THE IMPORTANCE OF FRACTURE TOUGHNESS IN THE ESTIMATION OF SEISMIC ANISOTROPY AND STRESS ORIENTATION IN SHALE FORMATIONS

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ABSTRACT

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Shale resource plays are associated with low permeability, and hence hydraulic fracturing is required for their stimulation and production. The effectiveness of hydraulic fracturing depends on how accurately a horizontal well is placed in the formation of interest. The direction of maximum stress and the magnitude of seismic anisotropy play an important role in the placement of a horizontal well, and effective hydraulic fracture stimulations along its length. Therefore, their estimation from seismic data can provide valuable information. While the source of seismic anisotropy is non-unique, fracture induced anisotropy as well as stress induced anisotropy are considered for an estimation of maximum stress direction and magnitude of seismic anisotropy. In this paper, we first introduce the concept of fracture toughness, which refers to the the ability of a rock to resist fracturing and propagation of pre-existing fractures. We then propose a workflow that uses its azimuthal variation for estimating these two parameters on seismic datasets from the Anadarko Basin. After estimating the maximum stress direction and magnitude of seismic anisotropy the available borehole breakout data as well as microseismic data for area of study are brought into consideration for authenticating the maximum stress direction analysis. The dipole shear logs measured at one well are used to validate the estimation for magnitude of seismic anisotropy.

KEY WORDS: fracture toughness, fracture intensity, anisotropy, stress orientation, shale reservoirs, azimuthal-velocity variation, azimuthal-amplitude variation, maximum-stress, minimum-stress, Woodford.

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INTRODUCTION

The ability of a well to produce hydrocarbons from a tight shale play depends on the natural permeability of the reservoir, as well as induced permeability resulting from hydraulic fracturing. The natural permeability is governed by the existence of natural fractures in such a way that horizontal wells that cross maximum vertical fractures are more productive (Brown et al., 1990; Hall et al., 2002). Usually, horizontal wells are drilled in the direction of minimum horizontal stress, so that hydraulic fracturing takes place in the direction of maximum stress that ensures better reservoir contact and production (Crosby et al., 1998). The scenarios of drilling horizontal wells in a direction other than the preferred one and problems associated with them have been discussed by many authors (Plahn et al., 1995; Weng, 1993; Miller et al., 2011).

After enhancing the natural permeability by placing a horizontal well adequately (Miller et al., 2010), an efficient and effective hydraulic fracture stimulation needs to be considered for generating the induced permeability, which depends on how a complex fracture network is created by induced fractures (Miller et al., 2011; 2013; Gray et al., 2012). The complexity of induced hydraulic fractures is a function of in-situ stresses. In an isotropic in-situ stress field, a complex fracture network can probably occur, increasing the contact area and thus the induced permeability, resulting in higher production (Miller et al., 2011; 2013).

Therefore, for the development of shale reservoirs, it is vital to understand the orientation and intensity of natural fractures along with the stress field distribution in the area. While the direct determination of fractures and stress field distribution from seismic data is not possible, indirect methods are usually used for extraction of such information. These methods are based on the fact that Earth becomes anisotropic in the presence of fractures along with varying in-situ stress field, which can be observed seismically. As the source of seismic anisotropy is non-unique, few assumptions are introduced in order to address such complexity. In this context, the existence of a single set of vertically-aligned fractures in the subsurface is usually considered for deriving the fracture orientation and their intensity from seismic data. Furthermore, an isotropic rock under ambient stress with randomly-oriented and distributed cracks, whose shapes gets changed due to the differential principal stresses, are believed to be the source of stress induced anisotropy (Schoenberg and Sayers,1995) and can be used to extract information on the pattern of induced fractures.

The vertically-aligned parallel fractures or anisotropic in-situ stress field in the subsurface cause a variation of some physical attributes of a seismic wave, namely, amplitude, travel-time and velocity with azimuth that is evident on 5D-interpolated prestack time migrated seismic gathers, when sorted by common-offset and common-azimuth (COCA). Grechka and Tsvankin (1998) showed that azimuthal variation of NMO velocities can be expressed in the form of sinusoids or ellipses (in polar coordinates) whose ellipticity is proportional to fracture density and their major and minor axes delineate the orientation of the fractures. Besides, Ruger and Tsvankin (1997) demonstrated an approach of estimating fracture orientation and intensity by making use of azimuthal variation of amplitude. Thus, the analysis of variation of velocity with azimuth (VVAz) and the variation of amplitude with azimuth (AVAz) using wide-azimuth prestack seismic data can provide a promising way to estimate fracture properties.

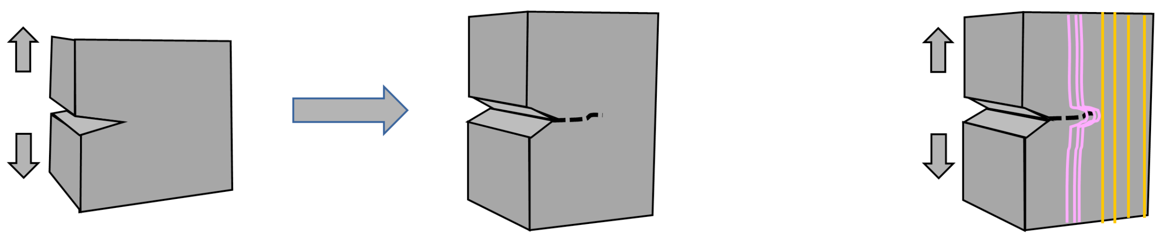
For such an analysis, the general requirements on input seismic data include wide azimuths, high fold, reasonably small bin size, an even distribution of offsets and azimuths, high signal-to-noise ratio and amplitudes preserved during processing. While the VVAz is a layer-based approach, AVAz is interface-based. Consequently, both these approaches exhibit different results for the same properties. Additionally, the poor data resolution offered by VVAz technique, and limitation of AVAz technique in terms of 90-degree ambiguity associated with the extracted fracture orientation make it very challenging to extract reliable estimation of fracture intensity and their orientation (Zheng, 2006).

