OPTIMIZATION OF STAGGERED GRID FINITE-DIFFERENCE COEFFICIENTS BASED ON CONJUGATE GRADIENT METHOD

KEJIE JIN[1,2], JIANPING HUANG[1,2], QIANG ZOU[3], ZIYING WANG[1,2], SIYOU TONG[4], BIN LIU[5] and ZIDUO HU[6]

[1] *School of Geosciences, China University of Petroleum (East China), Qingdao 266580, P.R. China. s19010010@s.upc.edu.cn*
[2] *Laboratory for Marine Mineral Resources, Pilot National Laboratory for Marine Science and Technology (Qingdao), Qingdao 266071, P.R. China. jphuang@upc.edu.cn*
[3] *Tarim Oilfield Branch, CNPC, Korla 841000, P.R. China.*
[4] *College of Marine Geosciences, Ocean University of China, Qingdao 266100, P.R. China.*
[5] *Research and Development Center Sinopec Geophysical Corp, Nanjing 210009, P.R. China.*
[6] *Northwest Branch, Research Institute of Petroleum Exploration & Development, PetroChina, Lanzhou 730020, P.R. China.*

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ABSTRACT

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The implementation of difference coefficients optimization strategy can effectively suppress numerical dispersion and improve the modeling accuracy. The conventional difference coefficients calculation method based on Taylor-series Expansion exists serious numerical dispersion. In this paper, we derive a new dispersion error function from the dispersion relation, and the optimal difference coefficients are obtained iteratively by using the conjugate gradient method, thus a staggered-grid difference coefficients optimization method based on the conjugate gradient is developed. We compare dispersion curves, snapshots and single shot records using low-velocity model, high-velocity model and Marmousi model, the results show that the new method can effectively reduce the numerical dispersion compared with the difference coefficients of the conventional Taylor-series Expansion method. The 8th-order optimized difference operators can achieve the modeling precision of 12th-order Taylor-series Expansion difference operators, which can effectively save calculation time and internal storage. The optimization method performs well for both simple model and complex model forward modeling.

KEY WORDS: finite-difference, staggered grid, numerical dispersion, difference coefficients, conjugate gradient.

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INTRODUCTION

The Finite-Difference (FD) method is widely used in seismic wave propagation numerical simulations for its easy implementation, fast computing speed and ease of extending to different complex media (Alterman and Karal, 1968; Igel, 1995; Virieux et al., 2011). The modeling accuracy of FD methods depends not only on the grid subdivision method and difference order, but also on the calculation of difference coefficients. Excessively fine grid subdivision and high-order difference will significantly increase the amount of calculation (Ren et al., 2012). Difference coefficients optimization can effectively suppress the numerical dispersion without increasing the amount of computing and improve the modeling accuracy. The conventional difference coefficients are obtained by Taylor-series Expansion, and the difference coefficients calculated in this way lead to high accuracy over a small wavenumber range. However, when the seismic wave frequency band is wider, strong numerical dispersion will appear, which is difficult to meet the requirements of numerical modeling accuracy (Liu, 2013; Yong et al., 2017). For this reason, the optimization method of difference coefficients has always been the focus of research.

Some scholars have proposed various optimization algorithms to get optimal FD coefficients. Etgen (2007) suggested using the phase velocity related to dispersion error to optimize the difference coefficients. Liu and Sen (2011) combined the time-space domain dispersion-relation-based FD scheme and the truncated FD scheme to obtain optimized spatial FD coefficients. Zhang and Yao (2013) proposed to calculate the optimal difference coefficients by simulated annealing method. Based on the least square theory, Liu (2013) obtained the FD coefficients of the second-order spatial derivative of the global optimization. Yang et al. (2014) used the dispersion relation and the least square method to derive the staggered-grid finite-difference (SFD) coefficients of arbitrary even-order precision of the first-order spatial derivative. Ren and Liu (2015) used the least squares algorithm to derive the optimal SFD coefficients by minimizing the relative error of time-space-domain dispersion relations over a given frequency range, and carried out numerical simulation of acoustic wave and elastic wave equations. Yang et al. (2017) obtained the optimized staggered-grid difference coefficients by using the minimax approximation method with a Remez algorithm. In particular, Yong et al. (2017) on the basis of the Equivalent Staggered Grid, derived plane wave solutions of displacement components in wavenumber domain from 2D elastic wave equations and established three time–space domain dispersion relations. Then applied Newton method to obtain optimal coefficients by minimizing the relative error between time dispersion and spatial dispersion.

Another method to optimize the difference coefficients is to introduce an appropriate window function. Fornber (1987) proved that the pseudo-spectral method is equivalent to the higher-order approximation of the FD method, and the difference coefficients can be obtained by truncating the spatial convolution sequence of the pseudo-spectral method through the time-domain window function. Zhou and Greenhalgh (1992) obtained the corresponding difference operators by using generalized weighted Hanning window truncation. Igel et al. (1995) deduced new FD operators using Gaussian window truncation. Chu and Stoffa (2012) calculated the optimal difference coefficients by using the scaled binomial window. Ren et al. (2018) converted the calculation of difference coefficients based on window function into the least squares problem and proposed the difference coefficients of window function optimized by least squares.

Due to the complexity of the media and the inherent problems of the finite-difference method, the above methods have some problems in the range of application and modeling accuracy and cannot completely eliminate the numerical dispersion. Therefore, it is still a hot topic to find an effective optimization method of difference coefficients to reduce numerical dispersion. Compared with the regular-grid FD method, under the same discrete condition, the SFD method has higher accuracy and better stability (Igel et al., 1995; Ren and Liu, 2015; Huang et al., 2015). For the numerical dispersion problem inherent in the finite-difference method, staggered-grid methods have greater precision and stability than conventional standard-grid FD methods (Yong et al., 2017). In this paper, based on the staggered-grid discrete scheme, we first construct an error function from the dispersion relation. Then we introduce the conjugate gradient method to obtain the optimized difference coefficients, while the initial value is selected from the coefficients of Taylor-series Expansion method. Finally, by means of dispersion analysis and numerical simulation, the effectiveness and certain advantages of conjugate gradient optimization method in reducing numerical dispersion are verified.

ACQUISITION OF DIFFERENCE COEFFICIENTS

Staggered-grid discrete format

Staggered-grid was initially proposed by Yee (1966), and Madariaga (1976) proposed an FD method for staggered-grid of first-order velocity-stress elastic wave equation. Virieux (1984, 1986) also used this differential grid scheme when simulating SH and P-SV waves in isotropic media. Tong et al. (2019) proposed new elastic equations transformed from traditional elastic wave equations for converted S-wave simulation combined with an SFD. For regular-grid, the value of the whole grid point is used to approximate the partial derivative. While staggered-grid modeling uses the value of the half grid point as shown in Fig. 1. This difference scheme can improve the local accuracy of numerical modeling and accelerate the convergence speed. For solving the first-order velocity-stress acoustic wave equation, the precision of staggered-grid is higher than that of regular-grid.

