0	11.3	2.3	1.8	0.5
400	8.1	8.9	2.9	2.8	0.7
602	7.5	7.4	8.5	5.0	2.8
800	17.0	32.3	17.2	13.1	6.9

Comparison of two estimated wavelet amplitude spectra

In order to verify the effectiveness of the SSMST, the spectral modeling method and method proposed in this paper are used to extract the seismic wavelet amplitude spectra, respectively. The results are shown in Fig. 8. In Fig 8, the solid blue line is the theoretical wavelet amplitude spectra, the red solid line is the estimated amplitude spectra extracted by spectral modeling and the yellow solid line is the estimated amplitude spectra extracted by method proposed in this paper.

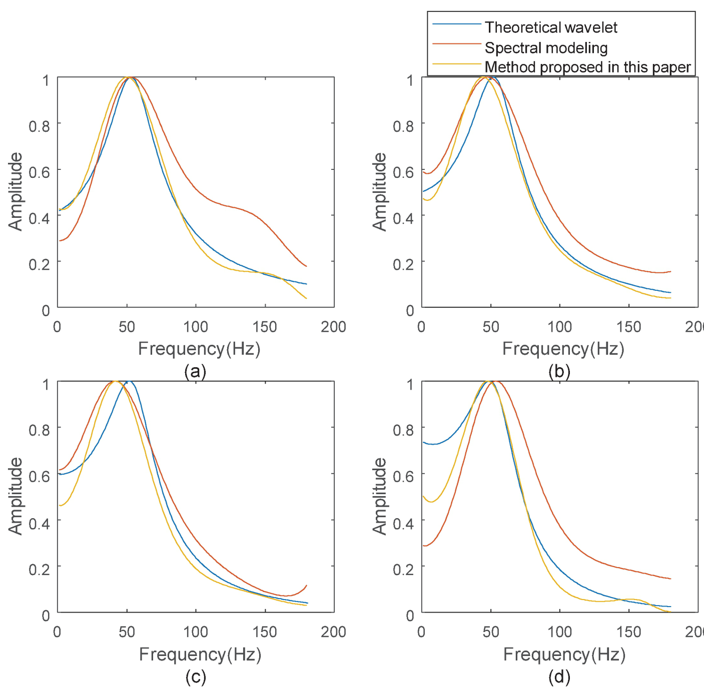

Fig. 8. Comparison of wavelet amplitude spectra estimated by two methods.
(a) t =191ms; (b) t = 400ms; (c) t=602ms; (d) t=800 ms.

Table 3. The mean square error of theoretical wavelet amplitude spectra and estimated wavelet amplitude spectra.

Time (ms)	191	400	602	800
Spectral modeling	4.7	1.6	5.7	7.8
Method proposed in this paper	0.5	0.7	2.8	6.9

It can be seen from Fig. 8 that estimated seismic wavelet amplitude spectra extracted by spectral modeling have low accuracy in the high-frequency because of the influence of the false frequency. At the same time, the accuracy of the estimated seismic wavelet amplitude spectra extracted by method proposed in this paper is relatively high in Fig. 8a and Fig. 8b. The accuracy of the estimated seismic wavelet amplitude spectra extracted by method proposed in this paper is relatively low in Fig. 8c and Fig. 8d. The reason is that both methods are based on the assumption that the amplitude spectra of the seismic wavelet is similar to the amplitude spectra of the Ricker wavelet. As the propagation progresses, the difference between the amplitude spectra of the wavelet and the amplitude spectra of the Ricker wavelet increases, which leads to a low accuracy.

To quantify the simulation experiment results, the mean square error of the extracted wavelet amplitude spectra and the theoretical wavelet amplitude spectra is calculated in turn. The results are shown in Table 3. As can be seen from the Table, the mean square error of the extracted wavelet amplitude spectra based on method proposed in this paper and the theoretical wavelet amplitude spectra is smaller, and the corresponding wavelet amplitude is more accurate.

Deconvolution result in frequency domain

Aiming at further verify the validity of the method, the deconvolution operator is used to deconvolute the non-stationary seismogram. The frequency domain deconvolution results corresponding to the spectral modeling and method proposed in this paper are obtained. The result is shown in Fig 9.

In Fig. 9, the blue solid line is the deconvolution results corresponding to the spectral modeling, and the red solid line is the deconvolution results of the method proposed in this paper. It can be seen from Fig. 9 that the deconvolution results corresponding to method proposed in this paper has a significant increase in the frequency amplitude in the high frequency portion and plays a role of high frequency compensation.