It is worth mentioning here that VVAz/AVAz approaches mentioned yield meaningful results only if vertical fractures are a source of anisotropy that cause azimuthal variation of velocity and amplitude. However, there might be scenarios where fractures are not present in the formation of interest, which is true in many shale plays, and thus the formations are stimulated (Gray et al., 2010). In such cases the local stress field is believed to be the cause of seismic anisotropy, and the information on stress-induced anisotropy can be extracted by studying the azimuthal response of conventional seismic data. Following linear slip theory (Schoenberg and Sayers, 1995), Gray et al. (2012) simplified Hooke's law (relates stress and strain) and determined the azimuthal response of seismic data in the form of differential horizontal stress ratio (DHSR), which can be used for identifying the parallel or complex patterns of fractures. Mathematically, DHSR is defined as

$$\mathrm{DHSR} = (\sigma_{max} - \sigma_{min}) / \sigma_{max} \quad , \tag{1}$$

where σ_{min}, σ_{max} are minimum and maximum horizontal stresses. Fractures will be parallel if DHSR is high, otherwise, the pattern is expected to be complex. This approach requires long-offset, wide-azimuth seismic data which is often not available. Also, as stated above, the method assumes that the source of anisotropy is in-situ stress field, which is difficult to confirm.

In view of the challenges in the estimation of fracture intensity, stress orientation as well as pattern of induced fractures determined by following the commonly used seismic methods, we propose the concept of fracture toughness (*FT*) for estimating the parameters of interest. For doing so, we turn to the basics of hydraulic fracturing of rocks which entail the initiation of fractures and their propagation as depicted in Fig. 1a. To initiate a fracture, priority should be given to a material which absorbs less energy before it gets fractured. Once the fracture is initiated, the stress state within the rock gets disturbed due to stress concentration at the crack tip as shown in Fig. 1c, where yellow vertical lines and red curved lines represent uniform stress condition and stress concentrated state, respectively. A rock can withstand fracture tip stresses up to a critical value, which is referred to as the critical stress intensity factor. This ability of a rock to resist fracturing and propagation of pre-existing fractures is known as *fracture toughness* (Eaton, 2018). Rocks with low fracture toughness promote fracture propagation.



(a) (b) (c)

Fig. 1. Hydraulic fracturing process comprising (a) crack initiation and (b) propagation. (c) Fracture initiation disturbs the stress state within the rock due to stress concentration at the crack tip (Modified after Rocha-Rangel, 2011).

USING *FT* FOR ESTIMATION OF MAXIMUM STRESS DIRECTION AND HYDRAULIC FRACTURE PATTERNS

It is common knowledge that the orientation and propagation directions of hydraulic fractures are controlled by in-situ stresses. Being tensile in nature, hydraulic fractures open in the direction of minimum stress due to least resistance offered by a formation in this direction (Hoeksema, 2013). Therefore, as per the definition of fracture toughness, it must be minimum in the direction of minimum horizonal stress and maximum in the direction of maximum horizonal stress. This suggests that, the azimuthal variation of fracture toughness should allow us to extract the maximum stress direction.

From the definition of fracture toughness, it is intuitive that a complex fracture network would be formed, if fracture toughness is the same in all directions, i.e., difference between maximum and minimum *FT* is low and induced fractures can propagate in any direction. But, if the difference is large, then fractures are likely to follow a particular direction and will tend to create planar fractures. Based on these arguments we introduce a new attribute, which is named differential horizontal fracture toughness ratio (*DHFTR*) and is defined as

$$DHFTR = (FT_{min} - FT_{max}) \,/\, FT_{max} \quad . \tag{2}$$

where FT_{min}, FT_{max} represent minimum and maximum fracture toughness. Hydraulic fractures will be parallel if *DHFTR* is low, otherwise, the pattern will be complex as shown in Fig. 2.

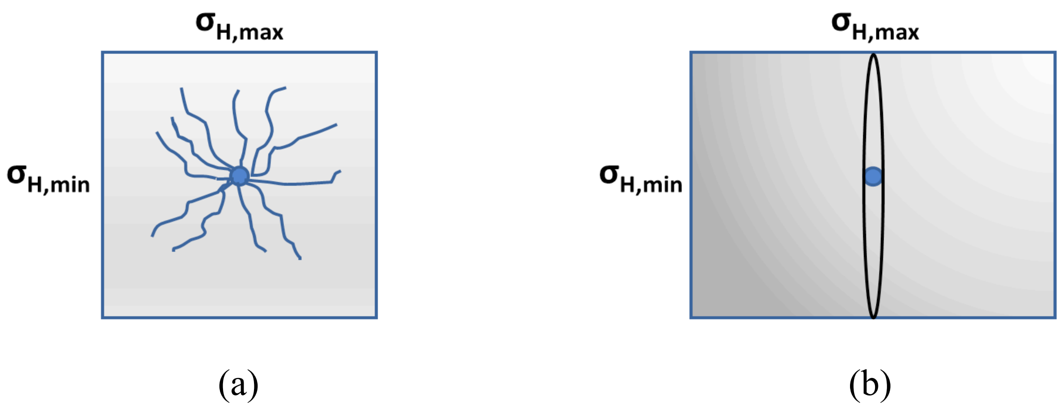


Fig. 2. A hypothetical situation when the formation offers (a) close to isotropic resistance to the fracture propagation, i.e., difference of *FT* (max, min) is low and fractures can propagate in all the direction; (b) different resistance in the different directions, i.e., difference of *FT* is high and fractures can propagate in the preferred direction.