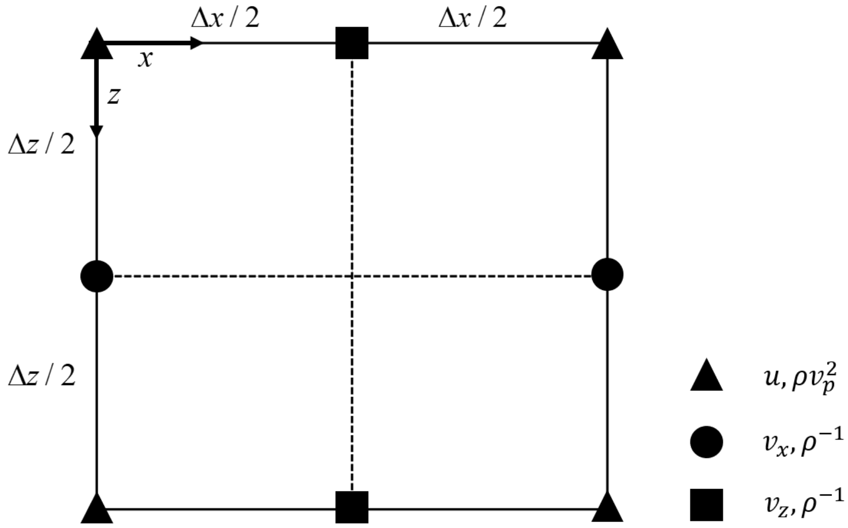


Fig. 1. Schematic diagram of staggered-grid of two-dimensional isotropic media acoustic wave equation.

The first-order velocity-stress equations describing 2D acoustic wave equation in heterogeneous isotropic media can be expressed as:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \rho {v_p}^2 (\dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_z}{\partial z}) \\ \dfrac{\partial v_x}{\partial t} = \dfrac{1}{\rho}\dfrac{\partial u}{\partial x} \\ \dfrac{\partial v_z}{\partial t} = \dfrac{1}{\rho}\dfrac{\partial u}{\partial z} \end{cases}, \tag{1}$$

where ρ is the density, u is the normal stress, the velocities of particles are expressed in terms of v_x and v_z, and the P-wave velocity is v_p.

The *2L*-order FD of first-order spatial derivative under the staggered grid is defined as eq. (2). What should be noted is that we only developed the equations for the *x*-direction because of the *z*-direction is essentially identical.

$$\frac{\partial u(x)}{\partial x} \approx \frac{1}{\Delta x}\sum_{m=1}^{L} a_m \left\{ u\left[x + \frac{(2m-1)}{2}\Delta x\right] - u\left[x - \frac{(2m-1)}{2}\Delta x\right]\right\}, \tag{2}$$

where Δx represents the grid step, L is half of the spatial order, $a_m\,(m=1,2,3...L)$ are the SFD coefficients. And i, j are coordinates of grid nodes in x- and z-directions respectively. Similarly, k is the time coordinate. Because the staggered grid uses the values of the half-grid points, the coordinates of the half-grid points are represented by $i+\frac{1}{2}, j+\frac{1}{2}$, and $k+\frac{1}{2}$ respectively. For the convenience of writing, in the following derivation, we express u, v_x and v_z in the following form: take u for example, if we express $u(x,z,t)$ as $U_{i,j}^{k}$, then $u(x+\frac{\Delta x}{2}, z+\frac{\Delta z}{2}, t+\frac{\Delta t}{2})$ can be expressed as

$$U_{i+\frac{1}{2},j+\frac{1}{2}}^{k+\frac{1}{2}}.$$

Supposing the discrete value of stress u is $U_{i,j}^{k+1/2}$, the discrete value of velocity component v_x is $P_{i+1/2,j}^{k}$, and the discrete value of velocity component v_z is $Q_{i,j+1/2}^{k}$. According to eq. (2), the staggered-grid discrete format of eq. (1):

$$\begin{aligned}
U_{i,j}^{k+1/2} &= U_{i,j}^{k-1/2} + \frac{\Delta t \rho v_p^{\,2}}{\Delta x}\left\{\sum_{m=1}^{L} a_m \left[P_{i+(2m-1)/2,j}^{k} - P_{i-(2m-1)/2,j}^{k}\right]\right\} \\
&+ \frac{\Delta t \rho v_p^{\,2}}{\Delta z}\left\{\sum_{m=1}^{L} a_m \left[Q_{i,j+(2m-1)/2}^{k} - Q_{i,j-(2m-1)/2}^{k}\right]\right\} \\
P_{i,j}^{k} &= P_{i,j}^{k-1} + \frac{\Delta t}{\Delta x \rho}\left\{\sum_{m=1}^{L} a_m \left[U_{i+(2m-1)/2,j}^{k-1/2} - U_{i-(2m-1)/2,j}^{k-1/2}\right]\right\} \qquad , \qquad (3) \\
Q_{i,j}^{k+1/2} &= Q_{i,j}^{k-1/2} + \frac{\Delta t}{\Delta z \rho}\left\{\sum_{n=1}^{N} a_m \left[U_{i,j+(2n-1)/2}^{k} - U_{i,j-(2n-1)/2}^{k}\right]\right\}
\end{aligned}$$

where $U_{i,j}^{k+1/2}$ is $u(x,z,t+\frac{\Delta t}{2})$, $P_{i+1/2,j}^{k}$ is $v_x(x+\frac{\Delta x}{2},z,t)$, $Q_{i,j+1/2}^{k}$ is $v_z(x,z+\frac{\Delta z}{2},t)$, $\Delta x, \Delta z$ are the sampling interval in x-direction and z-direction respectively, Δt is the time sampling interval. For eq. (3), the numerical solution of eq. (1) can be obtained only by asking for the corresponding difference coefficients a_m, so as to realize the numerical simulation of seismic waves in acoustic media.

Therefore, the key to realize seismic wave numerical simulation by using SFD method is to obtain the corresponding difference coefficients. At the same time, the difference coefficients also affect the accuracy of seismic wave numerical simulation. The method of calculating the SFD coefficients is described below.