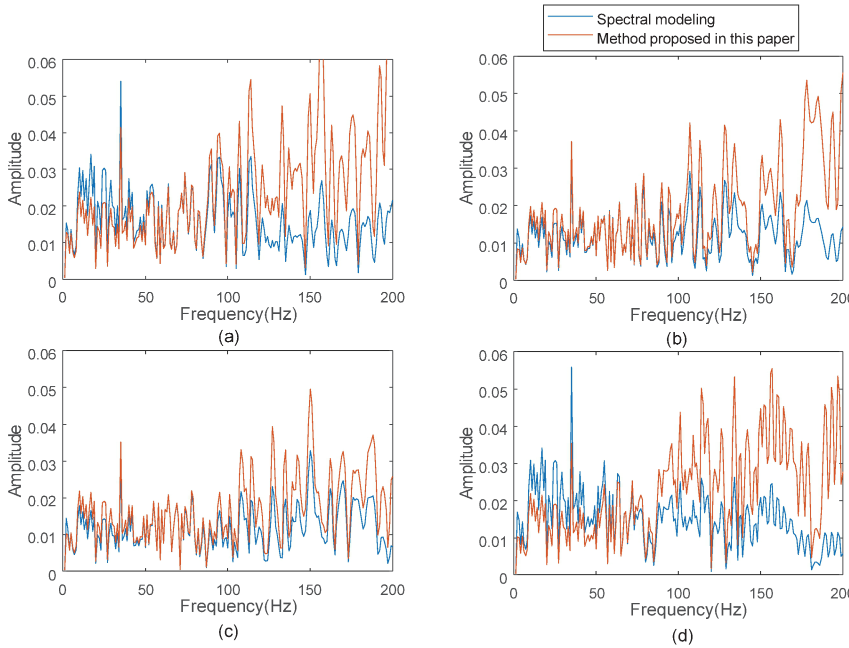

Fig. 9. Comparison of deconvolution results in frequency domain.
(a) t =191 ms; (b) t = 400 ms; (c) t = 602 ms; (d) t = 800 ms.

Comparison of two estimated wavelet

Seismic wavelets in time domain are obtained by combining the amplitude spectra and phase spectra extracted by phase-only filter. The results are shown in Fig. 10. The blue solid line is the theoretical wavelet, the yellow solid line is the seismogram wavelet whose amplitude spectra extracted by spectral modeling and the red solid line is the seismogram wavelet whose amplitude spectra extracted by method proposed in this paper. It can be seen from the Fig. 10 that the time domain wavelet corresponding to the proposed method is closer to the theoretical wavelet and the extraction precision is higher.

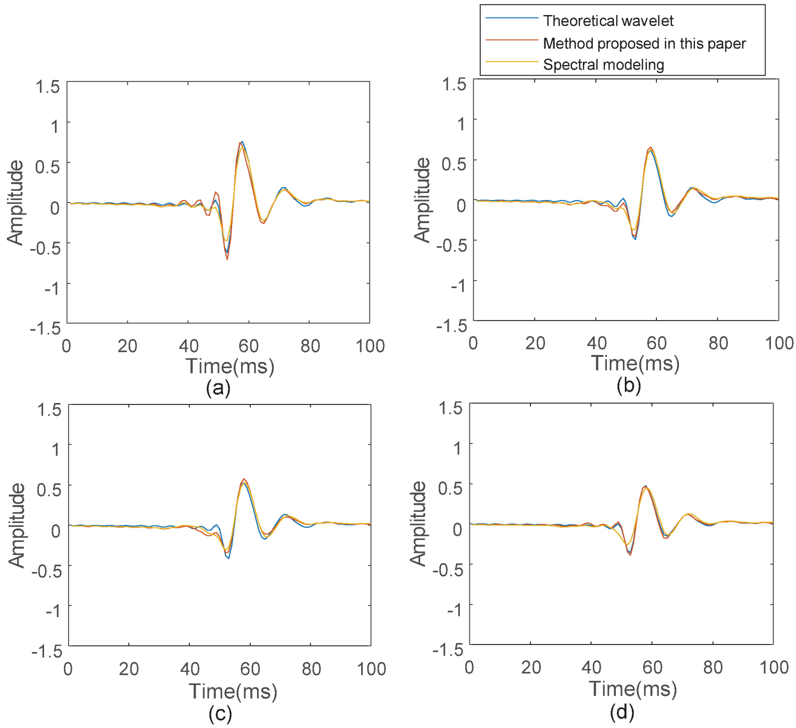

Fig. 10. Comparison of seismogram wavelet estimated by two methods. (a) t = 191 ms; (b) t = 400 ms; (c) t = 602 ms; (d) t = 800 ms.