ESTIMATION OF FRACTURE TOUGHNESS

There are both direct and indirect ways in which *FT* can be determined. The direct way is to make measurements on rock samples, which is difficult and more complex than other tests of rock mechanical properties. Therefore, a correlation of *FT* with Young's modulus, Poisson's ratio, tensile strength and compressive strength have been derived from experimental data obtained from different types of rocks (Barry et al., 1992). Sierra et al. (2010) published experimental data showing the relationship between *FT* and tensile strength, compressive strength, Young's modulus and Poisson's ratio for Woodford shale.

As rocks resist the propagation of preexisting cracks, a minimum pressure is required to overcome this resistance and make a fracture grow. Thus, the minimum pressure required to grow a fracture can be correlated with *FT* as the higher the *FT,* the higher the required minimum pressure will be. If somehow this pressure is estimated, it can be used as a proxy for *FT.* So, there are two different ways *FT* could be estimated. One way is to use its relationship with P- wave velocity and Young's modulus as published, and then take their optimal combination (RMS average, arithmetic mean, etc.). The second way is to estimate the minimum pressure required for fracture propagation. Based on the theory proposed by Griffith (1920,1924) to explain the rupture of brittle, elastic material, Sack (1946) derived an equation to predict the minimum pressure (critical) necessary to extend a fracture in a rock for hydraulic fracking considering the penny shaped cracks as

$$P_c = [\ \pi\alpha E / 2(1\ v^2)\ C \quad , \tag{3}$$

where α is the specific surface energy of the rock, E is Young's modulus, v is Poisson's ratio and C is crack length which can be eliminated using its relationship with volume of the crack (V) as (Sneddon,1946)

$$V = [16(1\ v^2)\ C^3 P_c] / 3E \quad . \tag{4}$$

The above equations can be combined to yield the critical pressure per unit volume of fracture and allow us to compute it using well-log data or seismic data. The estimation of *FT* via the above equations has been validated by authors using mud-log data as well as core data, where it was concluded that tight carbonate formations provide high resistance to fracture propagation and can be treated as fracture barriers. Additionally, it was illustrated that shales with high volume of clay offer minimum resistance to fracture propagation but absorb excessive energy before fracture get initiated and hence must be avoided for fracturing. However, a small amount of carbonate present in a shale formation makes it favorable for fracturing. The details of this analysis can be found in the literature (Sharma et al., 2019).

ESTIMATION OF STRESS ORIENTATION AND MAGNITUDE OF SEISMIC ANISOTROPY

Proposed workflow

In order to extract the maximum stress direction and magnitude of seismic anisotropy using the azimuthal variation of fracture toughness, a workflow is proposed as shown in Fig. 3. As per this workflow, pre-stack seismic inversion is carried out first on preconditioned azimuth-sectored gathers by following adequate simultaneous inversion parametrization. Knowledge of initial low frequency models, angle dependent wavelets and inversion parameters are key elements of simultaneous inversion, and their azimuth consistency is paramount in executing simultaneous inversion of individual azimuths. Once impedance inversion is completed, the fracture toughness volumes for individual azimuths are computed using the P-impedance and S-impedance volumes. The estimation of fracture toughness from pre-stack seismic data has been illustrated previously (Sharma et al., 2019) and thus we focus here is on its azimuthal variation. As fracture toughness of a material would be low and high in the direction of minimum and maximum stress, respectively, the maxima and minima of fracture toughness along with their azimuth are computed first. Thereafter, the azimuth corresponding to the maximum fracture toughness is selected as the direction of maximum horizontal stress. Finally, the difference between minimum and maximum fracture toughness is expected to provide the magnitude of stress-induced anisotropy and can be used in estimating the pattern of hydraulic fracturing.

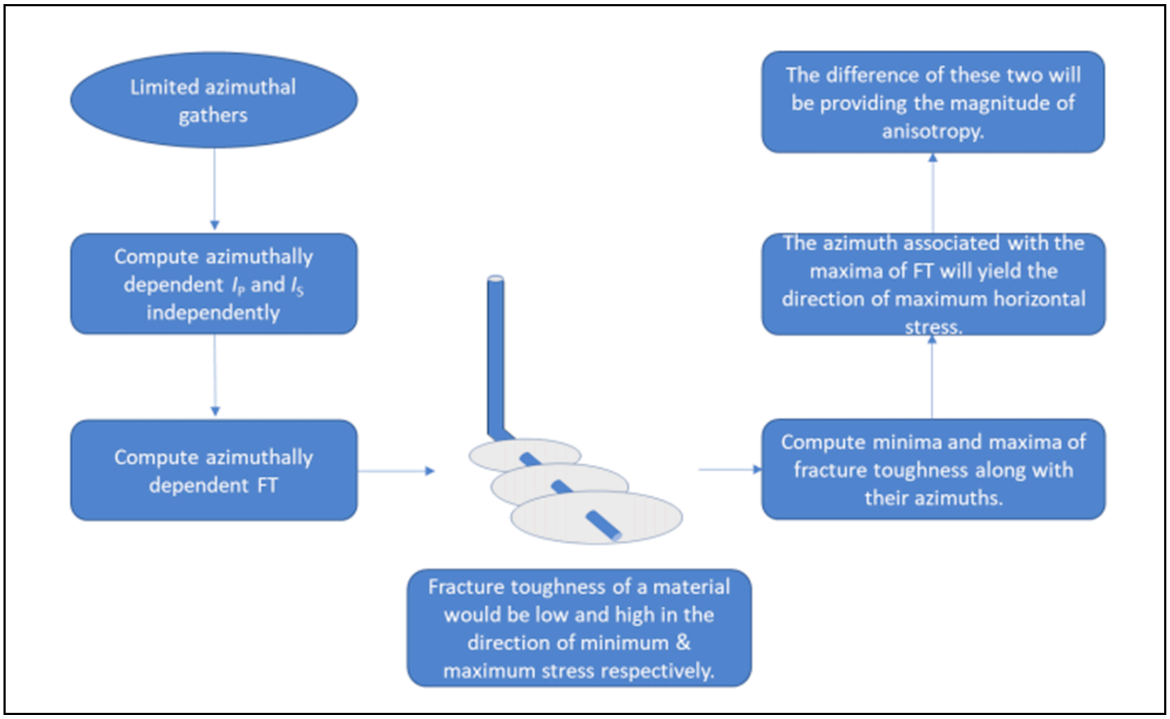


Fig. 3. Workflow for estimating maximum horizon stress direction as well as magnitude of stress-induced anisotropy using fracture toughness.