Coefficients obtained by Taylor-series Expansion method

The traditional difference coefficients are obtained by Taylor-series Expansion method. Suppose that the stress field $u(x)$, and at least *2L+1* of its derivatives are continuous in the interval including $x = x_0$. The finite *2L+1* order Taylor-series Expansion of $u(x)$ near at $x = x_0$ with $\Delta x = x - x_0$ at $x = x_0 \pm \frac{(2m-1)}{2}\Delta x$ is:

$$u\left[x_0 \pm \frac{(2m-1)}{2}\Delta x\right] = u(x_0) + \sum_{i=1}^{2L+1} \frac{(\pm\frac{2m-1}{2})^i (\Delta x)^i}{i!} u^{(i)}(x_0) + o(\Delta x^{2L+2}) \quad , \tag{4}$$

Defining $\left.\frac{\partial^m u(x)}{\partial x^m}\right|_{x=x_0} = u^{(m)}(x_0)$, then substitute the equations of eq. (4) into eq. (2), simplify, and get:

$$\Delta x u^{(1)}(x_0) \approx \sum_{m=1}^{L} (2m-1)\Delta x a_m u^{(1)}(x_0) + \sum_{m=1}^{L}\sum_{i=1}^{L-1} \frac{(2m-1)^{2i+1}\Delta x^{2i+1}}{(2i+1)!} a_m u^{(2i+1)}(x_0) \quad , \tag{5}$$

Expanding the first term on the right side of eq. (5) to get:

$$\sum_{m=1}^{L} (2m-1)\Delta x a_m u^{(1)}(x_0) = \left[a_1 + 3a_2 + 5a_3 + 7a_4 + \ldots + (2L-1)a_L\right]\Delta x u^{(1)}(x_0) \quad . \tag{6}$$

Similarly, expanding the second term on the right side of eq. (5):

$$\sum_{m=1}^{L}\sum_{i=1}^{L-1}\frac{(2m-1)^{2i+1}\Delta x^{2i+1}}{(2i+1)!}a_m u^{(2i+1)}(x_0)$$
$$=\sum_{m=1}^{L}\left\{\frac{(2m-1)^3\Delta x^3}{3!}a_m u^3(x_0)+\frac{(2m-1)^5\Delta x^5}{5!}a_m u^5(x_0)+\ldots+\frac{(2m-1)^{(2L-1)}\Delta x^{(2L-1)}}{(2L-1)!}a_m u^{(2L-1)}(x_0)\right\}$$
(7)

From the coefficients correspondence of $u^m(x_0)$ on the left and right sides of eq. (5) being equal, the following system of equations can be obtained:

$$\begin{pmatrix} 1 & 3^1 & \ldots & (2L-1)^1 \\ 1 & 3^3 & \ldots & (2L-1)^3 \\ \ldots & \ldots & \ldots & \ldots \\ 1 & 3^{2L-1} & \ldots & (2L-1)^{2L-1} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \ldots \\ a_L \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ \ldots \\ 0 \end{pmatrix} \quad . \tag{8}$$

The Taylor-series Expansion staggered-grid finite-difference (TESFD) coefficients a_m can be estimated by solving eq. (8). The Taylor-series Expansion omits the higher order terms, which causes numerical dispersion. The precision will decrease rapidly with the increase of wavenumber. In this paper, conjugate gradient method is used to iterate the difference operators based on Taylor-series Expansion method, and the optimal solution can be obtained by convergence within the given error range.

Coefficients optimization with conjugate gradient method

The Fourier transform of eq. (2) is applied to obtain the expression of the first-order spatial derivative of staggered-grid in the Fourier domain:

$$\frac{K\Delta x}{2}\approx\sum_{m=1}^{L}a_m\sin[K\Delta x(2m-1)/2] \quad , \tag{9}$$

where K is the horizontal wavenumber, and a_m（$m=1,2,3...L$）represent the coefficients of the staggered grid.

The error function is:

$$E(\beta)=\sum_{m=1}^{L}a_m\sin[(2m-1)\beta]/\beta-1 \quad , \tag{10}$$

where $\beta=K\Delta x/2$, taking $\beta\in[0,\pi/2]$, selecting a certain wavenumber sampling interval, β can be discretized into I equal parts, $\beta_i, i=1,2,...,I$. Therefore, eq. (10) is actually a system of linear equations composed of I equations, which can be written as:

$$E = Ax - b \quad , \tag{11}$$

A is the $L \times I$ real matrix related to β, x is the FD coefficient vectors: $x = (a_1, a_2 ... a_m)^T, b = (1,1...1)^T$. E is the error vector, and the conjugate gradient method can be used to find an optimal value of x by iterative convergence, so as to minimize the relative dispersion error E**.**

The iterative form of conjugate gradient method mathematically can be written as:

$$x_{k+1} = x_k + \alpha_k p_x \quad , \tag{12}$$

where x_k represents the k-th iteration value, α_k is the k-th search step size:

$$\alpha_k = \frac{p_k{}^T q_k}{p_k{}^T A p_k} \quad , \tag{13}$$

q_k is the residual vector of the k-th iteration, $q_k = b - Ax_k$, p_k is the search direction, and its calculation formula is as follows:

$$p_k = q_k + \frac{q_k{}^T q_k}{q_{k-1}{}^T q_{k-1}} p_{k-1} \quad . \tag{14}$$

In order to ensure the correctness of convergence direction, the staggered-grid difference operators based on Taylor-series Expansion method are selected as the initial value x_0, and the convergence speed is also accelerated. In the first iteration of conjugate gradient method, taking $q_0 = b - Ax_0$ as residual vector. And q_0 is taken as the initial search direction, namely $p_0 = q_0$, thereout the step size could be calculated and the result of the first iteration could be obtained. When a certain error range or iteration number is limited, the optimal conjugate gradient staggered-grid finite-difference (CGSFD) coefficients can be obtained.

Tables 1 and 2 list the SFD coefficients calculated by the Taylor-series Expansions method and the conjugate gradient method, respectively.

Table 1. First derivative difference coefficients of TESFD method.

Space order (*2L*)	a_1	a_2	a_3	a_4	a_5	a_6
L=2	1.1250000	-0.0416667				
L=3	1.1718750	-0.0651042	0.004688			
L=4	1.1962891	-0.0797526	0.0095703	-0.0006975		
L=5	1.2112427	-0.0897217	0.0138428	-0.0017657	0.0001187	
L=6	1.2213364	-0.0969315	0.0174477	-0.0029673	0.0003590	-0.0000218

Table 2. First derivative difference coefficients of CGSFD method.

Space order (*2L*)	a_1	a_2	a_3	a_4	a_5	a_6
L=2	1.1407061	-0.0472992				
L=3	1.1948784	-0.0777345	0.0077875			
L=4	1.2206376	-0.0957788	0.0160881	-0.0020056		
L=5	1.2355203	-0.1074073	0.0229949	-0.0048984	0.0006875	
L=6	1.2444627	-0.1148663	0.0281136	-0.0076920	0.0018120	-0.000272