Deconvolution results

To further verify the validity of the method, the deconvolution operator is used to deconvolute the non-stationary seismogram. The deconvolution results in time domain corresponding to the spectral modeling and method proposed in this paper are obtained, as shown in Fig. 11. Here, Fig. 11a is a reflection coefficient sequence, Fig. 11b is deconvolution results corresponding to spectral modeling, and Fig. 11c is deconvolution results corresponding to method proposed in this paper. It can be seen from Fig. 11 that most of the sharp pulse characteristics of the reflection coefficient sequence corresponding to the two methods are still recovered. But, the deconvolution results corresponding to the Fig. 11b diagram is not ideal recovery in the 800 ms partial, and the error is large. The method proposed in this paper has been relatively improved.

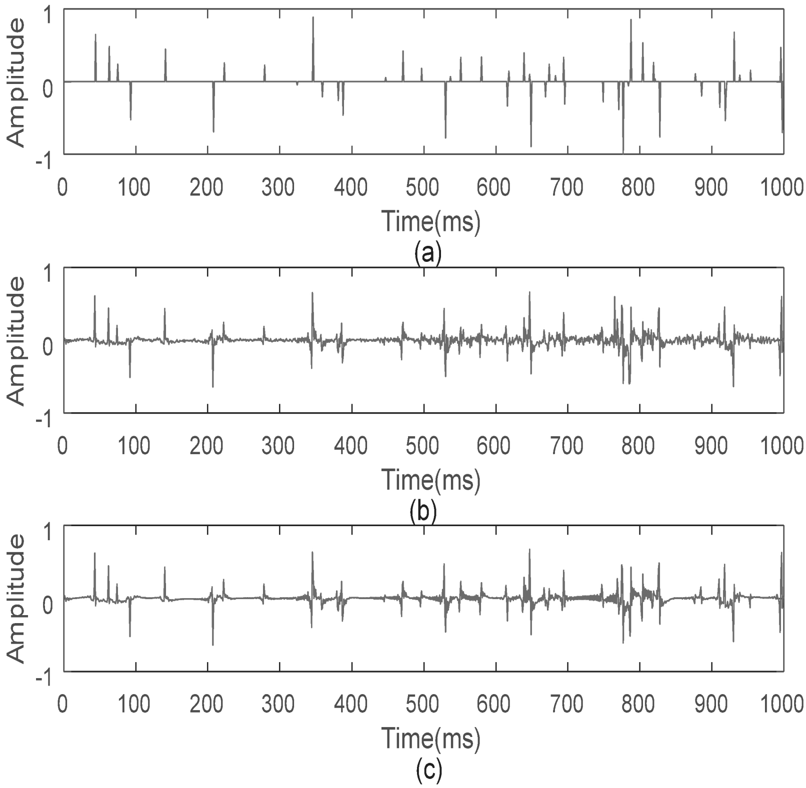

Fig 11. Comparison the result of deconvolution. (a) Reflection Coefficient; (b) Spectral modeling; (c) Method proposed in this paper.

This chapter is for the actual seismogram data and 150th seismogram data is extracted and shown in Fig. 12. The amplitude spectrum is extracted by method proposed in this paper and phase spectrum is extracted by phase-only filter. Seismic wavelet in time domain is obtained by combining amplitude spectrum and phase spectrum. The result is shown in Fig. 13, Fig. 13a, Fig. 13b, and Fig 13c correspond to time 200 ms, 400 ms, and 600ms, respectively. It can be seen from the figure that as the propagation progresses, the time domain wavelet changes significantly with the propagation time.

To further verify the accuracy and effectiveness of the extraction method, multiple deconvolution processing is performed on the actual seismogram. The processing results are shown in Fig. 14. Fig. 14a is the original seismogram profile, Fig. 14b is the result of multichannel deconvolution with spectral modeling and Fig. 14c is the result of multichannel deconvolution with method proposed in this paper. The results show that compared with the traditional spectral modeling method, using proposed method in this paper to deconvolve can improve the vertical resolution of seismic profile, the horizontal continuity and consistency of stratum. Therefore, the method in this paper can obtain higher resolution seismogram, which is of great significance in the subsequent work of full waveform inversion and migration imaging.

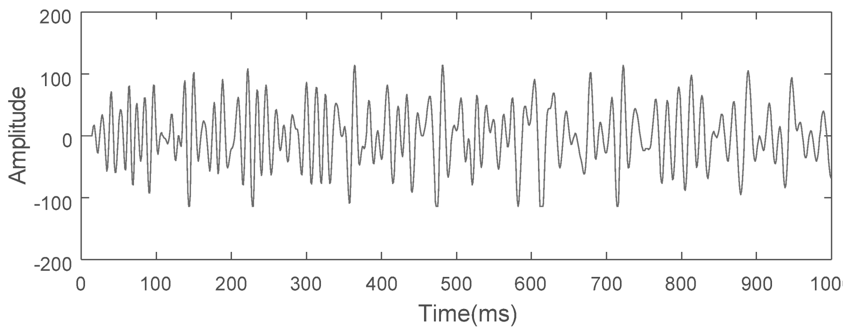

Fig 12. The 150th seismogram.