Application of FT

For application of the *FT* workflow, a dataset from the Anadarko Basin was selected. The Anadarko Basin in the continental US is a prolific oil- and gas-producing province. The stratigraphy of the broad zone of interest and the correlation of well curves with seismic is shown in Fig. 4. The different litho-units can be read off the formation tops located on the seismic section. The Meramec and Woodford shale are the formations of interest here. To execute the workflow mentioned in Fig.3, model-based simultaneous inversion was performed on the individual azimuth-sectored gathers by considering the same low-frequency models and angle dependent wavelets to make sure that azimuthal variation is preserved. Proper data conditioning was followed for individual azimuth-sectored gathers before putting them through simultaneous inversion. The main objective of data conditioning is to enhance the signal-to-noise ratio while the AVO response is preserved by following a workflow that starts with the stacking of azimuthal sectored (6 sectors of 30^o each) pre-stack data which yields the prestack migrated gathers. These are then put through a series of steps for signal-to-noise enhancement, comprising bandpass filtering, generating super-gathers, applying random noise attenuation and trim statics (Singleton, 2009; Yu et al., 2017). Such an inversion yields P- and S-impedance volumes for all the azimuths. Thereafter, fracture toughness volumes are also determined for different azimuths using impedances volume. Having computed these volumes, their maxima and minima corresponding to different azimuths are computed.

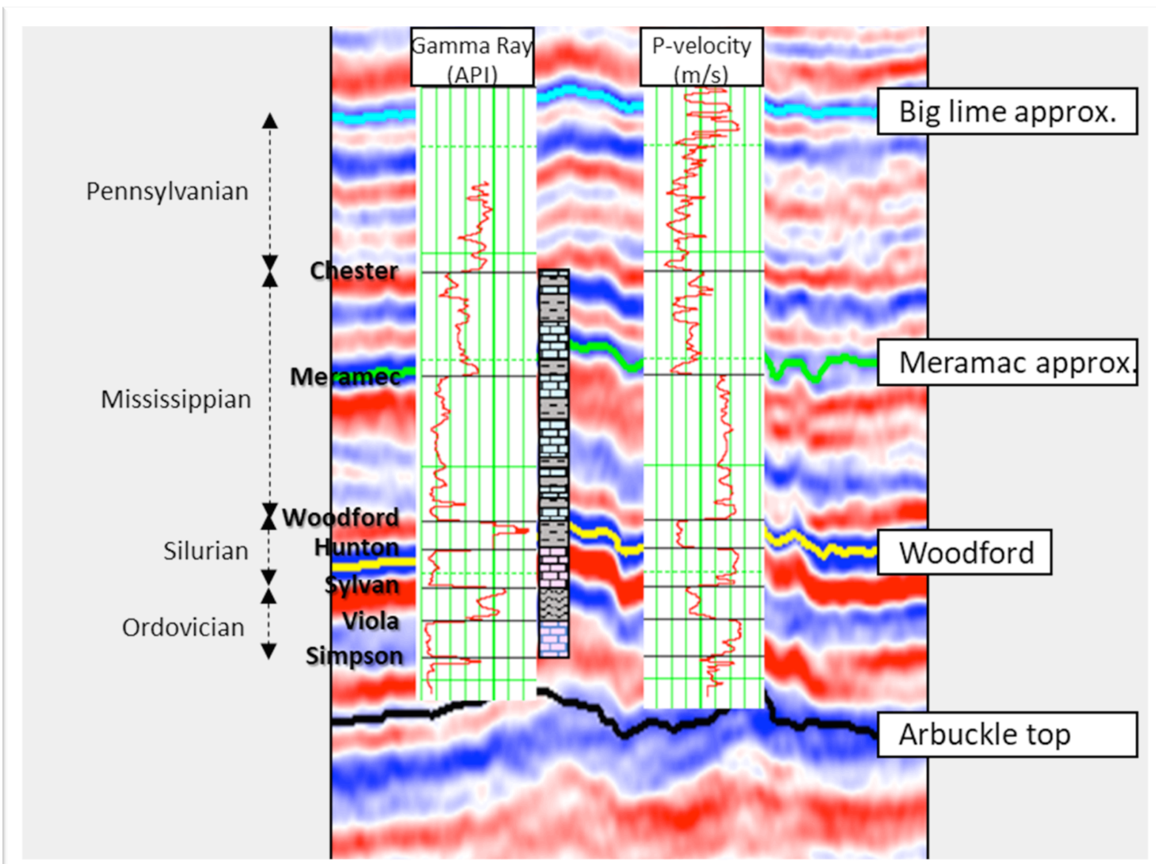


Fig. 4. Correlation of well log cures with seismic from the Anadarko Basin. The main stratigraphic units in the zone of interest are shown as indicated, where the litho-units can be read off from the marked formation tops.

As fracture toughness would be maximum in the direction of maximum horizontal stress direction, the azimuth associated with the maximum *FT* value at each time sample is computed and exported as a 3D volume which is taken as the orientation of maximum stress. Similarly, the difference between minimum and maximum fracture toughness is estimated using eq. (2) which is considered as magnitude of seismic anisotropy.