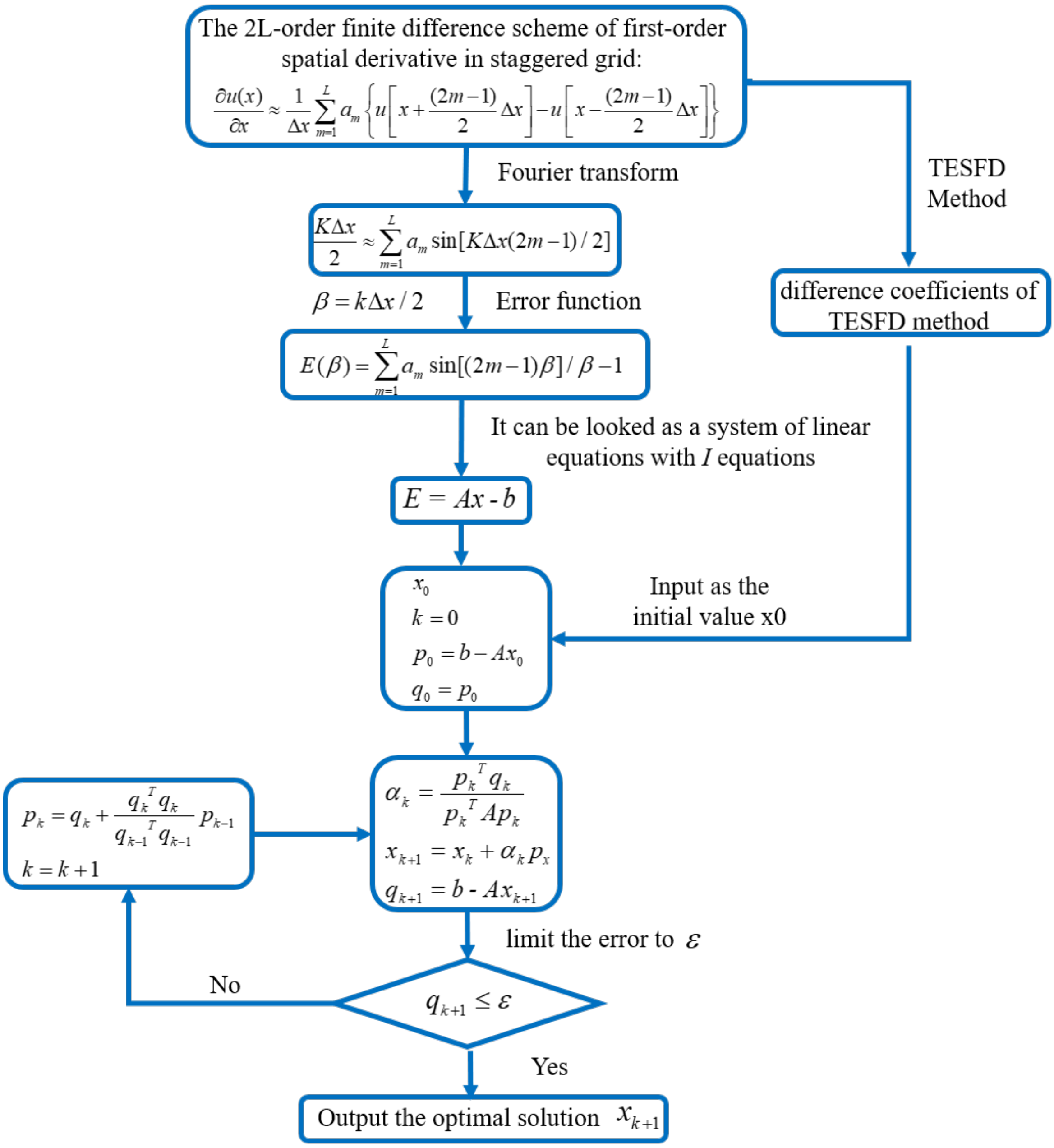


Fig. 2. Flow chart of optimization of difference coefficients by conjugate gradient method.

DISPERSION ANALYSIS

Numerical dispersion is an inherent shortcoming of the FD method. The error term produced when the differential equation is approximated by the difference equation makes the phase velocity change. As a result, the phase velocity is not equal to the group velocity, which results in numerical dispersion (Fei and Larner, 1995). The numerical dispersion is related to the precision of the FD method. In this section, the dispersion analysis of our method is implemented.

The relative error function of first-order staggered-grid can be expressed as:

$$E(K) = \sum_{m=1}^{L} a_m \sin[(m - \frac{1}{2})K\Delta x] / \frac{1}{2} K\Delta x \quad , \tag{15}$$

where a_m represents the FD coefficients, K is the horizontal wavenumber, Δx is the grid step, and L is half of the spatial operator length

Taking advantage of eq. (15) and taking $K\Delta x/2$ as the independent variable, taking $K \in [0, \pi/\Delta x]$ can obtain the dispersion curve of TESFD method and CGSFD method, as shown in Fig. 3.

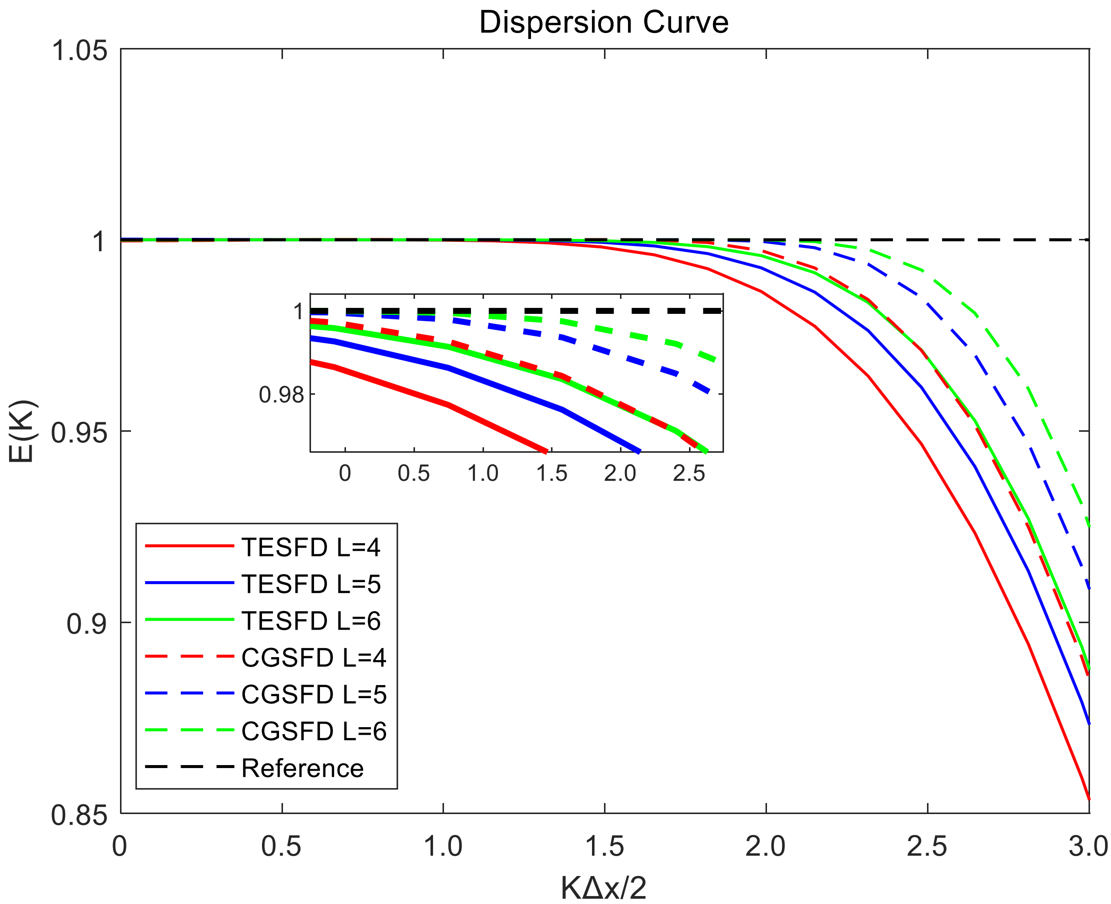


Fig. 3. Dispersion error curve.