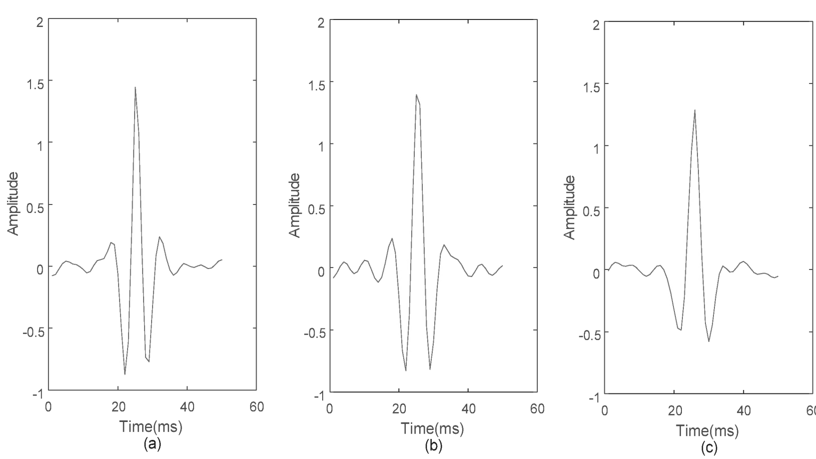

Fig. 13. Estimated wavelet of actual seismogram data. (a) t = 200 ms; (b) t = 400 ms; (c) t = 600 ms.

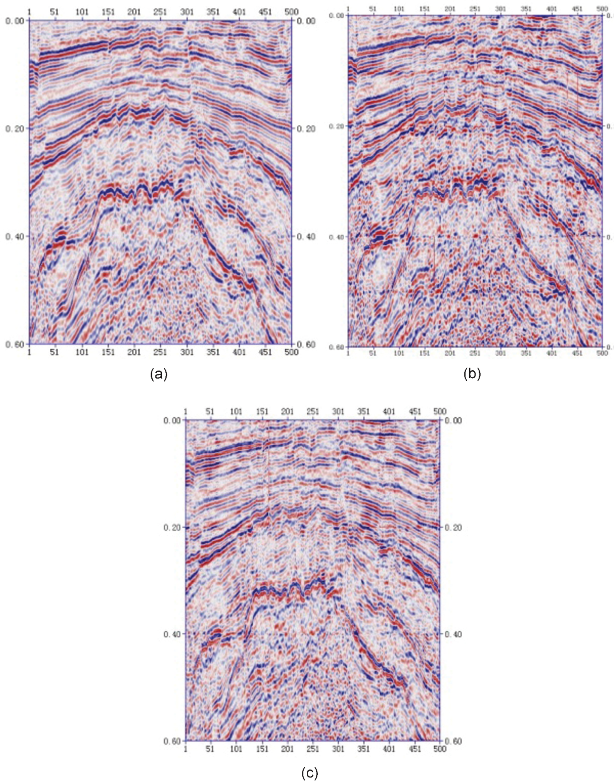

Fig 14. Time-varying wavelet deconvolution results of actual seismogram data. (a)Initial seismogram profile; (b) Result of multichannel deconvolution with spectral modeling; (c) Result of multichannel deconvolution with method proposed in this paper.

CONCLUSION

To solve the problem that there are false frequencies in the T-F spectra of seismogram and polynomial order needs to be set artificially, the reason of generating false frequencies was analyzed and SSMST was proposed to extract the T-F amplitude spectra of seismogram. Based on the understanding of the basic principle of spectral modeling method and the problem of spectral modeling method, a time-varying wavelet estimation method based on modified T-F spectral modeling is proposed. The false frequencies are effectively reduced and the appropriate polynomial order can be determined automatically. Compared with the spectral modeling, the method proposed in this paper has higher accuracy by simulation experiments and actual seismogram data processing. However, the proposed method assumes that the wavelet amplitude spectrum is a smooth unimodal curve similar to the Ricker. When there is a big difference between the wavelet amplitude spectrum and the Ricker, the effect of this method is poor. Therefore, the main work in future research is to study the time-varying wavelet extraction method with a wider range of application.

ACKNOWLEDGMENTS

This research was supported by the National Natural Science Foundation of China (40974072) and the China University of Petroleum (East China) graduate student innovation project fund (YCX2015050). The authors gratefully acknowledge this financial support.

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