For the purpose of comparison, VVAz and AVAz analysis were also performed on the dataset used here. Common-offset and common-azimuth (COCA) gathers were first generated for adding the stability in the data and bringing in more foldage into the analysis. Thereafter, a bandpass filter was applied on the COCA gathers for removing any high frequency noise and enhancing the signal. Being satisfied with the filtering results, residual azimuthal travel-time shifts needed to be determined using dynamic trim statics, which were the shifts required to align all azimuths on the individual reflection events. For doing so, time-variant time shifts were computed at the center of time windows defined by the available horizons using the stack power optimization. Therefore, the time shifts at each horizon are the key input for VVAz analysis. Usually, the maximum allowable shift is chosen in such a way that possible cycle skipping is limited to very large incident angles (offsets). Determination of time shifts at the center of different horizons may have the risk of introducing artifacts. In order to avoid such artifacts and attempt to glean more accurate azimuthal information from the COCA gathers, other horizons were introduced into the zone of interest. Even though the horizons were not trackable, some were generated with time shifts of the picked markers, and others generated as proportional slices. All these picked horizons were then used in gather flattening process, which improve the analysis. Having gained the confidence in the results of the gather flattening process, the determined time shifts mentioned above are then inverted for RMS anisotropic attributes ($V_{\text{NMO-fast}}$, $V_{\text{NMO-slow}}$, and ϕ_{fast}) for each CMP. Next, intermediate RMS attributes are converted to $V_{\text{INT-fast}}$, $V_{\text{INT-slow}}$ and orientation using the Generalized Dix equation (Grechka et al., 1999). Subsequently, flattened gathers were used in the curve fitting analysis based on the Rüger equation (Ruger and Tsvankin, 1997) which provides the estimation of fracture intensity and orientation.

COMPARISON AND CALIBRATION

After following the above approaches, three different estimations for the magnitude of seismic anisotropy and maximum stress direction were available for comparison. In order to calibrate the magnitude of seismic anisotropy, shear wave slowness data were available at one well. It is well known that shear wave gets split into two polarized shear waves once it enters an anisotropic medium. For fractured media one of these shear waves is usually aligned with the strike direction and becomes fast shear wave while other gets aligned perpendicular to strike direction and taken as slow shear-wave. The time differences between fast and slow shear-waves provide information about the intensity of seismic anisotropy.

Fig. 5 shows the measured shear-wave slowness data in terms of fast- and slow-shear curves on the first track and their difference is displayed on the second track which is taken as intensity of anisotropy. Figs. 6a to c show an in-line section passing through the well from the anisotropic intensity volumes extracted using VVAz, AVAz and proposed *FT* approach, respectively. In order to see how well the estimated anisotropic intensity matches with the measured intensity of shear-wave slowness data, the curve shown in Fig. 5 is overlaid on the displays. The low intensity values of seismic anisotropy are displayed in white and gray colours, while the orange colour represents the high intensity. Notice, a reasonable match between the intensity of seismic anisotropy estimated using proposed *FT* approach and that computed from measured shear-wave slowness data within the interval bounded by Meramec and Woodford horizons. Such a strong correlation lends the confidence in the proposed workflow.

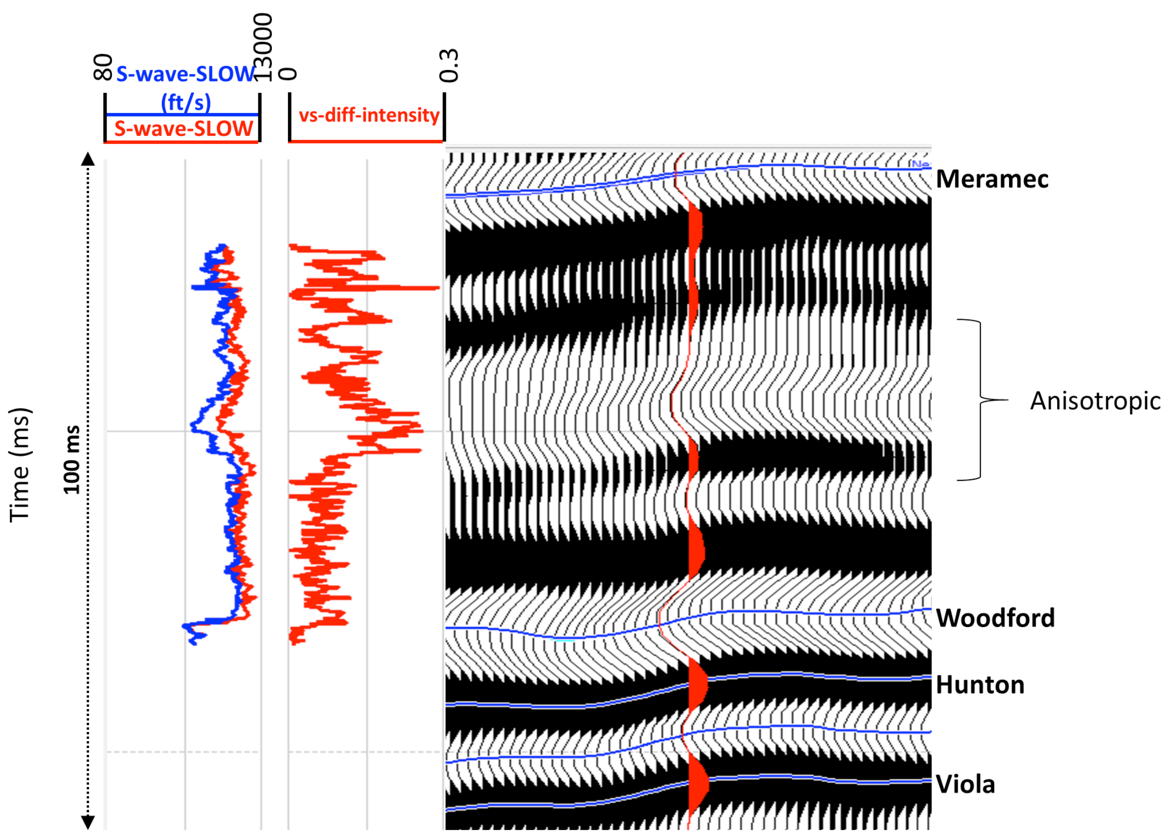


Fig. 5. The fast and slow shear wave slowness data available in one well over area of interest are shown in the first track, their difference is displayed in the second track. The middle portion of Meramec to Woodford interval seems to be anisotropic.