From Fig. 3, we can observe that: (1) With the increase of the wavenumber, dispersion curve gradually departure from 1, indicating that the numerical dispersion of the FD method generally increases with the increase of the wavenumber *K*. (2) The dispersion error of the two methods both decreases with the increase of the space order, which indicates that the increase of the space order can suppress the dispersion error. However, the increase of the space order will reduce the calculation efficiency. (3) Compared with TESFD method of the same space order (solid line), CGSFD method (dashed line) can effectively reduce the dispersion error, so as to suppress the numerical dispersion. (4) The dispersion curve of 8th-order spatial difference CGSFD method (red dashed line) and the dispersion curve of 12th-order spatial difference TESFD method (green solid line) almost coincide, suggesting that the 8th-order CGSFD method can achieve the precision of the 12th-order TESFD method. Reducing the spatial order means reducing the calculation amount. The calculation time of 350 ms Marmousi model finite-difference forward is 50.9 s and 70.5 s respectively, by using the 8th-order CGSFD method and the 12th-order TESFD method. The calculation time of the latter is 1.38 times of the former, suggesting that the proposed method requires less calculation time and higher calculation efficiency than the traditional Taylor-series Expansion method when the same precision is achieved.

NUMERICAL MODELING ANALYSIS

In order to verify the effectiveness of the difference coefficients obtained by the CGSFD method in reducing dispersion, the difference coefficients obtained by the TESFD method and the CGSFD method are used to simulate the wave field of the homogeneous isotropic model and the Marmousi model.

Homogeneous medium

To verify the correctness of the above dispersion analysis, the homogeneous medium is tested first, and the homogeneous medium with low velocity and high velocity is tested respectively.

Low-velocity medium

We first design a low velocity homogeneous model with the size of 201×201, time sampling interval is set $\Delta t = 1ms$, and the spacial sampling interval is set $\Delta x = \Delta z = 10m$, P-wave velocity of 2000 m/s and space-order of 10. The model equation is a first-order velocity-stress acoustic wave equation, as shown in eq. (1). The source signal is represented by a Ricker wavelet with a center frequency of 30 Hz and is located at the center of the model. The difference coefficients are calculated by TESFD method and CGSFD method respectively, and the simulation results are shown in Fig. 4.

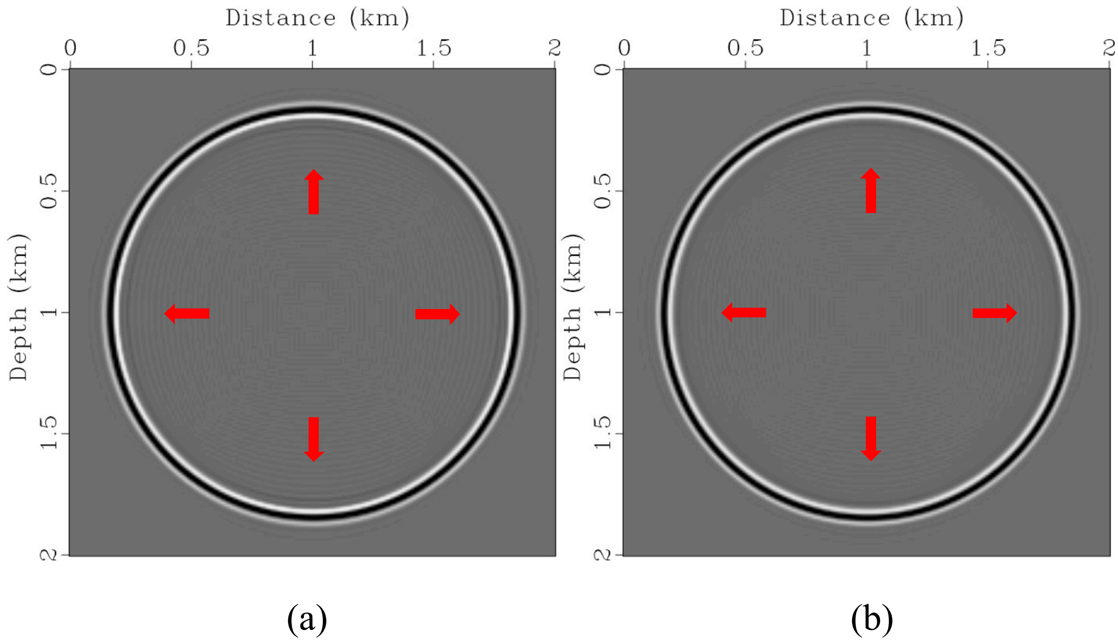
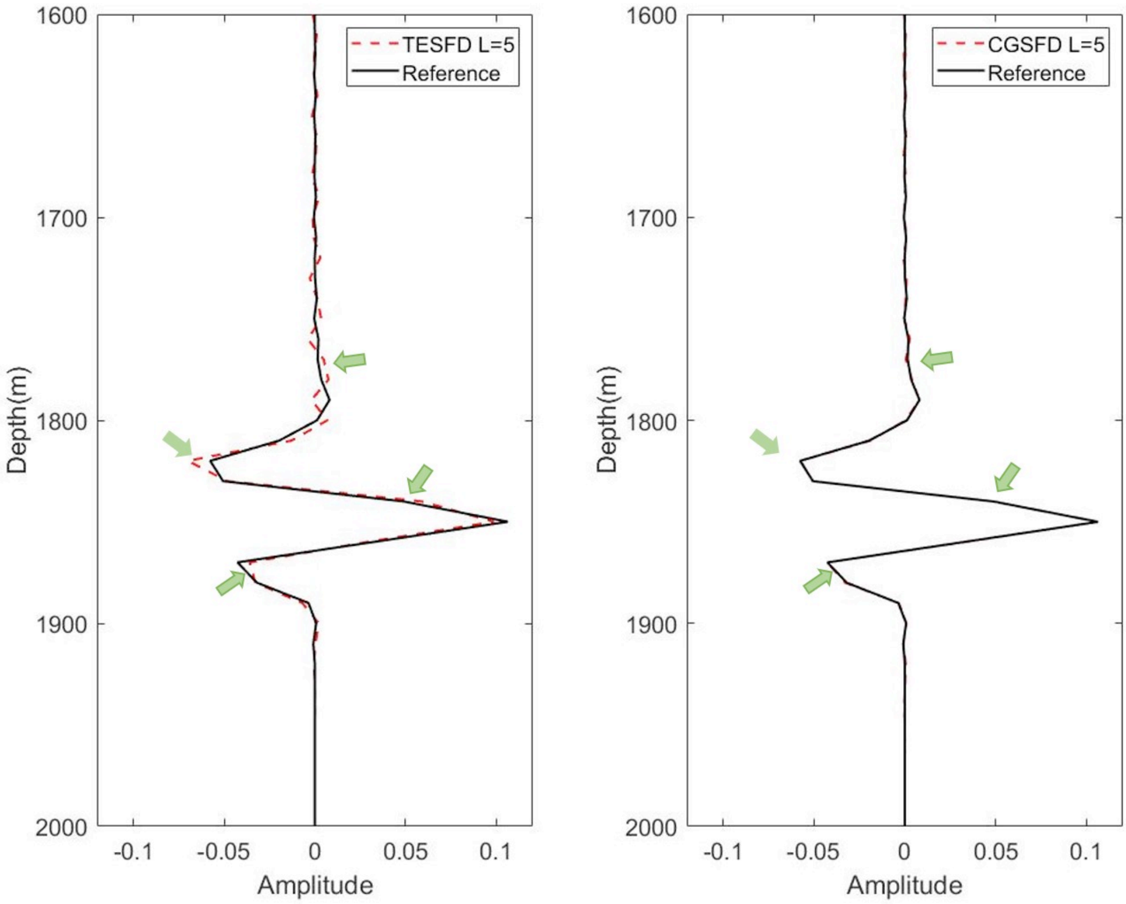


Fig. 4. Snapshots of the acoustic field at 450 ms of (a) TESFD method and (b) CGSFD method.

Figs. 4(a) and 4(b) are the snapshots computed by the TESFD and CGSFD methods, respectively, at the moment of 450 ms. The four boundaries of the wave field snapshot shown in Fig. 4(a) all have obvious numerical dispersion (indicated by the red arrow), while the numerical dispersion at the corresponding positions in Fig. 4(b) is almost invisible. For further verification, we extract the slices from the snapshots in Fig. 4 for comparative analysis. The reference solution is obtained by using the traditional high-order FD scheme L = 10, and the result is shown in Fig. 5.

As can be seen from Fig. 5, the coincidence degree between 10th-order CGSFD method and reference trace is higher than that of the 10th traditional TESFD method. Indicating that the CGSFD method can effectively suppress the numerical dispersion compared with the traditional method, which is consistent with the results of dispersion analysis and wave field snapshot above. It shows that the optimization method in this paper can effectively reduce the spatial dispersion in low velocity homogeneous medium.



(a) (b)

Fig. 5. Slice of the snapshots at x = 950 m, z = $1600 \sim 2000$ m of (a) TESFD method and (b) CGSFD method.

High-velocity medium

Then a high-speed homogeneous model is tested to verify the effectiveness of the CGSFD method. The high-speed homogeneous model with a size of 201×201, P-wave velocity is 4000 m/s and space order of 10. The time sampling interval is set $\Delta t = 1ms$, and the spatial sampling interval is set $\Delta x = \Delta z = 15m$. The source signal is represented by a Ricker wavelet with a center frequency of 40 Hz and is located at the center of the model. The difference coefficients are calculated by TESFD method and CGSFD method respectively, and the simulation results are shown in Fig. 6.

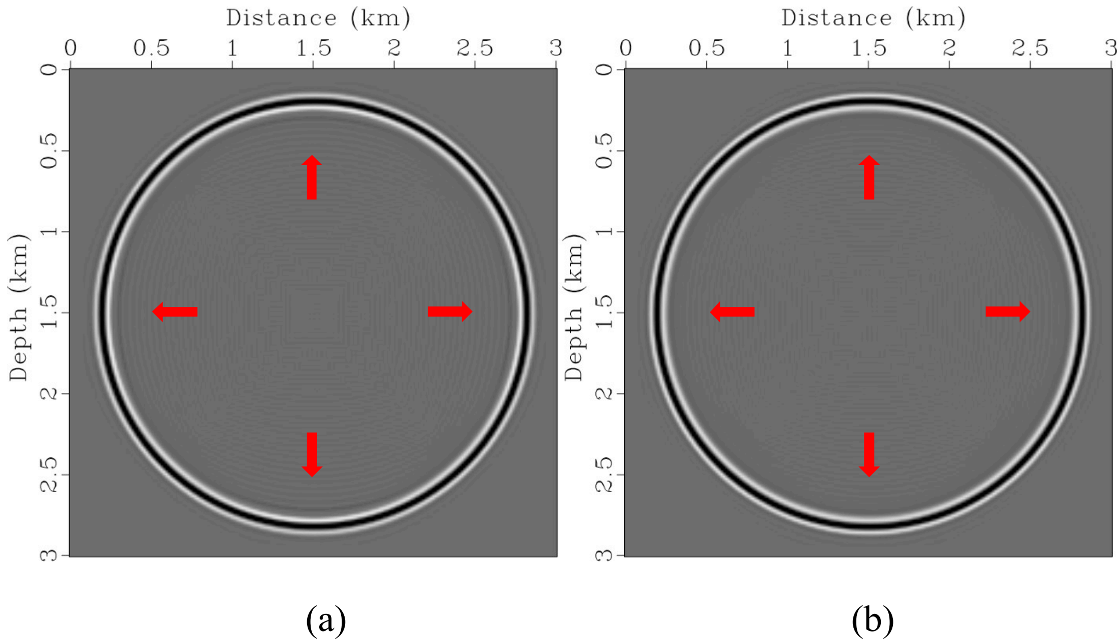


Fig. 6. Snapshots of the acoustic field at 350 ms of (a) TESFD method and (b) CGSFD method.

Figs. 6(a) and 6(b) are the snapshots computed by the TESFD and CGSFD methods, respectively, at the moment of 350 ms. From Fig. 6, we can clearly see that compared with the snapshot of the 10th-order TESFD method, the snapshot of the 10th-order CGSFD method effectively reduces the numerical dispersion in four boundaries (indicated by the red arrow). For further verification, we extract the slices from the snapshots in Fig. 6 for comparative analysis. The reference solution is obtained by using the traditional high-order FD scheme $L = 10$, and the result is shown in Fig. 7.