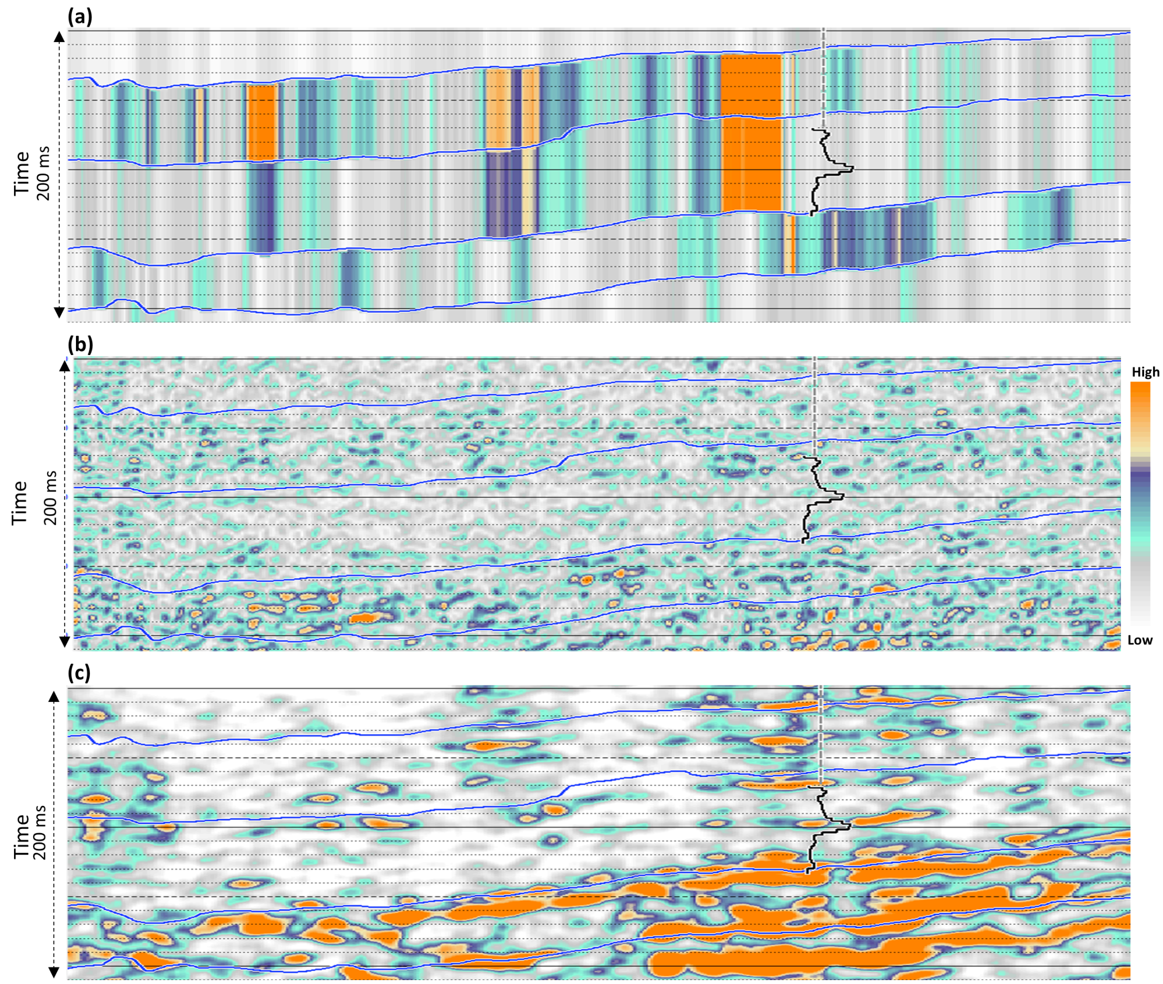


Fig. 6. An in-line section passing through the well from the anisotropic intensity volume extracted using (a) VVAz, (b) AVAz and (c) proposed *FT* approach. The measured intensity curve is overlaid. Notice a reasonable match between the measured and the estimated intensity of anisotropy when proposed workflow is followed [Fig. (c)].

Being encouraged with the intensity result, maximum stress orientation estimations using different approaches are compared next. A comparison is made between the fracture orientations determined from *FT* approach and AVAz approach, as their resolution is expected to be the same as of input seismic data. Due to the low resolution of the VVAz attributes, they were not included in this comparison. The vector display of anisotropy intensity (the length of needles) and maximum stress orientation (direction of needle) at 5 ms below the Woodford horizon is shown in Fig. 7a when the AVAz approach is followed. Equivalent displays of anisotropy and maximum stress orientation extracted from the azimuthal variation of *FT* are shown in Fig. 7b. For calibration purpose, borehole breakout and micro-seismic data over study area was available at the level of Woodford Shale (Zhang, 2016; Alt and Zoback, 2015) to verify the direction of maximum stress. Notice, while the maximum stress orientation extracted from the

AVAz approach does not show any stable orientation pattern, the general trend seen on the orientation display extracted from the azimuthal variation of *FT* is consistent. The exhibited trends seem to be E-W that match with micro-seismic data observed at different stages along the length of the borehole. Consequently, the minimum stress orientation is N-S, which matches very well with the borehole breakout dataset display in the rose diagram. Such a resemblance again between the orientation extracted from proposed approach and the one extracted from direct measurement lends the confidence in the analysis.

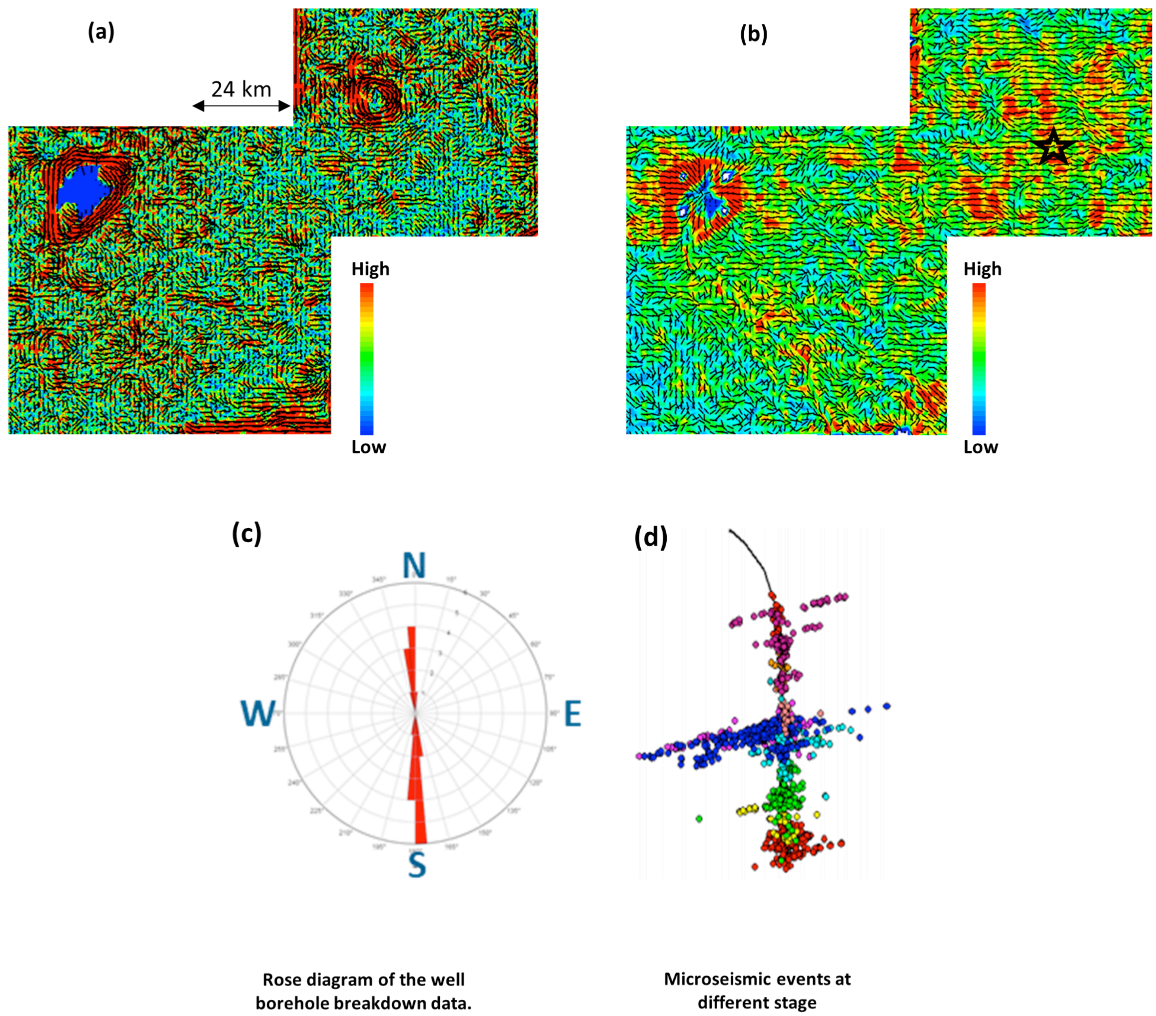


Fig. 7. (a) Intensity of anisotropy with overlay of orientation in the form of needles at 5ms below the Woodford horizon obtained from (a) AVAz analysis, (b) proposed FT approach. The consistent trend of minimum stress orientation (E-W) extracted using proposed *FT* approach seems to correlate well with the (c) borehole breakout dataset [location indicated with a star symbol in (b)] available for area of study (Zhang, 2016) shown in the rose diagram. Also, the strike direction of microseismic events indicates (d) E-W as the expected maximum direction. Such a similarity lends a confidence in the extraction of maximum stress orientation using proposed approach.

CONCLUSIONS

Considering the importance of maximum stress orientation and seismic anisotropy in placing a horizontal well in shale formation, a new workflow for estimation of these properties has been proposed, which entails using the azimuthal variation of fracture toughness. The application of the proposed workflow for determining these two properties for a dataset from Anadarko Basin has been demonstrated and compared with the other seismic methods such as VVAz and AVAz. The calibration of the maximum stress orientation and seismic anisotropy with the available borehole breakout data, microseismic data, as well as shear-wave slowness data shows that the proposed workflow looks very promising as it correlates well with the direct measurement of maximum stress direction and intensity of anisotropy.

REFERENCES

Alt, R.C. and Zoback, M.D., 2015. A detailed Oklahoma stress map for induced seismicity mitigation, Search and Discovery article #70198 accessed at http://www.searchanddiscovery.com/pdfz/documents/2015/70198alt/ndx_alt.pdf.html on 6th June 2019.