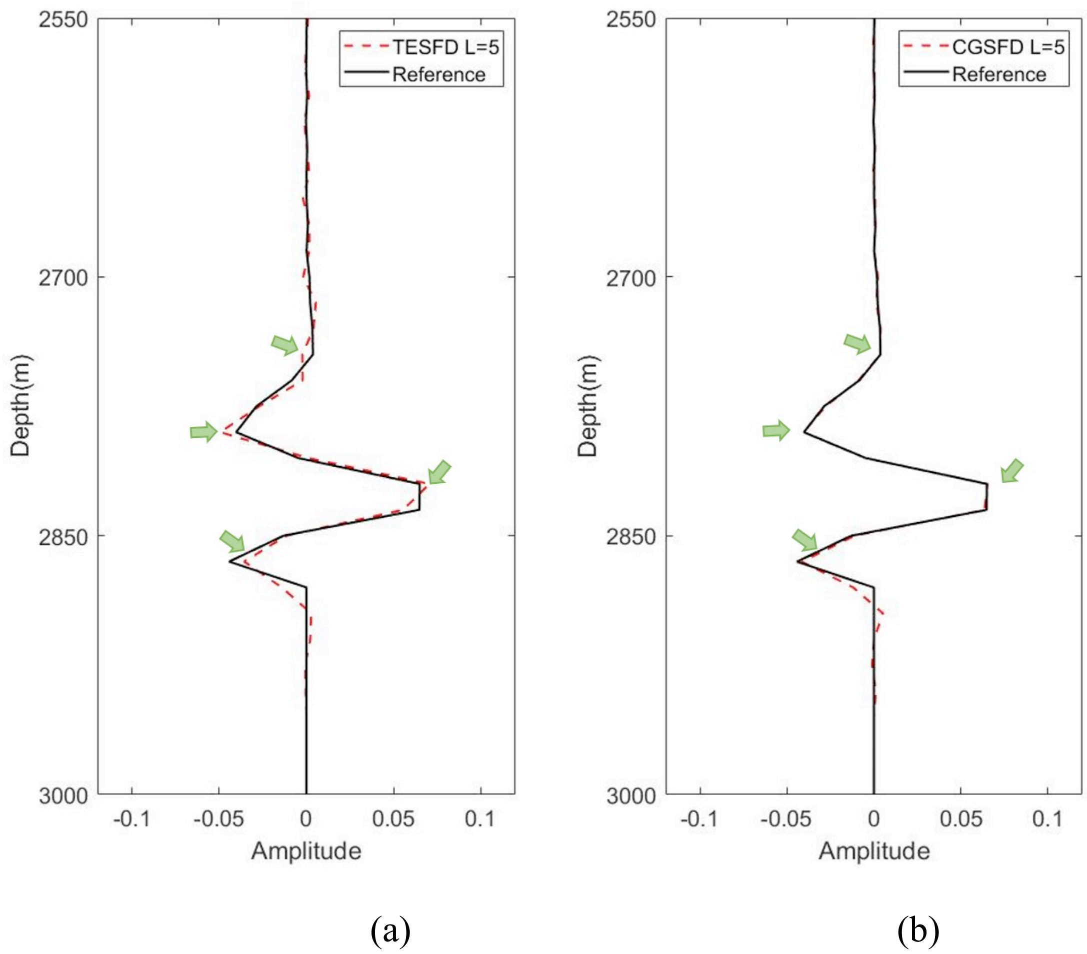


Fig. 7. Slice of the snapshots at x = $1425\,\mathrm{m}$, z = $2550 \sim 3000\,\mathrm{m}$ of (a) TESFD method and (b) CGSFD method.

As can be seen from Fig.7, in high-speed homogeneous medium, the degree of deviation of TESFD method from the reference trace is significantly more serious than that of CGSFD method. In other words, the CGSFD method can better match the reference channel compared with the traditional TESFD method. The result is consistent with the above dispersion analysis and snapshots, showing that the CGSFD method can reduce the spatial dispersion in high-velocity homogeneous medium.

Complex medium

In order to verify the effectiveness of the CGSFD method in the complex model, we perform numerical modeling on the Marmousi model with a relatively complex velocity field (Fig. 8).

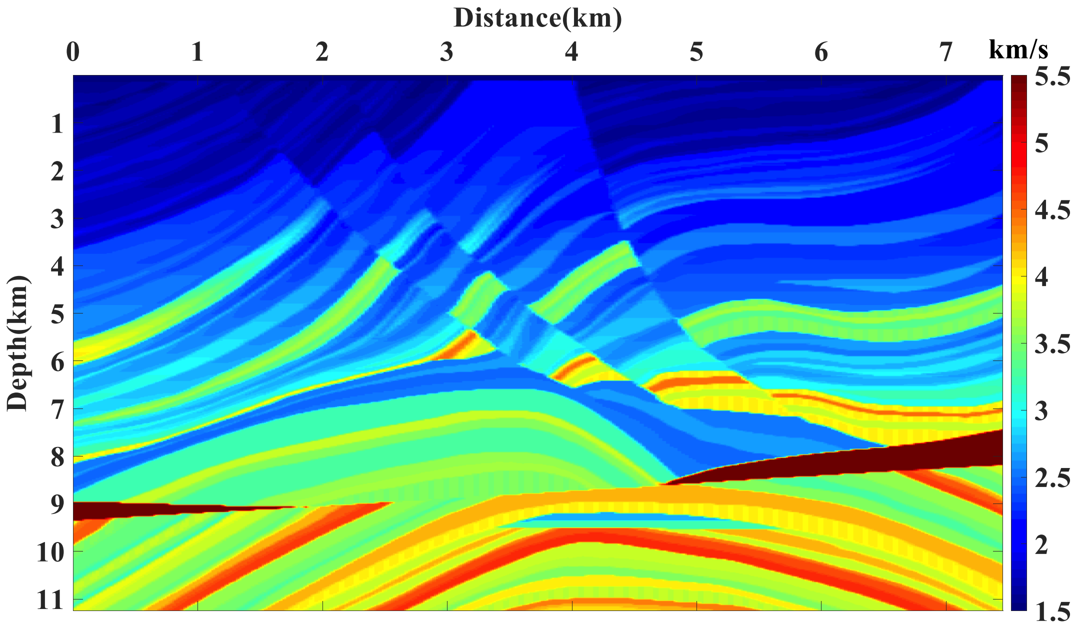
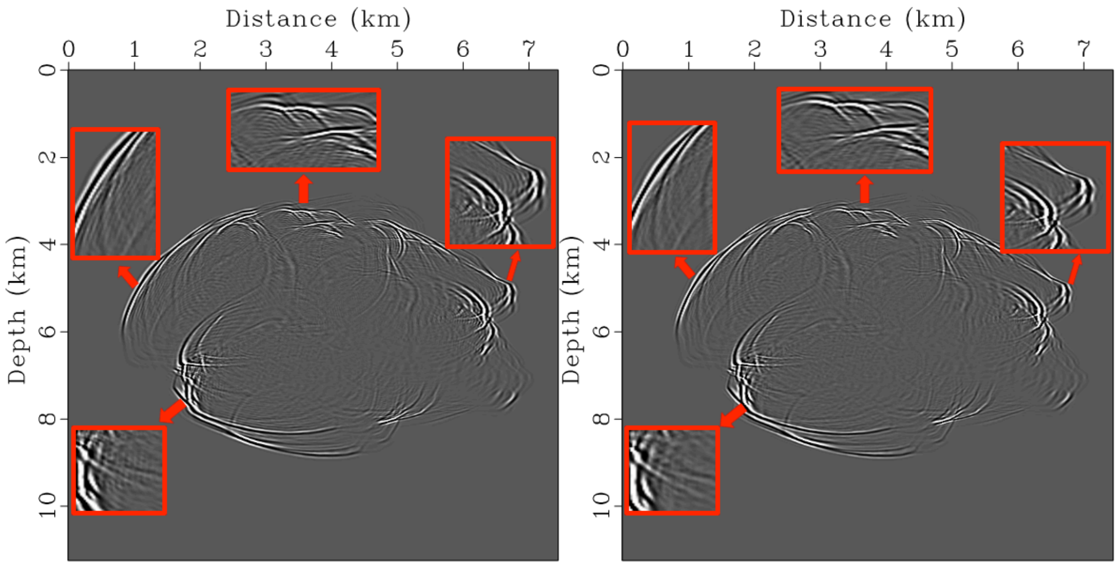


Fig. 8. Marmousi model.