Barry, N., Whittaker, N.R. and Singh, S.G., 1992. Rock Fracture Mechanics Principles Design and Applications. Elsevier Science Publishers, Amsterdam-New York.

Brown, E., Thomas, R. and Milne, A., 1990. The challenge of completion and stimulating horizontal wells: Oilfield Rev., 2(3): 52-64.

Crosby, D.G., Yang, Z. and Rahman, S.S., 1998. Transversely fractured horizontal wells. SPE Ann. Techn. Conf., SPE, 50093.

Eaton, D.W., 2018. Passive Seismic Monitoring of Induced Seismicity. Cambridge University Press, Cambridge.

Gray, D., Anderson, P., Logel, J., Delbecq, F., Schmidt, D. and Schmid, R., 2012. Estimation of stress and geomechanical properties using 3D seismic data. First Break, 30: 59-68.

Gray, D., Schmidt, D. and Delbecq, F., 2010. Optimize shale gas field development using stresses and rock strength derived from 3D seismic data. SPE, 137315-MS.

Grechka, V. and Tsvankin, I., 1998. 3D description of normal-moveout in anisotropic, inhomogeneous media. Geophysics., 63: 1079-1092.

Grechka, V., Tsvankin, I. and Cohen, J. K., 1999. Generalized Dix equation and analytic treatment of normal-moveout velocity for anisotropic media, Geoph. Prosp., 47: 117-148.

Griffith, A.A., 1920. The phenomena of rupture and flow in solids, Philos. Trans.. Roy. Soc., A 221: 163-198

Griffith, A.A., 1924. The theory of rupture. In: Biezeno, C.G. and Burgers, J.M. (Eds), Proc. 1st Internat. Congr. Appl. Mech., Delft. Drukkerij J. Waltman Jr.: 54-63.

Hall, S.A., Kendall, J.M. and Barkved, O.I., 2002. Fractured reservoir characterization using P-wave AVOA analysis of 3D OBC data. The Leading Edge, 21: 777-781.

Hoeksema., R.N., 2013. Elements of hydraulic fracturing. Oilfield Rev., Summer, 25(2): 51-52.

Miller, C., Water, G. and Rylander, E., 2011. Evaluation of production log data from horizontal wells drilled in organic shales. SPE, 144326.

Miller, C., Rylander, E. and Calvej, J.L., 2010. Detailed rock evaluation and strategic reservoir stimulation planning for optimal production in horizontal gas shale wells. AAPG Internat. Conf., Calgary, AB, Canada.

Miller, C., Hamilton, D., Sturm, S., Waters, G., Taylor, T., Calvez, J.L. and Singh, M., 2013. Evaluating the impact of mineralogy, natural fractures and in situ stresses on hydraulically induced fracture system geometry in horizontal shale wells. SPE, 163878.

Miller, C., Water, G. and Rylander, E., 2011. Evaluation of production log data from horizontal wells drilled in organic shales. SPE, 144326.

Plahn, S.V., Nolte, K.G., Thompson, L.G. and Miska, S., 1995. A quantitative investigation of the fracture pump-in/flowback test. SPE Ann. Techn. Conf., SPE 30504.

Rocha-Rangel, E., 2011. fracture toughness determinations by means of indentation fracture, nanocomposites with unique properties and applications in medicine and industry. Cuppoletti, J. (Ed.), ISBN: 978-953-307-351-4, InTech, Available from: https://www.intechopen.com/books/nanocomposites-with-unique-properties-and-applications-in-medicine-and-industry/fracture-toughness-determinations-by-means-of-indentation-fracture.

Rüger, A. and Tsvankin, I., 1997. Using AVO for fracture detection: Analytic basis and practical solutions. The Leading Edge, 10: 1429-1434.

Sack, R.A., 1946. Extension of Griffith's theory of rupture to three dimensions. Proc., Phys. Soc. London, 58: 729.

Schoenberg, M., and Sayers, C.M., 1995. Seismic anisotropy of fractured rock. Geophysics, 60: 204-211.

Sierra R., Tran, M. and Abousleiman, Y., 2010. Woodford Shale mechanical properties and the impacts of lithofacies. Proc. Symp. on the 44th US Rock Mechanics Symposium and 5th US-Canada Rock Mechanics Symposium, Salt Lake City, Utah, 27-30 June, ARMA-10-461,10p.

Sharma, R.K., Chopra, S. and Lines, L.R., 2019. Replacing conventional brittleness indices determination with new attributes employing true hydrofracturing mechanism. Expanded Abstr., 89th Ann. Internat. SEG Mtg., San Antionio, 3235-3239.

Sneddon, I.N., 1946. The distribution of stress in the neighborhood of a crack in an elastic solid. Proc. Roy. Soc.: A, 187: 229.

Sneddon, I.N. and Elliott, H.A., 1946. The opening of a Griffith crack under internal pressure. Quart. Appl. Math., 4: 262.

Singleton, S., 2009. The effects of seismic data conditioning on prestack simultaneous impedance inversion, The Leading Edge, 28: 772-781.

Weng, X., 1993. Fracture initiation and propagation from deviated wellbores. SPE Ann. Techn. Conf. SPE, 26597.

Yu, G., Zhang, Y. Wang, X., Liang, X., Liu, B., Singleton, S. and Smith, M., 2017. Seismic data gather conditioning for prestack seismic data inversion. Expanded Abstr., 87th Ann. Internat. SEG Mtg., Houston: 36, 5074-5078.

Zhang, J., 2016. Comprehensive reservoir characterization of the Woodford shale in parts of Garfield and Kingfisher counties, Oklahoma. M.Sc. thesis, Univ. of Oklahoma.

Zheng, X., 2006. Seismic azimuthal anisotropy and fracture analysis from p-p reflection data. Ph.D. thesis, Univ. of Calgary, Calgary.