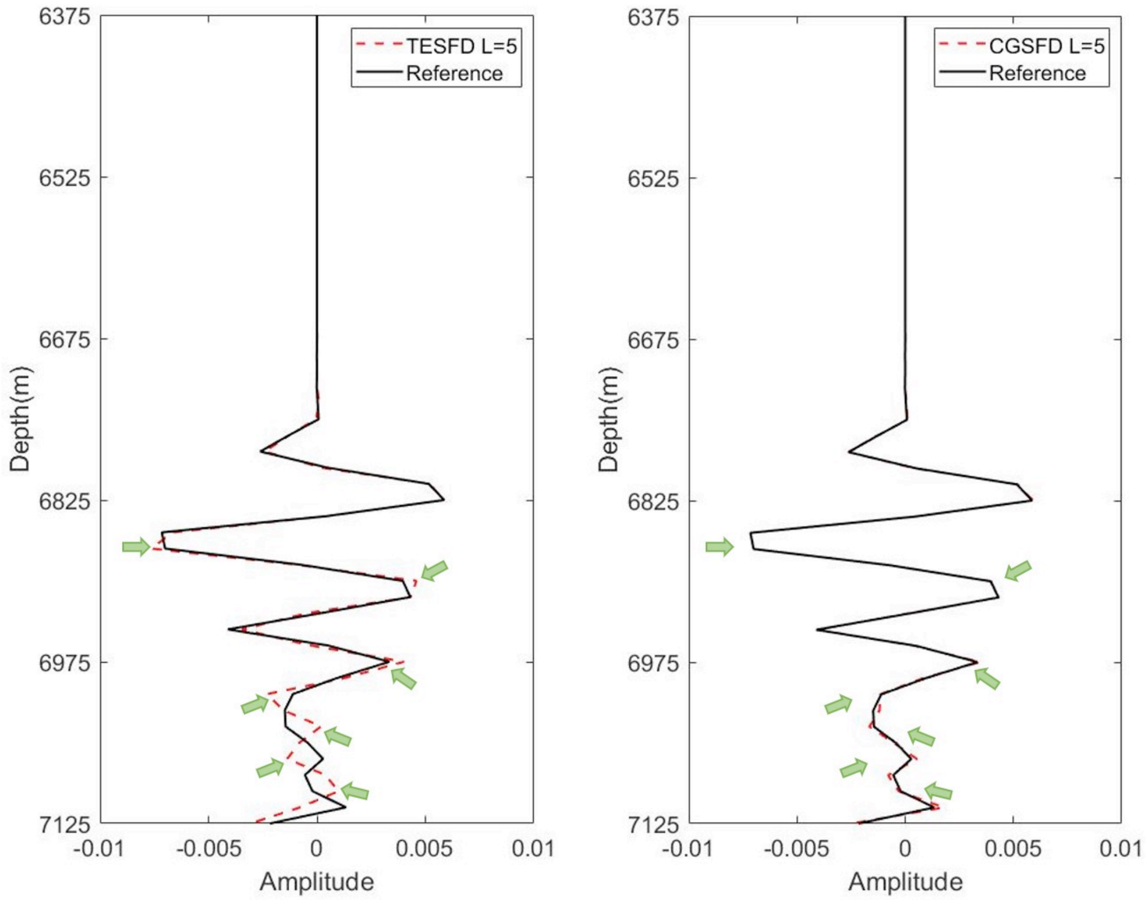
The Marmousi model is a complex model with the size of 497×750, and the simulation parameters are as follows: time sampling interval is $\Delta t = 1ms$, spatial sampling interval is $\Delta x = \Delta z = 15m$ and the P-wave velocity is illustrated in Fig. 8. A Ricker wavelet with a center frequency of 30Hz is used as the source signal, and the simulation results of 10th-order TESFD method and 10th-order CGSFD method are shown in Figs. 9(a) and 9(b), respectively.



(a) (b)

Fig. 9. Snapshots of the acoustic field at 350 ms of (a) TESFD method and (b) CGSFD method.

Due to the complexity of Marmousi model, it is difficult for us to see obvious differences directly between the two snapshots, so we made local amplification for some areas with obvious dispersion improvement (as shown in the red box in Fig. 9). As can be seen from the red box, the CGSFD method is still effective in reducing the numerical dispersion in some areas with significant dispersion. In order to further compare the effects of the two methods, slices in Fig. 9 are extracted for comparative analysis, and the results are shown in Fig. 10.



(a) (b)

Fig. 10. Slice of the snapshots at $x = 7500\,\mathrm{m}$, $z = 6375 \sim 7125$ m of (a) TESFD method and (b) CGSFD method.

In the areas indicated by the green arrow of Fig.10, there are significant differences between the TESFD method and the reference trace, while the CGSFD method is almost identical to the reference trace. It shows the CGSFD method can also effectively suppress numerical dispersion in complex media. Therefore, under certain accuracy requirements, the space order required by the CGSFD method is lower than that of the TESFD method for difference forward modeling. Reducing the space order will reduce computation amount and improve efficiency.

CONCLUSION

In order to ameliorate the numerical dispersion problem inherent in finite-difference method, we first establish an error function from the dispersion relation based on the staggered-grid scheme. Then we take the Taylor-series Expansion FD coefficients as the initial value. Finally, the conjugate gradient method is applied to iteratively calculates the optimized difference coefficients. Through the results of dispersion analysis and numerical simulation, the following understandings are obtained:

(1) In the case of the same space order, the difference operators obtained by the CGSFD method have higher accuracy than the difference operators obtained by the traditional TESFD method. As a result, the CGSFD method has better spatial dispersion suppression effect in forward simulation than the TESFD method. The optimized coefficients are easy to use because the CGSFD method does not need to change the structure of the finite-difference algorithm. This means that the CGSFD method can improve the calculation accuracy almost without increasing the calculation amount.

(2) Under the specific computing accuracy requirements, the low-order CGSFD method can be used to achieve the accuracy of the higher-order traditional TESFD method. Reducing the space order will reduce computation amount. It is shown that the CGSFD method requires less computation than TESFD method under the same accuracy.

(3) When iterative optimization is carried out by mathematical method, the choice of initial value will affect the correctness and efficiency of iteration. Our method selects the difference operators of Taylor-series Expansion method as the initial value in conjugate gradient iteration, which not only ensures the correctness of iteration direction, but also speeds up the speed of iterative convergence.

(4) The CGSFD method constructs the dispersion error function from the expression of the first-order spatial derivative of staggered-grid, which can effectively reduce the spatial dispersion. While the optimization of time domain difference is not considered, so there is no suppression effect on time dispersion.

In addition, the method presented in this paper can also be applied effectively in the forward simulation of complex models and can be easily extended to the study of reverse time migration imaging and full waveform inversion.

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